

PROBLEMS IN PHYSICS

Book has been primarily designed to prepare students for competitive examinations such as IIT-JEE, PMT, AMIE, etc. A variety of problems involving various concepts and an interplay of different formulae and models, are included to enable students gain a thorough understanding of the subject. All topics normally covered at the Class XII level of equivalent curriculum are included so that the book can benefit these students equally.

Chapter on units, dimensions, errors and significant figures is included.

Each important topic is covered in 26 separate chapters and explained in a step-by-step manner that would enable students to thoroughly prepare for the competitive examination.

Each chapter begins with basic definitions of physical quantities involved, their units, and the relevant formulae to solve the problems.

Each chapter is followed by carefully chosen illustrations to explain physical principles involved along with simplified numerical calculations in most cases.

At the end of each chapter, a set of exercises involving conceptual as well as numerical calculations has been provided in the form of multiple choice questions following the latest trend of different examinations.

Estimated time required to solve each problem is given to provide a good grounding of the subject in a set-time.

Answers to all the exercises are given at the end of each chapter.

Three Self-Assessment Papers has also been included at the end of the book along with their answers.

The author is a Professor of Physics at the Indian Institute of Technology, Kanpur. He has been associated with undergraduate teaching for over thirty years. He has also contributed to various national journals.

**WHEELER
PUBLISHING**

of A. H. Wheeler & Co. Ltd.

ISBN 81-85814-21-X

Rs 125

PROBLEMS IN PHYSICS

A complete package for:

- IIT Joint Entrance Examination
- Combined Engineering Entrance
- Pre-Medical Test
- AMIE ■ NDA ■ NET
- Class XII Students

D SARAN

Acknowledgement

I wish to express my sincere thanks to my esteemed colleagues, Prof. Y.R. Waghmare and Prof. H.S. Mani (at present, Director, Mehta Research Institute, Allahabad) for stimulating me to take up the work of writing *Problems in Physics*. Their constant encouragement was an unfailing source of inspiration to me. I am also grateful to Profs M.S. Kalra, Harish Verma (Science College, Patna), Dr Raghvendra Tewari and Mrs Urmila Kalra for their valuable help and suggestions. I am especially indebted to Mr Himanshu Shanker for going through the entire manuscript and for help in checking the answers to the problems. I must also thank Mr Vivek Saraf and Ajay Kumar Singh for revising some chapters of the book. The financial assistance provided by the Quality Improvement Programme established at the Indian Institute of Technology, Kanpur, by the Ministry of Education and Culture, Government of India, is also gratefully acknowledged. Finally, I am thankful to Mr G.R. Hoshing for typing the manuscript and to Mr R.K. Bajpai for making the drawings.

And finally, I acknowledge the most valuable of everything from my close and dear ones which cannot be put in words here.

D.S.

Contents

<i>Preface</i>	v
<i>Acknowledgement</i>	vi
1. Units and Dimensions, Errors and Significant Figures	1–10
2. Vectors	11–23
3. Newton's Laws of Motion	24–56
4. Work, Energy and Power	57–67
5. Impulse, Momentum and Conservation of Momentum	68–79
6. Rotational Motion of Rigid Bodies	80–104
7. Simple Harmonic Motion	105–121
8. Gravitation	122–137
9. Fluid Statics	138–161
10. Fluid Dynamics	162–174
11. Elasticity	175–187
12. Heat	188–218
13. Kinetic Theory of Gases	219–234
14. Thermodynamics	235–267
15. Wave Motion and Sound	268–297
16. Reflection, Refraction and Optical Instruments	298–337
17. Interference and Diffraction of Light	338–361
18. Photometry	362–373
19. Electrostatics—Coulomb's Law, Electric Fields and Potentials	374–414
20. Capacitors and Dielectrics	415–435
21. Current, Resistance and Circuits	436–459
22. Electromagnetics	460–490
23. Varying Currents and Alternating Currents	491–510
24. The Atom	511–541
25. Nuclei and Radioactivity	542–559
26. Electronics: Thermionic and Solid State	560–580
<i>Self-Assessment</i>	581–603

Units and Dimensions, Errors and Significant Figures

The International System of Units is an expanded version of the rationalized metric-kilogram-second-ampere (MKSA) system of units and is based on six fundamental or basic units. The six basic units are the units of length (L), mass (M), time (T), current (I), temperature (K) and luminous intensity. The units associated with these are the metre, kilogram, second, ampere, kelvin degree and candela respectively. Two supplementary units are the radian and the steradian for plane angle and solid angle respectively. All other units are derived units.

The units of mass, length and time indicate their nature and not their magnitude. For instance, the unit of area is the product of two unit lengths, we have the unit of area represented by L^2 or the unit of volume as L^3 . We express this by saying that the unit of area is of two dimensions in length and the unit of volume, of three dimensions in length. We write them as $M^0L^2T^0$ and $M^0L^3T^0$ respectively.

ILLUSTRATIONS

$$\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{l/L} = \frac{(M \cdot a)/A}{l/L} = \frac{(M)(LT^{-2})/L^2}{L/L}$$

Units: N/m^2 , dimensions: $ML^{-1}T^{-2}$.

$$\begin{aligned} \text{Surface tension} &= \frac{\text{energy}}{\text{area}} \quad \text{or} \quad \frac{\text{force}}{\text{length}} \\ &= \frac{M \cdot a}{L} = \frac{MLT^{-2}}{L} = MT^{-2} \quad \text{or} \quad ML^0T^{-2} \end{aligned}$$

Units: N/m ; dimensions: ML^0T^{-2} .

$$\text{Power} = \frac{W}{t} = \frac{F \cdot d}{t} = \frac{MLT^{-2} \cdot L}{T} = ML^2T^{-3}$$

Units: J/s , dimensions: ML^2T^{-3} .

$$\text{Capacitance } C = \epsilon_0 \frac{A}{d}$$

where ϵ_0 is the permittivity constant of the medium. Its dimensions may be worked out as follows.

$$C = (\text{Coul}^2/\text{N} - \text{m}^2) (L^2/L) = \frac{Q^2 L^2}{(\text{MLT}^{-2} \cdot L^2) L} = Q^2 L^{-2} T^2 M^{-1}$$

Units: Farad, dimensions: $M^{-1} L^{-2} T^2 Q^2$

$$\text{Electric energy density} = \frac{\text{energy}}{\text{volume}} = \frac{F \cdot d}{L^3} = \frac{\text{MLT}^{-2} \cdot L}{L^3} = \text{ML}^{-1} \text{T}^{-2}$$

Units: J/m^3 ; dimensions: $\text{ML}^{-1} \text{T}^{-2}$.

Illustration to Show the Validity of a derived equation

The velocity of propagation of an electromagnetic wave in free space is given by

$$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

where ϵ_0 and μ_0 are permittivity and permeability of free space respectively. From Coulomb's Law,

$$\epsilon_0 = \frac{Q_1 Q_2}{4\pi F \cdot R^2}$$

and hence the dimensions are

$$\text{M}^{-1} \text{L}^{-3} \text{T}^2 \text{Q}^2$$

From Ampere's Law,

$$\mu_0 = \frac{4\pi F \cdot R^2}{(I_1 \cdot dl_1)(I_2 \cdot dl_2)}$$

and hence the dimensions are

$$\text{MLQ}^{-2}$$

Therefore, the dimensions of $1/\sqrt{\mu_0 \epsilon_0}$ are

$$\sqrt{\frac{1}{(\text{M}^{-1} \text{L}^{-3} \text{T}^2 \text{Q}^2)(\text{MLQ}^{-2})}} = \text{LT}^{-1}$$

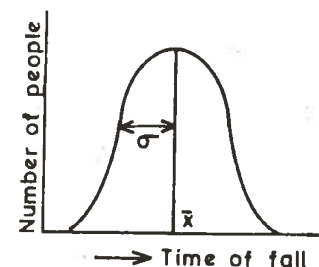
which are the same as the dimensions of velocity.

Errors of a Repeated Measurement

Let us consider an example where the time of fall of an object is measured with the help of a stopwatch. We soon realise that there is a lot of inconsistency in our observations. It may be difficult to know exactly when to start the

watch and exactly when to stop it. Our reaction time is slow and this leads to a large error.

Since we are determined to find the time, we ask twenty other students to measure the time of fall, and if we plot the results we get a distribution of the type shown in the figure. This is called Gaussian or normal distribution. It seems that most random processes lead to Gaussian distribution and so we assume this in our illustrations.



(a) Mean value

If $x_1, x_2, x_3, \dots, x_n$ are n measured values of a particular quantity, the mean value is defined as

$$\bar{x} = \frac{1}{n} (x_1 + x_2 + \dots + x_n) = \sum_{i=1}^n \frac{x_i}{n}$$

(b) Standard deviation

An important measure of the spread of experimental data is the standard deviation, σ defined as

$$\sigma^2 = \frac{1}{n-1} [(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2]$$

$$\text{or } \sigma = \left[\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \right]^{1/2}$$

(c) Standard error in the mean

The standard error or, as it is sometimes called, the probable error of the mean, α , of a set of data is given by

$$\alpha = \frac{\sigma}{\sqrt{n}}$$

ILLUSTRATION

The average speed of a racing car is measured by 10 different judges. The values found are 193.2, 195.6, 195.4, 194.7, 195.1, 193.8, 195.5, 194.7, 194.2, 195.3 miles per hour. What value should be recorded for the average speed of the car?

We find here that

$$\bar{x} = 194.75$$

$$\sigma = 0.80$$

$$\alpha = 0.25$$

We are 68% confident that the average speed is 194.8 ± 0.3 miles/h. If we measure a quantity Z , and the

Absolute error in $Z = \Delta Z$

Relative or fractional error in $Z = \frac{\Delta Z}{Z}$

Percentage error in $Z = 100 \times \frac{\Delta Z}{Z}$

'Discrepancy' means the difference between the result of an experiment and the result of one which has been accepted as more reliable i.e. the 'true' result.

Combination (Propagation) of Errors

Suppose we make a measurement x with an absolute error Δx and a measurement y with an absolute error Δy . These numbers combine to give an answer Z . What is the uncertainty ΔZ in Z ? The following rules are used in combining errors.

1. Addition: $Z = X + Y$
Standard error $\Delta Z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Limit error $\Delta Z = |\Delta x| + |\Delta y|$
2. Subtraction: $Z = X - Y$
Standard error $\Delta Z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Limit error $\Delta Z = |\Delta x| + |\Delta y|$
3. Linear Combination: $Z = ax + by$
Standard error $\Delta Z = \sqrt{(a\Delta x)^2 + (b\Delta y)^2}$
Limit error $\Delta Z = |a\Delta x| + |b\Delta y|$
4. Multiplication: $Z = XY$
Standard error $\Delta Z = \sqrt{y^2(\Delta x)^2 + x^2(\Delta y)^2}$
Limit error $\Delta Z = |y\Delta x| + |x\Delta y|$
5. Division: $Z = X/Y$
Standard error $\Delta Z = \sqrt{x^2(\Delta y)^2/y^4 + (\Delta x)^2/y^2}$
Limit error $\Delta Z = |x\Delta y/y^2| + |\Delta x/y|$

Let us take an example where the relation is

$$A = B^x C^y D^z$$

Here, the absolute errors in B , C and D are ΔB , ΔC and ΔD respectively. The relative error can be expressed as

$$\frac{\Delta A}{A} = x \frac{\Delta B}{B} + y \frac{\Delta C}{C} + z \frac{\Delta D}{D}$$

The density of a material is expressed as

$$\rho = \frac{M}{\pi L(d/2)^2}$$

where M , L and d are mass, length and diameter of the material in the form of a small cylinder. Therefore,

$$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + 2 \frac{\Delta d}{d} + \frac{\Delta L}{L}$$

Measurement	Absolute error	Relative error
L	ΔL	$\Delta L/L$
d	Δd	$\Delta d/d$
M	ΔM	$\Delta M/M$

ILLUSTRATIONS

1. In an experiment on determination of density of a metal given in the form of a small cylindrical rod, the following observations are recorded:

1. Mass of the cylinder = 12.7 g (used a physical balance whose least count is 0.1 g)

2. Radius of the cylinder = 0.574 cm (used a micrometer with LC = 0.001 cm)

3. Length of the cylinder = 3.28 cm (used a Vernier callipers, LC = 0.01 cm)

Calculate the density, estimate the error and write the result in proper significant figures.

Solution

$$\rho = \frac{M}{\pi R^2 L}$$

$$\frac{\Delta \rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta L}{L} + 2 \frac{\Delta R}{R}$$

$$= \frac{0.1}{12.7} + \frac{0.01}{3.28} + 2 \times \frac{0.001}{0.574}$$

$$= \frac{1}{127} + \frac{1}{328} + \frac{1}{287} \approx 0.014 \approx 1.4\%$$

$$\Delta \rho = \rho \times \frac{1.4}{100}; \rho = \frac{12.7}{\pi (0.574)^2 \times 3.28} \text{ g/cc}$$

$$\rho = 3.74 \text{ g/cc}$$

$$\therefore \rho = 3.74 \pm 0.05 \text{ g/cc}$$

2. In the determination of value of g by simple pendulum, the time period is measured by a stopwatch whose least count is 0.5 s and the length of the thread is measured with a meter scale, having a least count of 1 mm. The diameter of the bob is measured with the Vernier callipers with least count 0.01 cm. The following observations are recorded

1. Length of the thread = 105.3 cm

2. Diameter of the bob = 2.45 cm

6 Problems in Physics

3. Time period = 2.07 s

4. Number of oscillations = 10

Calculate the value of g , estimate error and write the result in proper significant figures.

Solution

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad g = 4\pi^2 \frac{L}{T^2}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \frac{\Delta T}{T}$$

$$= \frac{(0.1 + 0.005)}{(105.3 + 1.225)} + 2 \frac{\Delta T}{T}$$

$$= \frac{0.105}{106.525} + 2 \times \frac{0.5}{2.07 \times 10}$$

$$= 0.00098 + 0.048 \approx 5\%$$

$$g = \frac{4 \times \pi^2 \times 106.5}{(2.07)^2} = \frac{4204.45}{4.28} = 982 \text{ cm/s}^2$$

$$\Delta g = g \times \frac{0.048}{100} = 982 \times \frac{0.048}{100} = 0.47$$

$$\therefore g = 982 \pm 0.47 \text{ cm/s}^2$$

Let us see the result if we increase the number of oscillations from 10 to 20.

$$\frac{\Delta g}{g} = 0.00098 + \frac{2 \times 0.5}{20 \times 2.07} \approx \frac{1}{41.4} \approx 0.024$$

We observe that by increasing the number of oscillations from 10 to 20, the relative error in g falls down from about 5% to 2%.

Significant Figures

In carrying out computations, only significant figures should be retained. Significant figures are those figures which are known to be reasonably trustworthy; they are figures actually obtained from the measuring instruments, including zeros and estimated figures. The position of the decimal point does not affect the number of significant figures; if a zero is used merely to locate the decimal point, it is not significant, but if it actually represents a value read on the instrument, or estimated, then it is significant. The following rules may be used for the retention of significant figures in a computation.

1. In addition and subtraction, do not carry the result beyond the first column that contains a doubtful figure. This means that all figures lying to the right of the last column in which all figures are significant should be dropped.

2. In multiplication and division, retain in the result only as many significant figures as the least precise quantity that the data contains.

Units and Dimensions, Errors and Significant Figures 7

3. In dropping figures which are not significant, the last figure retained should be unchanged if the first figure dropped is less than 5. It should be increased by 1 if the first figure dropped is 5 or greater.

These rules give the number of significant figures which should appear in the final result. But in computing, in all the intermediate steps, it is better to carry one more significant figure than is measured in the final result.

The following examples illustrate the principles just mentioned.

(a) In obtaining the sum of these numbers

806.5		806.5
32.03	These should be	32.0
0.0652	written as	0.1
125.0		125.0
963.5952	(wrong)	963.6 (right)

Then the sum is expressed to the correct number of significant figures.

(b) The sides of a plate are measured with the help of Vernier callipers and the length and width are recorded as 7.62 cm and 3.81 cm. The area of the plate is calculated to be 29.0322 sq. cm. This number has six significant figures but the original quantities are known only to three significant figures. Therefore, only three significant figures should be retained in the result which should be written as 29.0 sq. cm.

EXERCISES

- Three measurements are made as 18.425 cm, 7.21 cm and 5.0 cm. The addition should be written as
 - 30.635
 - 30.64
 - 30.63
 - 30.6
- On subtracting 0.2 J from 7.26 J, the result should be written as
 - 7.1 J
 - 7.06 J
 - 7.0 J
 - 7 J
- The number of significant figures in a measurement of 0.0353 m is
 - 5
 - 4
 - 3
 - 2
- The number of significant figures in 1.118×10^{-3} V are
 - 7
 - 4
 - 3
 - 2
- In the multiplication of 107.88 and 0.610, the correct answer is
 - 65.8068
 - 65.807
 - 65.81
 - 65.8
- Subtract 0.2 kg from 34 kg. The result in terms of proper significant figures is
 - 33.8 kg
 - 33.80 kg
 - 34 kg
 - 34.0 kg
- If 97.52 is divided by 2.54, the correct result in terms of significant figures is
 - 38.4
 - 38.3937

8. 9.8 is divided by 9.3. The correct result should be written as
 (a) 1.054 (b) 1.05376
 (c) 1.05 (d) 1.0538
9. The units and dimensions of electric field intensity are
 (a) $\frac{\text{volt}}{\text{metre}}, \text{ML}^2\text{T}^{-2}\text{Q}^{-2}$ (b) $\frac{\text{volt}}{\text{metre}}, \text{MLT}^{-2}\text{Q}^{-1}$
 (c) $\frac{\text{coul}}{\text{metre}^2}, \text{MLT}^{-2}\text{Q}^{-1}$ (d) $\frac{\text{newton}}{\text{metre}}, \text{MLT}^{-1}\text{Q}^{-1}$
10. The units and dimensions of impedance are
 (a) mho, $\text{ML}^2\text{T}^{-1}\text{Q}^{-2}$ (b) ohm, $\text{ML}^2\text{T}^{-1}\text{Q}^{-2}$
 (c) ohm, $\text{ML}^2\text{T}^{-2}\text{Q}^{-1}$ (d) ohm, $\text{MLT}^{-1}\text{Q}^{-1}$
- 11.* The three columns give physical quantity, its units and its dimensions. Match the three items from the three columns:
- | Physical quantity | Units | Dimensions |
|-----------------------------|--------------------|---|
| A. Permittivity | A. mho/m | A. $\text{ML}^{-1}\text{T}^{-2}$ |
| B. Permeability | B. A/m^2 | B. $\text{ML}^2\text{T}^{-2}\text{Q}^{-1}$ |
| C. Capacitance | C. V | C. ML^2T^{-3} |
| D. Impedance | D. mho | D. $\text{MT}^{-1}\text{Q}^{-1}$ |
| E. Conductance | E. Ω | E. $\text{MLT}^{-2}\text{Q}^{-1}$ |
| F. Current density | F. J/m^3 | F. $\text{M}^{-1}\text{L}^{-3}\text{TQ}^2$ |
| G. Conductivity | G. W | G. $\text{L}^{-2}\text{T}^{-1}\text{Q}$ |
| H. Electric field intensity | H. V/m | H. $\text{M}^{-1}\text{L}^{-2}\text{TQ}^2$ |
| I. Power | I. Wb/m^2 | I. $\text{ML}^2\text{T}^{-1}\text{Q}^{-2}$ |
| J. Potential difference | J. F | J. $\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2$ |
| K. Energy density | K. F/m | K. $\text{M}^{-1}\text{L}^{-3}\text{T}^2\text{Q}^2$ |
| L. Magnetic flux density | L. H/m | L. MLQ^{-2} |
12. The dimensions of gravitational constant are
 (a) $\text{M}^{-1}\text{L}^3\text{T}^{-2}$ (b) ML^2T^{-3}
 (c) $\text{M}^{-1}\text{L}^{-2}\text{T}^{-1}$ (d) $\text{M}^{-1}\text{L}^{-3}\text{T}^2$
13. The dimensions of coefficient of viscosity are
 (a) MLT^{-1} (b) $\text{ML}^{-1}\text{T}^{-1}$
 (c) $\text{M}^{-1}\text{LT}^{-2}$ (d) $\text{M}^{-1}\text{L}^{-1}\text{T}$
14. The value of acceleration due to gravity in the cgs system is 980 cm/s^2 . The value of g in kilometres per minute square will be
 (a) 3.528 (b) 335.28
 (c) 35.28 (d) 0.3528
15. The density of a material is 8 g/cc . In a system in which the unit of length is 5 cm and unit of mass is 20 g, the density of material will be
 (a) 40 (b) 50
 (c) 24 (d) 80
16. E, m, L, G denote energy, mass, angular momentum and gravitational constant respectively. The dimensions of EL^2/m^2G^2 will be that of
 (a) angle (b) length
 (c) mass (d) time
17. The value of the multiplication 3.124×4.576 correct to three significant figures is
 (a) 14.295 (b) 14.3
 (c) 14.295424 (d) 14.305
18. The value of the expression $\frac{7.13 \times 21.54}{415.3}$ correct to three significant figures is
 (a) 0.3698 (b) 0.37
 (c) 0.370 (d) 0.369

19. The radius of a thin wire is 0.16 mm. The area of cross-section taking significant figures into consideration in square millimetres is
 (a) 0.0804 (b) 0.080
 (c) 0.08 (d) 0.080384
20. In an experiment, the length of a cylinder using vernier callipers is measured as 2.51 cm, 2.49 cm, 2.58 cm, 2.48 cm and 2.55 cm respectively. The average length of the cylinder in centimetres is
 (a) 2.5283 (b) 2.53
 (c) 2.52 (d) 2.528
21. In the above experiment, the absolute error in centimetres is
 (a) 0.035 (b) 0.03
 (c) 0.05 (d) 0.052
22. The percentage error in the measurements in the above experiment is
 (a) 14% (b) $\pm 0.0138\%$
 (c) $\pm 0.138\%$ (d) $\pm 1.38\%$
23. $[\text{ML}^2\text{T}^{-2}]$ are the dimensions of
 (a) work (b) momentum
 (c) power (d) pressure
24. $[\text{ML}^2\text{T}^{-2}]$ are the dimensions of
 (a) couple (b) momentum
 (c) power (d) pressure
25. $[\text{ML}^{-1}\text{T}^{-2}]$ are the dimensions of
 (a) stress (b) strain
 (c) energy (d) work
26. The dimensions of work, energy and couple are
 (a) ML^{-2}T^2 (b) ML^2T^{-2}
 (c) $\text{ML}^{-2}\text{T}^{-1}$ (d) ML^2T^{-1}
27. If the units of length and force are increased four times, the unit of energy is increased
 (a) 4 times (b) 16 times
 (c) 8 times (d) no increase
28. Out of the following expressions for the couple per unit twist, time period of a compound pendulum, frequency of a stretched wire and time period of a simple pendulum respectively, the dimensionally inconsistent one is
 (a) $C = \frac{\eta \pi r^4}{2l}$ (b) $T = 2\pi \sqrt{\frac{k^2}{g} + \frac{l}{g}}$
 (c) $f = \frac{1}{l} \sqrt{\frac{T}{\rho}}$ (d) $T = 2\pi \sqrt{\frac{l}{g}}$
- where η is coefficient of rigidity, r is the radius of the wire, K is the radius of gyration, and ρ is mass per unit length.
29. The moment of momentum is called
 (a) couple (b) torque
 (c) impulse (d) angular momentum
30. The number of significant figures in $11.118 \times 10^{-3} \text{ V}$ is
 (a) 3 (b) 4
 (c) 6 (d) 5
31. Four lengths are measured as 18.425 cm, 7.21 cm, 5.4 cm, and 10.3571 cm. Taking significant figures into account, the sum of the lengths should be
 (a) 40.0321 cm (b) 41.03 cm
 (c) 41.032 cm (d) 41.0 cm

10 Problems in Physics

32. Taking into account the significant figures, the product of 109.832 and 0.6107 should be written as
 (a) 67.0744 (b) 67.1
 (c) 67.07 (d) 67.074402
33. 14.28 is divided by 0.714. The answer should be written as
 (a) 20.0 (b) 20.00
 (c) 20 (d) 20.000
- ✓ 34. The position of a particle moving along the y-axis is given as $y = At^2 - Bt^3$ where y is measured in metres and t in seconds. The dimensions of B are
 (a) LT^{-2} (b) LT^{-3}
 (c) MLT^{-2} (d) MLT^{-3}
35. In the above motion, the time when the particle attains maximum positive- y position shall be ($A = 3 \text{ m/s}^2$, $B = 1 \text{ m/s}^3$)
 (a) 1.5 s (b) 4.0 s
 (c) 2.0 s (d) 3.0 s
36. If ρ is the mean density of the earth, r its radius, g the acceleration due to gravity and G the gravitational constant, on the basis of dimensional consistency, the correct expression is
 (a) $\rho = \frac{3G}{4\pi rg}$ (b) $\rho = \frac{rG}{12\pi g}$
 (c) $\rho = \frac{3g}{4\pi rG}$ (d) $\rho = \frac{3r}{4\pi gG}$

Answers

- | | | | | |
|---|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (c) | 4. (b) | 5. (d) |
| 6. (c) | 7. (a) | 8. (c) | 9. (b) | 10. (b) |
| 11. AKK BLL CJJ DEI EDH FBG GAF HHE IGC JCB KFA LID | | | | |
| 12. (a) | 13. (b) | 14. (c) | 15. (b) | 16. (a) |
| 17. (b) | 18. (c) | 19. (c) | 20. (d) | 21. (a) |
| 22. (d) | 23. (a) | 24. (a) | 25. (a) | 26. (b) |
| 27. (b) | 28. (b) | 29. (d) | 30. (d) | 31. (b) |
| 32. (c) | 33. (a) | 34. (b) | 35. (c) | 36. (c) |

Note: Only problem 11 which is marked with a star should take about 8 minutes to work out. The others should not take more than 2 minutes.

2 Vectors

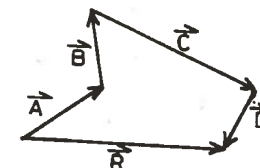
Introduction and Important Laws of Vector Algebra

Vector Addition When vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ are to be added, we draw the vectors as shown in the figure. We have a general rule

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

This can be extended to addition of n such vectors.

The product of a vector $\vec{A}$ and a scalar p is a vector $p\vec{A}$ where only the magnitude changes by a factor of p but the direction is unaltered.



Laws of Vector Algebra

- | | |
|--|-----------------------------------|
| 1. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ | Commutative law of addition |
| 2. $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ | Associative law of addition |
| 3. $p(q\vec{A}) = (pq)\vec{A} = q(p\vec{A})$ | Associative law of multiplication |
| 4. $(p + q)\vec{A} = p\vec{A} + q\vec{A}$ | Distributive law |
| 5. $p(\vec{A} + \vec{B}) = p\vec{A} + p\vec{B}$ | Distributive law |

Unit Vectors

$$\text{Unit vector } \hat{a} = \frac{\vec{A}}{|\vec{A}|}$$

Components of a Vector

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\text{Position vector } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Dot or Scalar Product of Two Vectors

$$\vec{A} \cdot \vec{B} = AB \cos \theta,$$

The following laws are valid

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

Commutative law of dot product

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Distributive law

$$p(\vec{A} \cdot \vec{B}) = p\vec{A} \cdot \vec{B} = \vec{A} \cdot (p\vec{B}) = (\vec{A} \cdot \vec{B})p \quad \text{where } p \text{ is a scalar}$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1; \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then

$$\vec{A} \cdot \vec{B} = A_1B_1 + A_2B_2 + A_3B_3$$

$$\vec{A} \cdot \vec{A} = A^2 = A_1^2 + A_2^2 + A_3^2$$

$$\vec{B} \cdot \vec{B} = B^2 = B_1^2 + B_2^2 + B_3^2$$

If $\vec{A} \cdot \vec{B} = 0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Cross or Vector Product of Two Vectors

$\vec{A} \times \vec{B} = AB \sin \theta \hat{u}$, (here $\hat{u}$ is determined by the right hand rule and θ is the smaller included angle) $0 \leq \theta \leq \pi$

where $\hat{u}$ is a unit vector indicating the direction of $\vec{A} \times \vec{B}$. If $\vec{A} = \vec{B}$ or if $\vec{A}$ is parallel to $\vec{B}$, then $\sin \theta = 0$ and $\vec{A} \times \vec{B} = 0$. The following laws are valid:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \quad (\text{Commutative Law for Cross Product fails})$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad \text{Distributive Law}$$

$$p(\vec{A} \times \vec{B}) = p\vec{A} \times \vec{B} = \vec{A} \times p\vec{B} = (\vec{A} \times \vec{B})p \quad \text{where } p \text{ is a scalar}$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0; \hat{i} \times \hat{j} = \hat{k}; \hat{j} \times \hat{k} = \hat{i}; \hat{k} \times \hat{i} = \hat{j}$$

If $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$|\vec{A} \times \vec{B}| =$ the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$. If $\vec{A} \times \vec{B} = 0$ and $\vec{A}$ and $\vec{B}$ are not null vectors, then $\vec{A}$ and $\vec{B}$ are parallel.

Triple Product Scalar triple product is defined as

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} \quad \text{where } \begin{aligned} \vec{A} &= A_1\hat{i} + A_2\hat{j} + A_3\hat{k} \\ \vec{B} &= B_1\hat{i} + B_2\hat{j} + B_3\hat{k} \\ \vec{C} &= C_1\hat{i} + C_2\hat{j} + C_3\hat{k} \end{aligned}$$

It represents the volume of a parallelepiped having $\vec{A}$, $\vec{B}$ and $\vec{C}$ as edges. Vector triple product is defined as

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

Since $(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{B} \cdot \vec{C})\vec{A}$, it is clear that $\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$.

Derivatives of Vectors Let vector $\vec{A}$ be a function of r written as $\vec{A}(r)$. Then,

$$\frac{d\vec{A}}{dr} = \frac{dA_1}{dr}\hat{i} + \frac{dA_2}{dr}\hat{j} + \frac{dA_3}{dr}\hat{k}$$

$$\text{also } \frac{d^2\vec{A}}{dr^2} = \frac{d^2}{dr^2}A_1\hat{i} + \frac{d^2}{dr^2}A_2\hat{j} + \frac{d^2}{dr^2}A_3\hat{k}$$

Integral of Vectors If $\vec{A}(r) = A_1(r)\hat{i} + A_2(r)\hat{j} + A_3(r)\hat{k}$ be a vector function of r , the indefinite integral of $\vec{A}(r)$ is defined as

$$\int \vec{A}(r) dr = \hat{i} \int A_1(r) dr + \hat{j} \int A_2(r) dr + \hat{k} \int A_3(r) dr$$

$$\text{Velocity } \vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$|\vec{v}| = \left| \frac{d\vec{r}}{dt} \right| = \sqrt{\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2}$$

$$\text{Acceleration } \vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$|\vec{a}| = \sqrt{\left(\frac{d^2x}{dt^2} \right)^2 + \left(\frac{d^2y}{dt^2} \right)^2 + \left(\frac{d^2z}{dt^2} \right)^2}$$

Relative Velocity and Acceleration If two particles P_1 and P_2 are moving with velocities $\vec{V}_1$ and $\vec{V}_2$ and accelerations $\vec{a}_1$ and $\vec{a}_2$ respectively, then

$$\vec{V}_{P_2/P_1} = \vec{V}_2 - \vec{V}_1 \quad \text{and} \quad \vec{a}_{P_2/P_1} = \vec{a}_2 - \vec{a}_1$$

Tangential and Normal Acceleration

$$\vec{a} = \frac{dV}{dt} \vec{T} + \frac{V^2}{R} \vec{N}$$

Circular Motion

$$V = \frac{dS}{dt} = R \cdot \frac{d\theta}{dt} = R\omega$$

$$\frac{dV}{dt} = \frac{d^2S}{dt^2} = R \frac{d^2\theta}{dt^2} = R\alpha$$

ILLUSTRATIONS

1. Find a unit vector in the direction of the resultant of vectors $\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{C} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

Solution

$$\begin{aligned} \vec{R} &= \vec{A} + \vec{B} + \vec{C} = 2\hat{i} - \hat{j} + \hat{k} + \hat{i} + \hat{j} + 2\hat{k} + 3\hat{i} - 2\hat{j} + 4\hat{k} \\ &= 6\hat{i} - 2\hat{j} + 7\hat{k} \end{aligned}$$

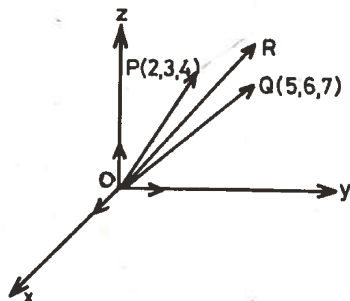
$$\begin{aligned} \text{Unit vector in the direction of } \vec{R} &= \frac{\vec{R}}{|\vec{R}|} = \frac{6\hat{i} - 2\hat{j} + 7\hat{k}}{\sqrt{36 + 4 + 49}} \\ &= \frac{6\hat{i} - 2\hat{j} + 7\hat{k}}{\sqrt{89}} \end{aligned}$$

2. If $P(2, 3, 4)$ and $Q(5, 6, 7)$ are two points in space, represent their position vectors $\vec{OP}$ and $\vec{OQ}$ in terms of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ where O is the origin of a rectangular coordinate system. Find their resultant $\vec{OR}$ and the unit vector in the direction of the resultant.

Solution

$$\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{OQ} = 5\hat{i} + 6\hat{j} + 7\hat{k}$$



$$\vec{OP} + \vec{OQ} = 7\hat{i} + 9\hat{j} + 11\hat{k} = \vec{OR}$$

$$|\vec{OR}| = \sqrt{251}$$

$$\text{Unit vector in the direction of } \vec{OR} = \frac{\vec{OR}}{|\vec{OR}|} = \frac{7\hat{i} + 9\hat{j} + 11\hat{k}}{\sqrt{251}}$$

3. Find the angular relationship between the two vectors

$$\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k} \quad \text{and} \quad \vec{B} = 2\hat{i} - 2\hat{j} + \hat{k}$$

Solution

$$\vec{A} \cdot \vec{B} = AB \cos \theta \Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\cos \theta = \frac{(2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} - 2\hat{j} + \hat{k})}{|\vec{A}| |\vec{B}|} = 0$$

$$\theta = 90^\circ$$

Therefore, the two vectors are perpendicular to each other.

4. If a force $\vec{F} = (3\hat{i} + 5\hat{j} - 2\hat{k})$ N acts at a point defined by $(7\hat{i} - 2\hat{j} + 5\hat{k})$ m, find the torque: (a) about the origin, and (b) about the point $(0, -1, 0)$.

Solution

$$\begin{aligned} \text{(a) Torque, } \vec{\tau} &= \vec{r} \times \vec{F} = (7\hat{i} - 2\hat{j} + 5\hat{k}) \times (3\hat{i} + 5\hat{j} - 2\hat{k}) \\ &= 35\hat{k} + 14\hat{j} + 6\hat{k} + 4\hat{i} + 15\hat{j} - 25\hat{i} \\ &= -21\hat{i} + 29\hat{j} + 41\hat{k} \text{ N} \cdot \text{m} \end{aligned}$$

(b) When the origin is at $(0, -1, 0)$, the radius vector becomes $7\hat{i} - \hat{j} + 5\hat{k}$
Therefore $\vec{\tau} = (7\hat{i} - \hat{j} + 5\hat{k}) \times (3\hat{i} + 5\hat{j} - 2\hat{k})$

$$\begin{aligned} &= 35\hat{k} + 14\hat{j} + 3\hat{k} + 2\hat{i} + 15\hat{j} - 25\hat{i} \\ &= -23\hat{i} + 29\hat{j} + 38\hat{k} \text{ N} \cdot \text{m} \end{aligned}$$

5. If $\vec{A} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j} - 4\hat{k}$, find

(a) $\vec{A} \cdot (\vec{B} \times \vec{C})$ (b) $\vec{C} \cdot (\vec{A} \times \vec{B})$ (c) $\vec{A} \times (\vec{B} \times \vec{C})$

Solution

$$\begin{aligned} \text{(a) } \vec{B} \times \vec{C} &= (\hat{i} - 2\hat{j} + \hat{k}) \times (-\hat{i} + \hat{j} - 4\hat{k}) \\ &= \hat{k} + 4\hat{j} - 2\hat{k} + 8\hat{i} - \hat{j} - \hat{i} = 7\hat{i} + 3\hat{j} - \hat{k} \end{aligned}$$

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = (2\hat{i} + \hat{j} - 3\hat{k}) \cdot (7\hat{i} + 3\hat{j} - \hat{k})$$

$$= 14 + 3 + 3 = 20$$

$$(b) \quad \vec{A} \times \vec{B} = (2\hat{i} + \hat{j} - 3\hat{k}) \times (\hat{i} - 2\hat{j} + \hat{k})$$

$$= -5(\hat{i} + \hat{j} + \hat{k})$$

$$\vec{C} \cdot (\vec{A} \times \vec{B}) = (-\hat{i} + \hat{j} - 4\hat{k}) \cdot -5(\hat{i} + \hat{j} + \hat{k})$$

$$= -5 + 5 + 20 = 20$$

$$(c) \quad \vec{A} \times (\vec{B} \times \vec{C}) = (2\hat{i} + \hat{j} - 3\hat{k}) \times (7\hat{i} + 3\hat{j} - \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 7 & 3 & -1 \end{vmatrix} = (-1 + 9)\hat{i} + (2 - 21)\hat{j} + (6 - 7)\hat{k}$$

$$= 8\hat{i} - 19\hat{j} - \hat{k}$$

6. A man in a boat on one side of a river wishes to reach a point directly opposite on the other side of the river. Assuming that the width of the river is D and that the speed of the boat and the current are V_b and V_c respectively ($V_b > V_c$), find the angle at which he should start and also find the time he would take to cross the river.

Solution

From the figure, we see that

$$\sin \theta = \frac{V_c}{V_b} \quad \text{or} \quad \theta = \sin^{-1} \frac{V_c}{V_b}$$

$$\text{Also, } t(\vec{V}_b + \vec{V}_c) = \vec{AB}$$

$$t^2[V_b^2 + V_c^2 + 2V_b V_c \cos(\theta + 90^\circ)] = D^2$$

$$\text{or } V_b^2 - V_c^2 = \frac{D^2}{t^2} \Rightarrow \sqrt{V_b^2 - V_c^2} = \frac{D}{t}$$

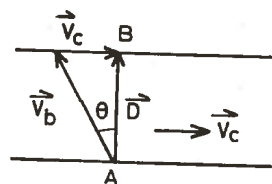
$$\therefore \text{Time taken to cross} = \frac{D}{\sqrt{V_b^2 - V_c^2}}$$

7. Find: (a) the radial acceleration along $\vec{r}$, and (b) acceleration perpendicular to $\vec{r}$, of a particle which moves on an elliptical path given by

$$\vec{r} = a \cos \omega t \hat{i} + b \sin \omega t \hat{j}$$

Solution

$$\frac{d\vec{r}}{dt} = -a\omega \sin \omega t \hat{i} + b\omega \cos \omega t \hat{j}$$



$$\frac{d^2\vec{r}}{dt^2} = -a\omega^2 \cos \omega t \hat{i} - b\omega^2 \sin \omega t \hat{j}$$

$$= -\omega^2 [a \cos \omega t \hat{i} + b \sin \omega t \hat{j}] = -\omega^2 \vec{r}$$

$$\text{Radial acceleration} = \vec{a} \cdot \hat{r} = -\omega^2 \vec{r}$$

$$\text{Acceleration perpendicular to } \vec{r} = \vec{a} \cdot \hat{\theta} = 0$$

EXERCISES

- The condition that $(\vec{a} \cdot \vec{b})^2 = (\vec{a}^2 \cdot \vec{b}^2)$ is satisfied when
 - $\vec{a} \parallel \vec{b}$
 - $\vec{a} \neq \vec{b}$
 - $\vec{a} \cdot \vec{b} = 1$
 - $\vec{a} \perp \vec{b}$
- The condition under which the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ are parallel is
 - $\vec{a} \perp \vec{b}$
 - $|\vec{a}| = |\vec{b}|$
 - $\vec{a} \neq \vec{b}$
 - $\vec{a} \parallel \vec{b}$
- The condition under which the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ should be at right angles to each other is
 - $\vec{a} \neq \vec{b}$
 - $\vec{a} \cdot \vec{b} = 0$
 - $|\vec{a}| = |\vec{b}|$
 - $\vec{a} \cdot \vec{b} = 1$
- If vectors $\vec{a} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + x\hat{k}$ are to be perpendicular to each other, the value of x should be
 - 2
 - 2
 - 3
 - 3
- The correct expression in the following is
 - $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$
 - $\vec{a} \cdot (\vec{b} \cdot \vec{c}) + \vec{b} \cdot (\vec{c} \cdot \vec{a}) + \vec{c} \cdot (\vec{a} \cdot \vec{b}) = 0$
 - $\vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{c} \times \vec{a}) + \vec{c} \cdot (\vec{a} \times \vec{b}) = 0$
 - $\vec{a} \cdot (\vec{b} + \vec{c}) + \vec{b} \cdot (\vec{c} + \vec{a}) + \vec{c} \cdot (\vec{a} + \vec{b}) = 0$
- The area of a triangle bounded by vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is
 - $\frac{1}{6} |\vec{a} + \vec{b} + \vec{c}|$
 - $\frac{1}{6} |\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}|$
 - $\frac{1}{6} |[(\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) + (\vec{a} \times \vec{b})]|$
 - $\frac{1}{6} |(\vec{a} \cdot \vec{b})(\vec{b} \cdot \vec{c})(\vec{c} \cdot \vec{a})|$
- The vector sum of three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is zero. If $\hat{i}$ and $\hat{j}$ are unit vectors in the directions of $\vec{a}$ and $\vec{b}$ respectively, then

- (a) $\vec{c}$ is in the plane of $\hat{i}$ and $\hat{j}$
 (b) $\vec{c}$ is along $(\hat{i} \times \hat{j})$
 (c) $\vec{c}$ is along $\hat{i}$
 (d) $\vec{c}$ is along $\hat{j}$
8. If four non-zero vectors satisfy $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ with $|\vec{a}| \neq |\vec{d}|$ and $|\vec{b}| \neq |\vec{c}|$, then
 (a) $(\vec{a} - \vec{d})$ and $(\vec{b} - \vec{c})$ are perpendicular
 (b) $(\vec{a} - \vec{d})$ and $(\vec{b} - \vec{c})$ are parallel
 (c) $(\vec{a} - \vec{d})$ must equal $(\vec{b} - \vec{c})$
 (d) $(\vec{a} - \vec{d})$ must equal $-(\vec{b} - \vec{c})$
9. A man running along a straight road with uniform velocity $\vec{u} = u\hat{i}$ feels that the rain is falling vertically down along $-\hat{j}$. If he doubles his speed, he finds that the rain is coming at an angle θ with the vertical. The actual direction and speed of the rain with respect to the ground is
 (a) $u\hat{i} - u \tan \theta \hat{j}$ (b) $u\hat{i} - \frac{u}{\tan \theta} \hat{j}$
 (c) $u\hat{i}$ (d) $u\hat{j}$
10. A particle moves such that $x = 3 + t + 2t^2$, $y = -1 + t + t^2$, $z = 3 \sin \pi t$ where x, y, z are measured in metres and t in seconds. The acceleration at $t = 3$ s is
 (a) $36\hat{i} + 2\hat{j}$ m/s² (b) $2\hat{i} + 36\hat{j}$ m/s²
 (c) $2\hat{i} + 36\hat{j} + \hat{k}$ m/s² (d) $2\hat{i} + 36\hat{j} + \pi\hat{k}$ m/s²
11. The acceleration of a particle is given by

$$\vec{a} = \left[2\hat{i} + 6t\hat{j} + \frac{2\pi^2}{9} \cos \frac{\pi t}{3} \hat{k} \right] \text{ m/s}^2$$

 The initial conditions are

$$\vec{r}(0) = x(0)\hat{i} + y(0)\hat{j} + z(0)\hat{k} = 0$$

$$\vec{v}(0) = \frac{dr}{dt}(0) = \frac{dx}{dt}(0)\hat{i} + \frac{dy}{dt}(0)\hat{j} + \frac{dz}{dt}(0)\hat{k} = (2\hat{i} + \hat{j}) \text{ m/s}$$

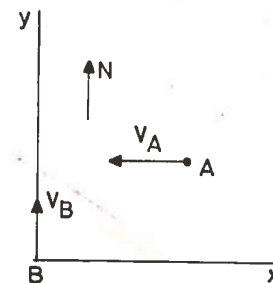
 The position vector at $t = 2$ s is
 (a) $(3\hat{i} + 8\hat{j} + 10\hat{k})$ m (b) $(8\hat{i} + 10\hat{j} + 3\hat{k})$ m
 (c) $(10\hat{i} + 3\hat{j} + 8\hat{k})$ m (d) $(3\hat{i} + 10\hat{j} + 8\hat{k})$ m
12. A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dt} = \frac{dy}{dt} = c$ where a, b and c are constants. The acceleration of the particle is
 (a) $(6ac^2x + 2bc^2)\hat{k}$ (b) $(6bc^2x + 2ac^2)\hat{k}$
 (c) $(ax^2 + by^2)\hat{k}$ (d) $(2ax^2 + 6by^2)\hat{k}$
13. When $t = 0$, a particle at $(1, 0, 0)$ moves towards $(4, 4, 12)$ with a constant speed of 65 m/s. The position of the particle is measured in metres and the

time in seconds. Assuming constant velocity, the position of the particle for $t = 2$ s is

- (a) $(13\hat{i} - 120\hat{j} + 40\hat{k})$ m (b) $(40\hat{i} + 31\hat{j} - 120\hat{k})$ m
 (c) $(13\hat{i} - 40\hat{j} + 12\hat{k})$ m (d) $(31\hat{i} + 40\hat{j} + 120\hat{k})$ m $(31\hat{i} + 40\hat{j} + 120\hat{k})$
14. The path of a particle is described by $x = 5t^2 + 2t + 3$, $y = t + 4$, $z = 2e^t$ where x, y and z are measured in metres and t is measured in seconds. The acceleration of the particle at $t = 2$ s is
 (a) $2e^2\hat{i} + 10\hat{k}$ m/s² (b) $10\hat{i} + 2e^2\hat{k}$ m/s²
 (c) $10\hat{i} + 2\hat{j} + e^2\hat{k}$ m/s² (d) $\hat{i} + e^2\hat{j} + e^2\hat{k}$ m/s²
15. A particle moves in such a manner that $x = A \sin \pi t$, $y = Bt^3 - 2t$, $z = ct^2 - 4t$, where x, y and z are measured in metres and t is measured in seconds, and A, B and C are unknown constants. Given that the velocity of the particle at $t = 2$ s is $\vec{v} = 3\pi\hat{i} + 22\hat{j}$, the velocity of the particle at $t = 4$ s is
 (a) $4\hat{i} + 3\pi\hat{j} + 94\hat{k}$ m/s (b) $-4\hat{i} + 94\hat{j} - 3\pi\hat{k}$ m/s
 (c) $3\pi\hat{i} + 94\hat{j} + 4\hat{k}$ m/s (d) $3\pi\hat{i} - 94\hat{j} - \hat{k}$ m/s
16. A particle starts from rest at the origin with a constant acceleration

$$\vec{a} = 2\hat{i} + 8\hat{j} - 6\hat{k} \text{ m/s}^2$$

 Its position at $t = 5$ s is
 (a) $(25\hat{i} + 100\hat{j} - 75\hat{k})$ m (b) $(25\hat{i} - 100\hat{j} - 75\hat{k})$ m
 (c) $(100\hat{i} - 25\hat{j} + 75\hat{k})$ m (d) $(25\hat{i} - 100\hat{j} + 75\hat{k})$ m
17. An aircraft A is flying west with a velocity $V_A = 900$ km/h while aircraft B is flying north with a velocity $V_B = 600$ km/h at approximately the same height. The magnitude of the velocity which A appears to have to a passenger riding in B is
 (a) 811.5 km/h
 (b) 541 km/h
 (c) 1082 km/h
 (d) 1623 km/h
18. Consider the two vectors $\vec{A}$ and $\vec{B}$. The magnitude of their sum, i.e. $|\vec{A} + \vec{B}|$, if $|\vec{A}| > |\vec{B}|$
 (a) is equal to $|\vec{A}| + |\vec{B}|$
 (b) cannot be less than $|\vec{A}| + |\vec{B}|$
 (c) cannot be greater than $|\vec{A}| + |\vec{B}|$
 (d) must be equal to $|\vec{A}| - |\vec{B}|$
19. ABCD is a quadrilateral. Forces $\vec{BA}$, $\vec{BC}$, $\vec{CD}$ and $\vec{DA}$ act at a point. Their resultant is
 (a) $2\vec{AB}$ (b) $2\vec{DA}$
 (c) $2\vec{BC}$ (d) $2\vec{BA}$



20. The two vectors $\vec{A}$ and $\vec{B}$ that are perpendicular to each other are

- (a) $\vec{A} = 3\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} - 2\hat{j} + \hat{k}$
 (b) $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} - 2\hat{j} + \hat{k}$
 (c) $\vec{A} = 2\hat{i} - 3\hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} - 2\hat{j} - \hat{k}$
 (d) $\vec{A} = \hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} - 2\hat{j} + \hat{k}$

21. The two vectors $\vec{A}$ and $\vec{B}$ that are parallel to each other are

- (a) $\vec{A} = 3\hat{i} + 6\hat{j} + 9\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$
 (b) $\vec{A} = 3\hat{i} - 6\hat{j} + 9\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$
 (c) $\vec{A} = 2\hat{i} + 3\hat{j} + 3\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$
 (d) $\vec{A} = 2\hat{i} + 6\hat{j} - 9\hat{k}$, $\vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$

22. The vector that is added to $(\hat{i} - 5\hat{j} + 2\hat{k})$ and $(3\hat{i} + 6\hat{j} - 7\hat{k})$ to give a unit vector along the x-axis is

- (a) $3\hat{i} + \hat{j} + 5\hat{k}$ (b) $\hat{i} + 3\hat{j} + 5\hat{k}$
 (c) $-3\hat{i} - \hat{j} + 5\hat{k}$ (d) $3\hat{i} + \hat{j} - 5\hat{k}$

23. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, the cosine of angle θ between them is equal to

- (a) $5/14$ (b) $1/7$
 (c) $3/14$ (d) $1/14$

24. A particle moves from the point $(1, 0, 2.5)$ to the point $(-2, 3, 4)$ m when a force $\vec{F} = (\hat{i} + 4\hat{k})$ N acts on it. The work done on it is

- (a) 6 J (b) 30 J
 (c) 3 J (d) 9 J

25. If two non-parallel vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude, then the vectors $(\vec{A} - \vec{B})$ and $(\vec{A} + \vec{B})$ will be

- (a) parallel to each other
 (b) parallel but oppositely directed
 (c) perpendicular to each other
 (d) inclined at an angle θ always less than 90°

26. The radius vector of a point is $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k})$ metres and a force $\vec{F} = (4\hat{i} + 5\hat{j})$ acts at that point. The moment of the force in newton-metres is

- (a) $-15\hat{i} + 12\hat{j} + 13\hat{k}$ (b) $15\hat{i} - 12\hat{j} + 13\hat{k}$
 (c) $-15\hat{i} - 12\hat{j} + 13\hat{k}$ (d) $15\hat{i} + 12\hat{j} + 13\hat{k}$

27. If none of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are zero and if $\vec{A} \times \vec{B} = 0$ and $\vec{B} \times \vec{C} = 0$, the value of $\vec{A} \times \vec{C}$ is

- (a) unity (b) zero
 (c) B^2 (d) $AC \cos \theta$

28. The vector having module equal to 3 and perpendicular to the two vectors $\vec{A} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ is

- (a) $\pm(2\hat{i} - \hat{j} - 2\hat{k})$ (b) $\pm(3\hat{i} + \hat{j} - 2\hat{k})$
 (c) $\pm(3\hat{i} + \hat{j} - 3\hat{k})$ (d) $\pm(3\hat{i} - \hat{j} - 3\hat{k})$

29. If the vectors $(\hat{i} + \hat{j} + \hat{k})$ and $3\hat{i}$ form two sides of a triangle, the area of the triangle is

- (a) $\sqrt{3}$ (b) $2\sqrt{3}$
 (c) $\frac{3}{\sqrt{2}}$ (d) $3\sqrt{2}$

30. A proton is moving with a velocity $(3\hat{i} + \hat{j}) \times 10^5$ m/s in an electric field of intensity $3(\hat{i} - \hat{j} + \hat{k}) \times 10^4$ V/m and a magnetic field of $(0.2\hat{j} + 0.3\hat{k})$ tesla. The magnitude of force experienced by the proton is

- (a) 26×10^{-15} N (b) 26×10^{-10} N
 (c) 26×10^{10} N (d) 26×10^8 N

31. The volume of a parallelepiped bounded by vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be obtained from the expression

- (a) $(\vec{A} \cdot \vec{B}) \times \vec{C}$ (b) $(\vec{A} \times \vec{B}) \cdot \vec{C}$
 (c) $(\vec{A} \cdot \vec{C}) \times \vec{B}$ (d) $(\vec{A} \times \vec{B}) \times \vec{C}$

32. One vertex of a parallelepiped is at the point $(1, -1, -2)$ of rectangular cartesian coordinates. If three adjacent vertices are at $(0, 1, 3)$, $(3, 0, -1)$ and $(1, 4, 1)$, the volume of the parallelepiped is

- (a) 400 units (b) 80 units
 (c) 40 units (d) 120 units

33. A scalar triple product $(\vec{A} \times \vec{B}) \cdot \vec{C}$ vanishes when

- (a) $\vec{A}$, $\vec{B}$ and $\vec{C}$ are not in the same plane
 (b) $\vec{A}$, $\vec{B}$ and $\vec{C}$ are in the same plane
 (c) $\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$
 (d) $\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{C}$

34. Three forces $\vec{F}_1 = (3\hat{i} + 2\hat{j} - \hat{k})$ N, $\vec{F}_2 = (3\hat{i} + 4\hat{j} - 5\hat{k})$ N and

$\vec{F}_3 = A(\hat{i} + \hat{j} - \hat{k})$ N act simultaneously on a particle. In order that the particle remains in equilibrium, the value of A should be

- (a) -6 (b) 6
 (c) 9 (d) -9

35. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two vectors and θ is the angle between them, then the value of $\cos \theta$ is

- (a) $l_1 l_2 + m_1 m_2 + n_1 n_2$ (b) $l_1 m_1 + m_1 n_1 + n_1 l_1$
 (c) $l_2 m_2 + m_2 n_2 + n_2 l_2$ (d) $m_1 l_2 + l_1 m_2 + n_1 m_2$

36. $|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2$ is equal to

- (a) $(\vec{A} + \vec{B})^2$ (b) $(\vec{A} - \vec{B})^2$
 (c) $A^2 + B^2$ (d) $A^2 B^2$

37. The value of p so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar should be

- (a) 16 (b) -4
 (c) 4 (d) -8

38. Three forces $\vec{F}_1 = 2\hat{i} + 3\hat{j}$, $\vec{F}_2 = 2\hat{i} - \hat{j} - \hat{k}$, and $\vec{F}_3 = 4\hat{i} + 2\hat{j} + \hat{k}$ act simultaneously on a particle. The force $\vec{F}_4$ that must be applied in order to keep the particle in equilibrium should be
- (a) $(8\hat{i} + 4\hat{k})$ (b) $-8\hat{i} + 2\hat{j} + \hat{k}$
 (c) $-(8\hat{i} + 4\hat{j})$ (d) $8\hat{i} - 4\hat{j} + \hat{k}$
39. A force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ N acts at a point (7, 3, 1). The torque about the origin (0, 0, 0) will be
- (a) $14\hat{i} - 38\hat{j} + 16\hat{k}$ (b) $14\hat{i} + 38\hat{j} - 16\hat{k}$
 (c) $14\hat{i} - 38\hat{j} - 16\hat{k}$ (d) $14\hat{i} + 38\hat{j} + 16\hat{k}$
40. A proton of velocity $(3\hat{i} + 2\hat{j}) \times 10^8$ cm/s enters a magnetic field $(2\hat{j} + 3\hat{k}) \times 10^3$ gauss. Its acceleration will be
- (a) $(2\hat{i} - 3\hat{j} + 2\hat{k}) \times 3 \times 10^{15}$ cm/s²
 (b) $(2\hat{i} + 3\hat{j} + 2\hat{k}) \times 3 \times 10^{15}$ cm/s²
 (c) $(-2\hat{i} - 3\hat{j} + 2\hat{k}) \times 3 \times 10^{15}$ cm/s²
 (d) $(2\hat{i} - 3\hat{j} - 2\hat{k}) \times 3 \times 10^{15}$ cm/s²
41. A man wishes to swim across a river 0.5 km wide. If he can swim at the rate of 2 km/h and the river flows at the rate of 1 km/h, the angle (with respect to the flow of the river) along which he should swim so as to reach a point exactly opposite his starting point should be
- (a) 60° (b) 120°
 (c) 145° (d) 90°
42. To a motorist going north at 30 km/h the wind appears to blow north-west. Actually it blows from the west. The actual velocity of the wind is
- (a) $30\sqrt{2}$ km/h (b) $\frac{30}{\sqrt{2}}$ km/h
 (c) 30 km/h (d) $2\sqrt{30}$ km/h
43. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = 2\hat{i} + 3\hat{j}$, $\vec{C} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{D} = -3\hat{j} + \hat{k}$, then $\vec{AB}$ and $\vec{CD}$ are
- (a) equal in magnitude and direction
 (b) equal in magnitude only
 (c) perpendicular to each other
 (d) parallel to each other
44. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular if
- (a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
 (b) $\vec{A}$ and $\vec{B}$ are perpendicular but their magnitudes are arbitrary
 (c) $|\vec{A}| = |\vec{B}|$ and their directions are arbitrary
 (d) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and $|\vec{A}| = |\vec{B}|$

Answers

1 (a)	2 (d)	3 (c)	4 (b)	5 (a)
6 (c)	7 (a)	8 (b)	9 (b)	10 (a)
11 (b)	12 (a)	13 (d)	14 (b)	15 (c)
16 (a)	17 (c)	18 (c)	19 (d)	20 (b)
21 (a)	22 (c)	23 (d)	24 (c)	25 (c)
26 (a)	27 (b)	28 (a)	29 (c)	30 (a)
31 (b)	32 (c)	33 (b)	34 (a)	35 (a)
36 (d)	37 (b)	38 (c)	39 (a)	40 (a)
41 (b)	42 (c)	43 (d)	44 (c)	

Note: All questions are almost of the same standard and should take about three minutes each to answer.

3

Newton's Laws of Motion

Motion in a Straight Line, Motion in a Plane

Average and Instantaneous Velocity

$\vec{V}_{inst} = \lim_{\Delta t \rightarrow 0} \Delta \vec{x} / \Delta t$ and in the notation of calculus can be written as

$$\vec{V}_{inst} = \frac{d\vec{x}}{dt}$$

$$\vec{a}_{av} = \frac{\vec{V}_2 - \vec{V}_1}{\Delta t} \quad \text{and} \quad \vec{a}_{inst} = \frac{d^2\vec{x}}{dt^2}$$

Also $\vec{a} = \vec{V} \cdot \frac{d\vec{V}}{dx}$

When V is function of x , since $V = dx/dt$, $x(t)$ can be determined from,

$$\int dt = \int \frac{dx}{V(x)}$$

Motion with Constant Acceleration

$$a(t) = a = \text{constant}$$

$$V = V_0 + at$$

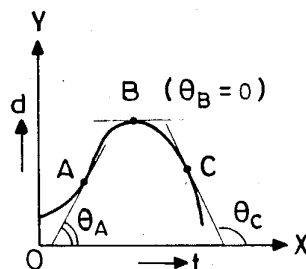
and $x = V_0t + \frac{1}{2}at^2$

Also $V^2 = V_0^2 + 2ax$

Graphical Representation

Displacement-Time Graph

If a particle is moving with constant velocity V then it will be represented

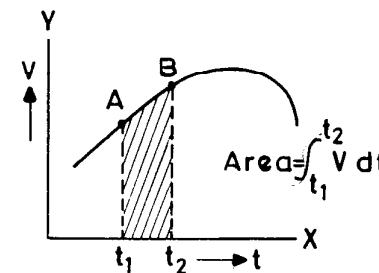
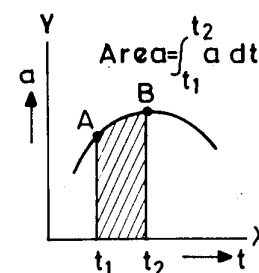
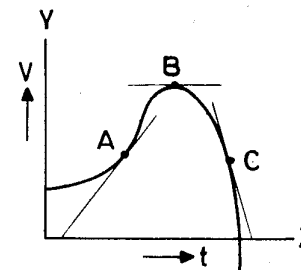


by a straight line in a displacement-time graph and its slope will give the velocity. Higher the velocity, greater will be the slope of the line. At A, the velocity $V_A = \tan \theta_A$. At B, the velocity $V_B = \tan \theta_B = 0$ and at C, $V_C = \tan \theta_C$, which is negative and it is concluded that the particle is travelling towards its initial position.

In this velocity-time graph, one can determine the acceleration at any point. At A, the acceleration is positive; at B, it is zero and at C, it is negative.

In an $a-t$ diagram, the area under the curve from t_1 to t_2 is equal to the change in velocity during this interval of time as shown in the figure.

Similarly, in a $V-t$ diagram, the area under the curve in the interval of time between t_1 and t_2 is equal to the change in the displacement in that interval.



Equations of Motion

$$\sum \vec{F} = m\vec{a} \quad \sum F_x = m \frac{d^2x}{dt^2}$$

(Newton's Second Law of Motion) $\sum F_y = m \frac{d^2y}{dt^2}$

$$\sum F_z = m \frac{d^2z}{dt^2}$$

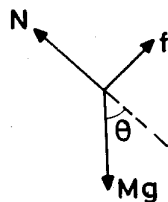
Normal and Tangential Components

$$\sum F_N = m \frac{V^2}{r} \quad \text{and} \quad \sum F_T = m \frac{dV}{dt}$$

Free Body Diagram

In order to apply the Second Law of motion, we represent the body as a particle and mark the real forces acting on it. For example, consider a block of mass M falling down an inclined plane of angle A where a frictional force f is also

acting between the block and the surface of the inclined plane. The free body diagram is shown here. N is the normal reaction of the inclined surface on the body, and Mg is the weight acting vertically downward. As the block is sliding down, the force of friction f acts up the plane.



Equations of Motion in Free Fall

$$V_y = V_{y,0} + gt$$

$$y = V_{y,0} t + \frac{1}{2} gt^2 \quad \text{and} \quad V_y^2 = V_{y,0}^2 + 2gy$$

Projectile Motion

$$V_x = V_0 \cos \theta_0 \quad \text{and} \quad V_y = V_0 \sin \theta_0 - gt$$

where V_0 is the initial velocity of projection and θ_0 is the angle of projection.

Equation of trajectory of the projectile

$$Y = (\tan \theta_0)X - \frac{g}{2(V_0 \cos \theta_0)^2} X^2$$

$$\text{Time to attain maximum height, } t = \frac{V_0 \sin \theta_0}{g}$$

$$\text{Maximum height attained} = Y_{\max} = (V_0 \sin \theta_0)t - \frac{1}{2} gt^2$$

$$\text{Time of flight} = 2t = \frac{2 \times V_0 \sin \theta_0}{g}$$

$$\begin{aligned} \text{Horizontal range of the projectile} &= R = (V_0 \cos \theta_0) \times \text{time of flight} \\ &= \frac{(V_0 \cos \theta_0)(2V_0 \sin \theta_0)}{g} \end{aligned}$$

Uniform Circular Motion

$$\vec{a} = -\hat{u}_r \frac{V^2}{r}$$

where $\hat{u}_r$ is the unit vector in the radial direction pointing away from the centre. The negative sign indicates that the acceleration is directed radially inward.

$$\vec{a} = \frac{d\vec{V}}{dt} = \hat{u}_\theta \frac{dV}{dt} + V \frac{d\hat{u}_\theta}{dt}$$

where $\hat{u}_\theta$ is the unit vector in the tangential direction

Force of Friction

$f_s \leq \mu_s N$ where μ_s is the coefficient of static friction and N is the normal reaction

$f_k = \mu_k N$ where μ_k is the coefficient of kinetic friction

ILLUSTRATIONS

1. A particle moves along a straight line in such a way that its distance x from a fixed point O on the line, after time t from the start, is given by

$$x = 2t^2 + 3t + 5$$

Find the velocity at the start and the distance covered in the 5th second.

Solution

$$\frac{dx}{dt} = 4t + 3$$

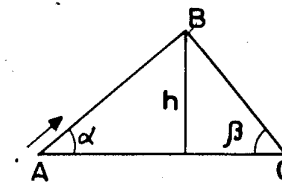
Therefore, at $t = 0$,
 $V = 3$ units

$$\text{Distance covered in 5 s} = 50 + 15 + 5 = 70 \text{ units}$$

$$\text{Distance covered in 4 s} = 32 + 12 + 5 = 49 \text{ units}$$

$\therefore$ Distance covered in 5th second = 21 units.

2. Two inclined planes are placed as shown in the figure. A particle is projected from the foot of the plane of angle α along its line with a velocity just sufficient to carry it to the top after which the particle slides down the other inclined plane. Find the total time it will take to reach the point C . (Assume that there is no friction)



Solution

Let V be the velocity of projection, t the time to reach B , and t' the time to reach from B to C .

$$AB = h \operatorname{cosec} \alpha$$

$$0 - V^2 = -2g (\sin \alpha) h \operatorname{cosec} \alpha$$

$$V = \sqrt{2gh}$$

$$t = \frac{\sqrt{2h}}{\sqrt{g}} \cdot \frac{1}{\sin \alpha}$$

$$t' = \frac{\sqrt{2h}}{\sqrt{g}} \cdot \frac{1}{\sin \beta}$$

$$\therefore \text{Total time} = \sqrt{\frac{2h}{g}} \left[\frac{1}{\sin \alpha} + \frac{1}{\sin \beta} \right]$$

3. A particle travels along a path given by

$$\vec{r} = 4 \cos \pi t \hat{i} + 6 \sin \pi t \hat{j} + 2t \hat{k}$$

Find the tangential component of acceleration and velocity when $t = 2.5$ s.

Solution

$$\hat{r} \text{ at } t = 2.5 \text{ s} = \frac{6\hat{j} + 5\hat{k}}{\sqrt{36 + 25}} = \frac{6\hat{j} + 5\hat{k}}{\sqrt{61}}$$

$$\vec{V} = \frac{d\vec{r}}{dt} = -4\pi \sin \pi t \hat{i} + 6\pi \cos \pi t \hat{j} + 2\hat{k}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = -4\pi^2 \cos \pi t \hat{i} - 6\pi^2 \sin \pi t \hat{j}$$

$$\vec{V} \text{ at } t = 2.5 \text{ s} = -4\pi \hat{i} + 2\hat{k} \text{ and } |\vec{V}|^2 = 16\pi^2 + 4 = 162$$

At $t = 2.5$,

$$\begin{aligned} \vec{a} &= -4\pi^2 \cos 2.5\pi \hat{i} - 6\pi^2 \sin 2.5\pi \hat{j} \\ &= 0 - 6 \times \pi^2 \hat{j} = -59.2 \hat{j} \end{aligned}$$

The unit vector along the velocity is $\frac{\vec{V}}{|\vec{V}|}$

Hence, the tangential component of the acceleration is

$$\begin{aligned} a_T &= \vec{a} \cdot \frac{\vec{V}}{|\vec{V}|} \\ &= \frac{(-59.2 \hat{j}) \cdot (-4\pi \hat{i} + 2\hat{k})}{\sqrt{162}} \\ &= 0 \end{aligned}$$

4. A proton of mass m kg moves along a horizontal path in vacuum at a speed V m/s and enters a uniform electric field that exerts a downward force F newton on it. The field extends over a region L m long. Through what angle is the proton deflected (neglect gravity).

Solution

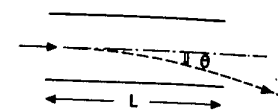
$$\vec{a} = \frac{-F}{m} \hat{j}$$

$$\vec{V} = \frac{-F}{m} t \hat{j} + V_x \hat{i}$$

$$V_y = -\left(\frac{F}{m}\right)t, V = V_x$$

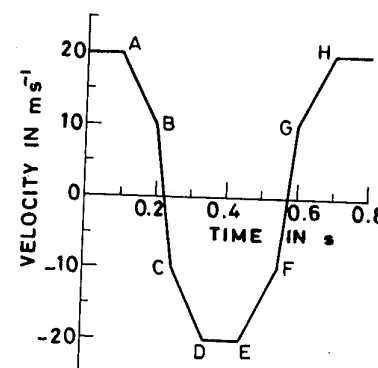
$$\therefore \tan \theta = \frac{-V_y}{V_x} = \frac{\left(\frac{F}{m}\right)t}{V} = \frac{F \cdot t}{mV} = \frac{F \cdot L}{mV^2}$$

$$\text{or } \theta = \tan^{-1}\left(\frac{FL}{mV^2}\right)$$



5. The velocity of a particle of mass 2 g is plotted as a function of time as shown in the figure. From the direct measurements in the graph, plot the force on the particle as a function of time and calculate

(a) maximum force (b) average force in time interval between 0 and 0.4 s.



Solution

To plot a graph between force and time, obtain the following data points from the given figure.

Time interval	Force
0 to 0.1 s	$F = 0$
0.1 to 0.2 s	$F = (0.002) \times \left(-\frac{10}{0.1}\right) = -0.2 \text{ N}$
0.2 to 0.25 s	$F = (0.002) \times \left(-\frac{20}{0.05}\right) = -0.8 \text{ N}$
0.25 to 0.35 s	$F = (0.002) \times \left(-\frac{10}{0.1}\right) = -0.2 \text{ N}$
0.35 to 0.45 s	$F = 0$
0.45 to 0.55 s	$F = (0.002) \times \left(\frac{10}{0.1}\right) = 0.2 \text{ N}$

0.55 to 0.6 s

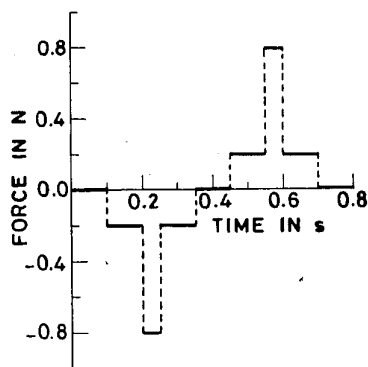
$$F = (0.002) \times \left(\frac{20}{0.05} \right) = 0.8 \text{ N}$$

0.6 to 0.7 s

$$F = (0.002) \times \left(\frac{10}{0.1} \right) = 0.2 \text{ N}$$

0.7 to 0.8 s

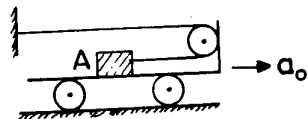
$$F = 0$$



The plot of F versus t is shown in the figure here. The maximum force is shown to be 0.8 N. Average force in the time interval 0 to 0.4 s is

$$\begin{aligned} & \frac{0.1 \times 0 + 0.1 \times (-0.8) + 0.05(-0.8) + 0.1(-0.2) + 0.05 \times 0}{0.4} \\ &= \frac{-0.02 - 0.04 - 0.02}{0.4} \\ &= \frac{-0.08}{0.4} = -0.2 \text{ N} \end{aligned}$$

6. A flat car is given an acceleration $a_0 = 2 \text{ m/s}^2$ starting from rest. A cable is connected to a crate A of weight 50 kg as shown. Neglect friction between the floor and the car wheels and also the mass of the pulley. Calculate corresponding tension in the cable if $\mu = 0.30$ between the crate and the floor of the car.



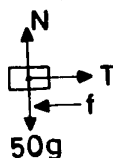
Solution

Referring to the free-body diagram, we have

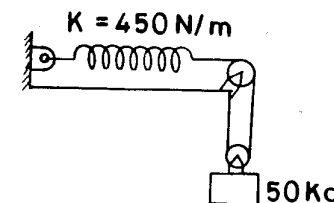
$$f = \mu N = 0.3 \times 50 \times 10 = 150 \text{ N} \quad [g = 10 \text{ m/s}^2]$$

$$a = a_0 + a_{\text{rel}} = 2 a_0 = 4 \text{ m/s}^2$$

$$\therefore T - 150 = 50 \times 4 \Rightarrow T = 350 \text{ N}$$



7. A block of mass 50 kg is released from the position of rest for which the spring is under a tension of 45 N. If the pulleys are massless and the block goes 150 mm down, find the velocity of the block at this instant.



Solution

$$F = KX = 450 X$$

$$45 \text{ N} = 450 X \Rightarrow X = 0.1 \text{ m}$$

$$V = \Delta K, \text{ change in kinetic energy;}$$

$$\therefore - \int_{y_1}^{y_2} 2F \cdot dy + 50(10)(y_2 - y_1) = \frac{1}{2} \times 50 \times (V^2 - 0)$$

(y_1 and y_2 correspond to the position of rest and when the block goes 150 mm down)

$$y_1 = \frac{X}{2} = \frac{0.1}{2} = 0.05 \text{ m}$$

$$y_2 = 0.150 + 0.05 = 0.20 \text{ m}$$

$$2F = 2 \times K \cdot 2y \text{ (where } y = X/2)$$

$$= 2 \times 450 \times (2y) = 4 \times 450y$$

$$\therefore -4 \times 450 \int_{0.05}^{0.2} y \, dy + 50 \times 10 \times 0.15 = \frac{1}{2} \times 50 \times V^2$$

$$\Rightarrow -1800 \left[\frac{y^2}{2} \right]_{0.05}^{0.20} + 75 = 25 V^2$$

$$\Rightarrow -900 [(0.20)^2 - (0.05)^2] + 75 = 25 V^2$$

$$\Rightarrow -900 [0.0375] + 75 = 25 V^2$$

$$\Rightarrow -33.75 + 75 = 25 V^2 \Rightarrow V^2 = 1.65 \Rightarrow V = 1.28 \text{ m/s.}$$

8. A car of mass m accelerates on a level road under the action of a driving force F from a speed V_1 to a higher speed V_2 in a distance s . If the engine develops a constant power output P , determine V_2 .

Solution

$$F = m \cdot a; \quad P = F \cdot V = ma \cdot V \Rightarrow a = \frac{P}{mV}$$

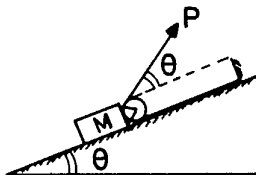
$$V \cdot dV = a \cdot ds; \quad mV^2 dV = P \cdot ds$$

$$\text{or} \quad \int_{V_1}^{V_2} mV^2 dV = \int_0^s P \cdot ds = P \cdot s$$

$$\text{or} \quad m \left[\frac{V^3}{3} \right]_{V_1}^{V_2} = P \cdot s$$

$$\frac{m}{3} [V_2^3 - V_1^3] = Ps \Rightarrow V_2 = \left(\frac{3Ps}{m} + V_1^3 \right)^{1/3}$$

9. A block of mass M is pulled by a force P by means of a pulley-string system as shown in the figure. Find the value of P if the block is to have an acceleration up the incline (neglect friction).



Solution

From the free-body diagram, we have

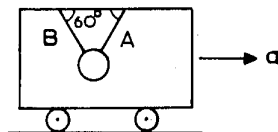
$$\begin{aligned} \sum F_y &= 0 \\ N + P \sin \theta - Mg \cos \theta &= 0 \end{aligned} \quad (1)$$

$$\begin{aligned} \sum F_x &= ma_x \\ P + P \cos \theta - Mg \sin \theta &= Ma \end{aligned} \quad (2)$$

$$a = \frac{1}{M} (P + P \cos \theta - Mg \sin \theta) \quad \text{and} \quad P = \frac{Mg \sin \theta}{1 + \cos \theta}$$

If friction is included, use equation 1 also to get the value of acceleration.

10. A steel ball is suspended from the ceiling of an accelerating carriage by means of two cords A and B. Determine the acceleration a of the carriage which will cause the tension in A to be twice that in B.



Solution

When the carriage is not accelerating the tension is T in both A and B.

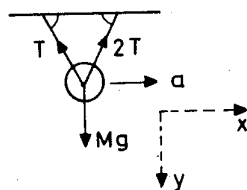
$$\begin{aligned} \sum F_x &= ma_x \\ 2T \sin 30 - T \sin 30 &= ma \end{aligned} \quad (i)$$

$$\begin{aligned} \sum F_y &= 0 \\ 2T \cos 30 + T \cos 30 - mg &= 0 \end{aligned} \quad (ii)$$

$$\text{or} \quad 3T \cos 30 = mg \Rightarrow T = \frac{mg}{3 \cos 30}$$

$$\therefore 2 \times \frac{mg}{3 \cos 30} \sin 30 - \frac{mg}{3 \cos 30} \sin 30 = ma$$

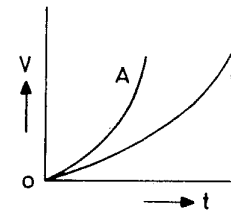
$$\frac{2}{3} g \tan 30 - \frac{1}{3} g \tan 30 = a \Rightarrow a = \frac{1}{3} g \tan 30 = \frac{g}{3\sqrt{3}}$$



EXERCISES

1. A body dropped from a certain height attains the same velocity as another falling with an initial velocity u from a height h below the first body. If g is the acceleration due to gravity, the square of the velocity is given by (A) gh (b) $4gh$ (c) $2gh$ (d) $gh/2$

2. A time-velocity graph of two vehicles A and B starting from rest at the same time is given in the figure. The statement that can be deduced correctly from the graph is

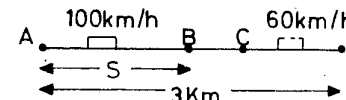


- (a) acceleration of A is greater than that of B
- (b) acceleration of B is greater than that of A
- (c) acceleration of A is increasing at a slower rate than that of B
- (d) velocity of B is greater than that of A

3. The magnitudes of the acceleration and deceleration of an express elevator are limited to $0.4g$ and the maximum vertical speed is 400 m/min . The minimum time t required for the elevator to go from rest at the 10th floor to a stop at the 30th floor, a distance of 100 m , is

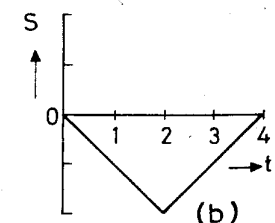
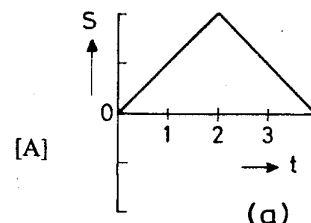
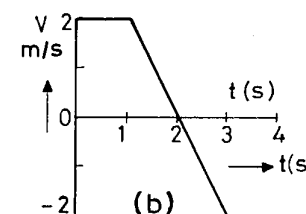
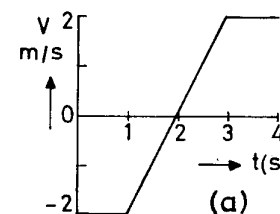
- (a) 1.67 s (b) 16.70 s
- (c) 167 s (d) 1670 s

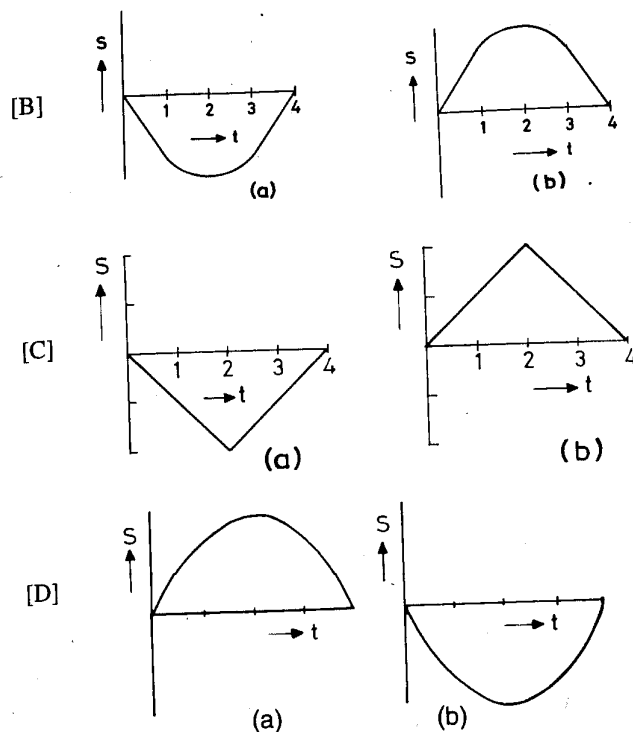
4. In travelling a distance of 3 km between points A and D, a car is driven at 100 km/h from A to B for t seconds and at 60 km/h from C to D for $t \text{ s}$. If the brakes are applied for 4 s between B and C to give the car a uniform deceleration, the value of t is



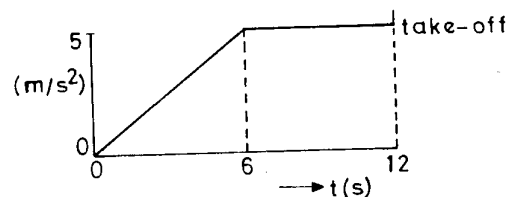
- (a) 655 s (b) 565 s
- (c) 65.5 s (d) 56.5 s

5. The $V-t$ curves for two different particle motions are shown. The corresponding $S-t$ curves, taking $S = 0$ when $t = 0$ for both instances will be

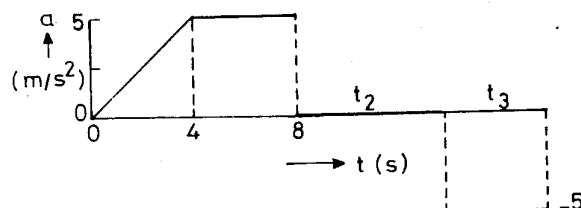




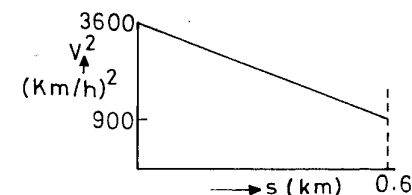
6. An experiment on the take-off performance of an aeroplane shows that the acceleration varies as shown in the figure, and that it takes 12 s to take-off from a rest position. The distance along the runway covered by the aeroplane is



- (a) 210 m (b) 2100 m
(c) 21000 m (d) 1200 m
7. The acceleration of a train between two stations 2 km apart is shown in the figure. The maximum speed of the train is



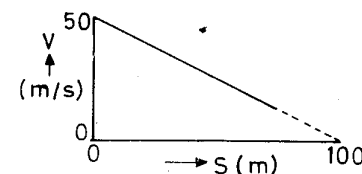
- (a) 60 m/s (b) 120 m/s
(c) 30 m/s (d) 90 m/s
8. The acceleration is constant when the relationship between
(a) the position coordinate s and the square of the velocity v is linear
(b) the position coordinate s and velocity v is linear
(c) the position coordinate s and the reciprocal of the velocity is linear
(d) the square of the position coordinate s and the velocity v is linear
9. A graph between the square of the velocity of a particle and the distance s moved is shown in the figure. The acceleration of the particle in kilometres per hour squared is



- (a) 2250 (b) 22.5
(c) -2250 (d) 225
10. A self-propelled vehicle of mass m whose engine delivers constant power P has an acceleration $a = P/mv$ (assume that there is no friction). In order to increase its velocity from v_1 to v_2 , the distance it has to travel will be

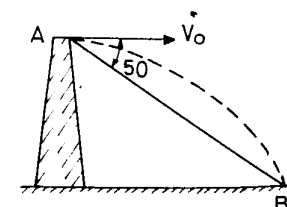
- (a) $\frac{3P}{m}(v_2^2 - v_1^2)$ (b) $\frac{m}{3P}(v_2^3 - v_1^3)$
(c) $\frac{m}{3P}(v_2^2 - v_1^2)$ (d) $\frac{m}{3P}(v_2 - v_1)$

11. If the velocity v of a particle moving along a straight line decreases linearly with its position coordinates s from 50 m/s to a value approaching zero at $s = 100$ m, the time it takes to reach the 100 m position will be



- (a) 10 s
(b) 5 s
(c) Infinity
(d) 0.5 s

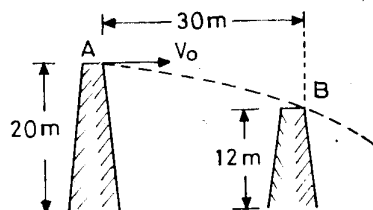
12. An object is thrown horizontally from a tower at A and hits the ground 3.5 s later at B. The line of sight from A to B makes an angle of 50° with the horizontal. The initial velocity of the object is



- (a) 144 m/s
(b) 14.41 m/s
(c) 7.2 m/s
(d) 28.8 m/s

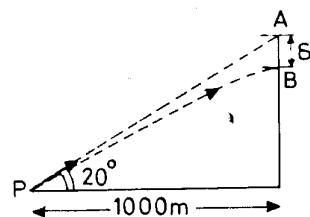
13. A boy wants to throw a ball from a point A so as to just clear the obstruction at B. The horizontal velocity with which the boy should throw the ball is

(a) 23.5 m/s
(b) 235 m/s
(c) 47 m/s
(d) 32.5 m/s



14. A shooter aims his rifle at an angle of 20° with the horizontal to hit an object at A but the bullet hits at the point B, δ below A. If the initial velocity of the bullet is 600 m/s, the value of δ is

(a) 1543 m
(b) 54.3 m
(c) 154 m
(d) 15.43 m

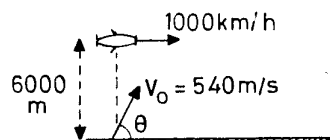


15. A particle moves along the positive branch of the curve $Y = X^2/2$ with X governed by $X = t^2/2$ where X and Y are measured in metres and t in seconds. At $t = 2$ s, the velocity of the particle is

(a) $2\hat{i} - 4\hat{j}$ m/s
(b) $2\hat{i} + 4\hat{j}$ m/s
(c) $4\hat{i} + 2\hat{j}$ m/s
(d) $4\hat{i} - 2\hat{j}$ m/s

16. An aircraft moving with a speed of 1000 km/h is at a height of 6000 m, just overhead of an anti-aircraft gun. If the muzzle velocity is 540 m/s, the firing angle θ should be

(a) 73°
(b) 30°
(c) 59°
(d) 45°



17. A bomber flying with a horizontal velocity of 500 km/h at an altitude of 5 km wants to hit a train moving with a constant velocity of 100 km/h in the same direction and in the same vertical plane. The angle θ between the line of sight of the target and the horizontal at the instant the bomb should be released is

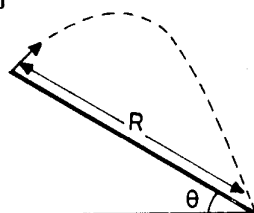
(a) 54.6°
(b) 56.4°
(c) 72.0°
(d) 27.4°

18. A particle moves along the parabolic path $y = ax^2$ in such a way that the x -component of the velocity remains constant, say c . The acceleration of the particle is

(a) $ac\hat{k}$
(b) $2ac^2\hat{j}$
(c) $ac^2\hat{k}$
(d) $a^2c\hat{j}$

19. A projectile is fired with a velocity V at right angle to the slope which is inclined at an angle θ with the horizontal. The expression for R is

(a) $\frac{2V^2}{g} \tan \theta \sec \theta$



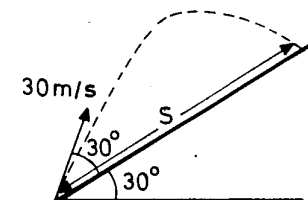
(b) $\frac{V^2}{g} \tan^2 \theta$

(c) $\frac{2V^2}{g} \tan \theta$

(d) $\frac{2V^2}{g} \sec \theta$

20. An object is projected up the incline at the angle shown in figure with an initial velocity of 30 m/s. The distance S up the incline at which the object lands is

(a) 612 m
(b) 61.2 m
(c) 6.12 m
(d) 122.4 m



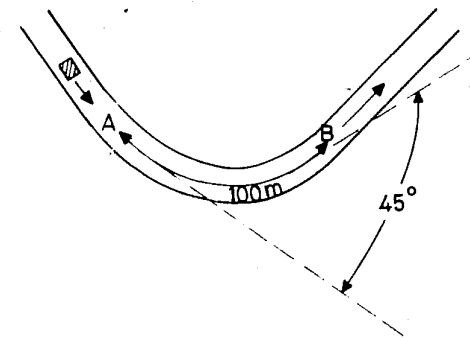
21. A racing car travelling at a constant speed has to pass through a horizontal turn where the radius of curvature of the road is 300 m. If the normal acceleration of the car cannot exceed $0.8g$, the maximum speed of the car without sliding can be

(a) 17.4 km/h
(b) 200 km/h
(c) 87.3 km/h
(d) 174.7 km/h

22. The muzzle velocity for a certain rifle is 600 m/s. If the rifle is pointed vertically upward and fired from an automobile moving horizontally at a speed of 72 km/h, the radius of curvature of the path of the bullet at maximum altitude is (neglect friction of the air)

(a) 408 m
(b) 204 m
(c) 40.8 m
(d) 144 m

23. A car moves round a turn of constant curvature between A and B (curve AB = 100 m) with a steady speed of 72 km/h. If an accelerometer were mounted in the car, the magnitude of acceleration it would record between A and B is



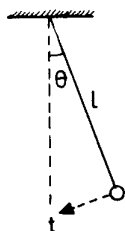
(a) zero
(b) 3.14 m/s^2
(c) 31.4 m/s^2
(d) 6.28 m/s^2

24. A point on the rim of a flywheel has a peripheral speed of 10 m/s at an instant when it is decreasing at the rate of 60 m/s^2 . If the magnitude of the total acceleration of the point at this instant is 100 m/s^2 , the radius of the flywheel is

- (a) 1.25 m (b) 12.5 m
(c) 25 m (d) 2.5 m

25. The tangential acceleration of the bob of a simple pendulum is $a_t = g \sin \theta$. For the simple pendulum shown, $l = 200$ mm, when $\theta = 30^\circ$, $d\theta/dt = -9$ rad/s. The magnitude of the total acceleration of the bob for this position is

- (a) 9.81 m/s^2
(b) 16.93 m/s^2
(c) 18.82 m/s^2
(d) 8.46 m/s^2



26. The speed of a body moving in a straight line changes from 25 m/s to 10 m/s in 3 s at a constant rate.
(a) during this time it travels 52.5 m
(b) its acceleration is 5 m/s^2
(c) its acceleration is greatest at the beginning because it is going fastest at that time
(d) the distance travelled during an interval of 3 s cannot be calculated from the given data
27. The velocity of a particle moving in the x-y plane is given by

$$\frac{dx}{dt} = 8\pi \sin 2\pi t; \quad \frac{dy}{dt} = 5\pi \cos 2\pi t$$

when $t = 0$, $x = 8$ and $y = 0$. The path of the particle is

- (a) a straight line (b) a circle
(c) an ellipse (d) a parabola
28. The velocity of a car travelling on a straight road is given by the equation $v = 6 + 8t - t^2$ where v is in metres per second and t in seconds. The instantaneous acceleration when $t = 4.5$ s is
(a) 0.1 m/s^2 (b) 1 m/s^2
(c) -1 m/s^2 (d) -0.1 m/s^2

29. A particle moves along a horizontal straight line with a velocity-time relationship as shown in figure. The total distance moved by the particle is

- (a) 39 m
(b) 13 m
(c) 26 m
(d) 2.6 m

30. When a particle is moving along a circular path with uniform speed, it has

- (a) radial velocity and radial acceleration
(b) radial velocity and transverse acceleration
(c) transverse velocity and radial acceleration
(d) transverse velocity and transverse acceleration

31. Planet A has a mass and radius twice that of Planet B. The escape velocity from Planet A is

- (a) twice that from B (b) four times that from B
(c) equal to that from B (d) half that from B

32. In the case of a satellite moving along a circular orbit, a larger orbit corresponds to

- (a) longer periods and slower velocity
(b) larger velocity and longer periods
(c) smaller periods and smaller velocity
(d) smaller periods and larger velocity

33. The time period of an earth's satellite is

- (a) directly proportional to the radius of the orbit
(b) inversely proportional to the radius of the orbit
(c) directly proportional to the square of the radius
(d) directly proportional to the square root of the cube of the radius

34. A satellite is moving round the earth (radius of earth = R) at a distance r from the centre of the earth. If g is the acceleration due to gravity, the acceleration of the satellite will be

- (a) g (b) $\sqrt{\frac{Rg}{r}}$
(c) $\frac{R^2}{r^2} g$ (d) $\sqrt{\frac{gr}{R}}$

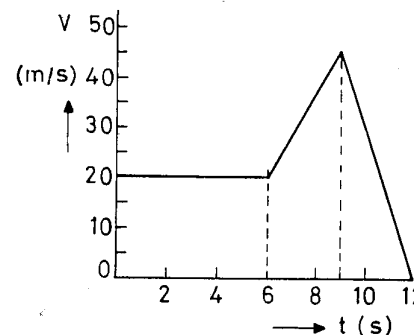
35. An earth satellite revolves in a circular orbit at a height of 300 km above the earth's surface. If the earth's radius is 6,000 km and g is 10 m/s^2 , the speed of the satellite will be very close to

- (a) 7500 m/s (b) 750 m/s
(c) 1500 m/s (d) 15000 m/s

36. For a spherically symmetric earth, the acceleration due to gravity should be about

- (a) 0.3 m/s^2 greater at the equator than at the poles
(b) 0.03 m/s^2 less at the equator than at the poles
(c) the same at the equator and at the poles
(d) 0.03 m/s^2 greater at the equator than at the poles

37. The graph in the figure shows the velocity of a body plotted as a function of time. The distance covered by the body in the first 12 seconds is



- (a) 360 m (b) 240 m
(c) 285 m (d) 500 m

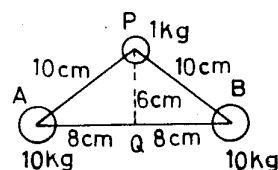
38. The motion of a body falling from rest in a resisting medium is described by the equation

$$\frac{dV}{dt} = A - BV \quad \text{where } A \text{ and } B \text{ are constants}$$

The velocity at any time t is

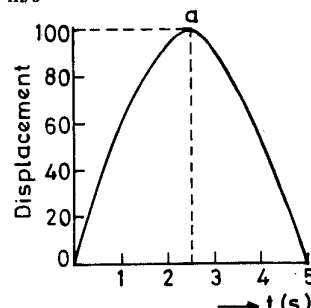
- (a) $A(1 - e^{B^2t})$ (b) $AB \cdot e^{-t}$
 (c) $\frac{A}{B}(1 - e^{-Bt})$ (d) $AB^2(1 - t)$
39. An elevator and its load have a total mass of 800 kg. If the elevator, originally moving downward at 10 m/s, is brought to rest with constant deceleration in a distance of 25 m, the tension in the supporting cable will be
 (a) 94 N (b) 1880 N
 (c) 940 N (d) 9440 N
40. A carriage is moving to the right with an acceleration a . A simple pendulum is hanging from the top of the ceiling, making an angle θ with the vertical. The relation between a and θ is
 (a) $a = g \sin \theta$ (b) $a = g \tan \theta$
 (c) $a = g \cos \theta$ (d) $a = g \sqrt{\tan \theta}$

41. Two spheres, each of mass 10 kg, are fixed at points A and B as shown in figure. A sphere of mass 1 kg is released from rest from the point P and acted upon only by the forces of gravitational attraction of spheres A and B. The magnitude and direction of the initial acceleration of the 1 kg sphere is

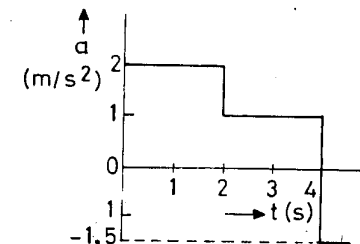


- (a) $G \times 1200 \text{ m/s}^2$ along PQ (b) $G \times 120 \text{ m/s}^2$ along QP
 (c) $G \times 1200 \text{ m/s}^2$ along QP (d) $G \times 120 \text{ m/s}^2$ along PQ
42. The mass of the moon is about $1/81$ and its radius $1/4$ that of the earth. The acceleration due to gravity on the surface of the moon is
 (a) 1.94 m/s^2 (b) 7.36 m/s^2
 (c) 5.82 m/s^2 (d) 3.88 m/s^2
43. A 4 kg block is accelerated upward by a cord whose breaking strength is 60 N. The maximum acceleration that can be given to the block without breaking the cord is
 (a) 10 m/s^2 (b) 15 m/s^2
 (c) 5 m/s^2 (d) 20 m/s^2

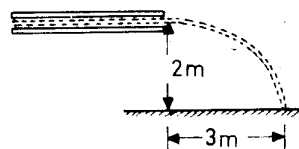
44. The figure shows the displacement-time graph of a body subject only to the force of gravity. This graph indicates that
 (a) at a , the acceleration is zero
 (b) at a , the velocity is maximum
 (c) at a , the displacement is zero
 (d) the acceleration is constant for all times shown



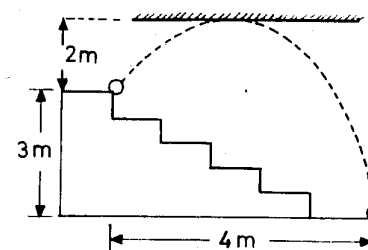
45. An elevator is moving up a shaft at 3 m/s. A bolt falls from the ceiling of the elevator and reaches the bottom of the shaft in 2 s. The distance from the bottom of the shaft to the elevator when the bolt fell was
 (a) 13.6 m (b) 1.36 m
 (c) 6.8 m (d) 4.5 m
46. A ball thrown upward at an angle of 30° to the horizontal lands on the top edge of a building 20 m away. The top edge is 5 m above the throwing point. The ball was thrown with a velocity of
 (a) 40 m/s (b) 60 m/s
 (c) 20 m/s (d) 30 m/s
47. The figure shows the acceleration versus time graph of a train. If it starts from rest, the distance it travels before it comes to rest is
 (a) 30 m (b) 26 m
 (c) 13 m (d) 40 m
48. A car is travelling on a straight road. The maximum velocity the car can attain is 24 m/s. The maximum acceleration and deceleration it can attain are 1.0 m/s^2 and 4 m/s^2 respectively. The shortest time the car takes to start from rest and come to rest in a distance of 200 m is
 (a) 22.4 s (b) 33.6 s
 (c) 11.2 s (d) 5.6 s
49. A car is travelling on a road. The maximum velocity the car can attain is 24 m/s and the maximum deceleration is 4 m/s^2 . If the car starts from rest and comes to rest after travelling 1032 m in the shortest time of 56 s, the maximum acceleration that the car attains is
 (a) 6 m/s^2 (b) 1.2 m/s^2
 (c) 12 m/s^2 (d) 3.6 m/s^2
50. The speed of an aeroplane at the instant it lands on a runway is 60 m/s. If the deceleration of the aeroplane is given as $a = -0.6 - 0.001v^2$, the distance that it covers on the runway before coming to a stop is
 (a) 98 m (b) 1800 m
 (c) 450 m (d) 973 m
51. In a 100 m race, a runner accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance at that velocity. If he finishes the race in 10.4 s, then his maximum velocity was
 (a) 15 m/s (b) 20 m/s
 (c) 10 m/s (d) 25 m/s
52. The nucleus of a helium atom travels along the axis of a straight hollow tube 4.0 m long. The tube is part of a particle accelerator. If the particle enters the tube with a speed of 1000 m/s and leaves at 9000 m/s, and assuming that the acceleration is uniform, the time the particle remains inside the tube is
 (a) $4.0 \times 10^{-3} \text{ s}$ (b) $8.0 \times 10^{-4} \text{ s}$
 (c) $1.0 \times 10^{-4} \text{ s}$ (d) $8.0 \times 10^{-3} \text{ s}$
53. A man sees an object from a 5 ft high window going up and then back down. If the total time the object is in sight is 1.0 s, the height above the window the object reaches is
 (a) $3/16 \text{ ft}$ (b) $1/16 \text{ ft}$
 (c) $3/8 \text{ ft}$ (d) $5/8 \text{ ft}$



54. A helicopter ascending at the rate of 12 m/s drops a food packet from a height of 80 m above the ground. The time the packet takes to reach the ground is
 (a) 5.5 s (b) 16.5 s
 (c) 11.0 s (d) 2.70 s
55. A particle moves in the x - y plane according to the equations $x = 4t^2 + 4t + 5$ and $y = -t^3 + 12t + 3$. At $t = 1$ s, the acceleration of the particle is
 (a) 15 m/s² (b) 5 m/s²
 (c) 20 m/s² (d) 10 m/s²
56. A particle moves in the x - y plane with velocity $V_x = 8t - 2$ and $V_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$ s, the equation of the path is
 (a) $x = y^2 - y + 2$ (b) $x = y + 2$
 (c) $x = y^2 + 2$ (d) $x = y^2 + y - 2$
57. The motion of a particle is defined by $x = a \cos \omega t$ and $y = a \sin \omega t$. The acceleration of the particle is
 (a) $a\omega$ (b) $a^2\omega$
 (c) $-a\omega^2$ (d) $a^2\omega^2$
58. A ball is thrown vertically up with a speed of 20 m/s. It is caught on its way down 5 m above the point from where it was thrown. The time lapse between the throw and the catch is
 (a) 1.9 s (b) 3.8 s
 (c) 8.3 s (d) 1.2 s
59. A ball is thrown from the top of a tower of height 80 m with a horizontal velocity of 30 m/s. The velocity with which it strikes the level ground is
 (a) 100 m/s (b) 50 m/s
 (c) 20 m/s (d) 80 m/s
60. A ball is thrown from the top of a tower with an initial velocity of 20 m/s at an angle of 40° with the horizontal so as to hit a point on a tall building 50 m away from the tower. The ball will hit the building
 (a) 105 m above the original level
 (b) 105 m below the original level
 (c) 10.5 m below the original level
 (d) just at the base of the building
61. A ball is thrown vertically upward with a speed V from a point h metres above the ground. The time taken for the ball to hit the ground is
 (a) $\left(\frac{V}{g}\right)\left[1 + \sqrt{1 + \left(\frac{2hg}{V^2}\right)}\right]$ (b) $\sqrt{1 + \left(\frac{2hg}{V^2}\right)}$
 (c) $\left(\frac{V}{g}\right)\sqrt{1 + \left(\frac{2hg}{V^2}\right)}$ (d) $\left(\frac{V}{g}\right)\sqrt{1 - \left(\frac{2hg}{V^2}\right)}$
62. Water is flowing from a horizontal pipe fixed at a height of 2 m from the ground. If it falls at a distance of 3 m as shown in figure, the speed of water when it leaves the pipe is
 (a) 47 m/s
 (b) 4.7 m/s
 (c) 7.4 m/s
 (d) 14.7 m/s



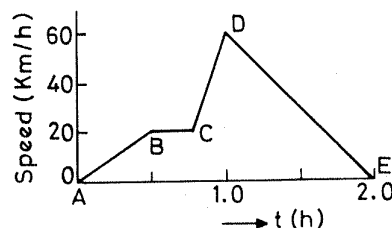
63. A boy throws a ball with a velocity V_0 at an angle α to the ground. At the same instant he starts running with uniform velocity to catch the ball before it hits the ground. To achieve this, he should run with a velocity of
 (a) $V_0 \cos \alpha$ (b) $V_0 \sin \alpha$
 (c) $V_0 \tan \alpha$ (d) $\sqrt{V_0^2 \tan \alpha}$
64. A ball is thrown from the top of a staircase which just touches the ceiling and finally hits the bottom of the steps. The initial speed of the ball is
 (a) 67 m/s
 (b) 76 m/s
 (c) 7.6 m/s
 (d) 6.7 m/s
65. A proton in a cyclotron moves in a circle of radius 0.80 m at a speed of 10^7 m/s. The acceleration of the proton and acceleration due to gravity have a ratio of approximately
 (a) 10^{10} (b) 10^{11}
 (c) 10^{13} (d) 10^{-13}
66. The rotor of a turbine rotates at the rate of 2000 rpm. If the diameter of the rotor is 5 m, the centripetal acceleration at the edge of the rotor is
 (a) 11×10^5 m/s² (b) 1.1×10^5 m/s²
 (c) 2.2×10^4 m/s² (d) 2.2×10^5 m/s²
67. An object of mass m is supported by a cord and the tension in the cord is recorded as 0.25 mg. The conclusion that can be drawn is
 (a) the object is at rest
 (b) it is moving with a constant velocity
 (c) it is accelerating upward with an acceleration $3g/2$
 (d) it is accelerating downward with an acceleration $0.75g$
68. The brakes of a car moving at 20 m/s along a horizontal road are suddenly applied, and it comes to rest after travelling some distance. If the coefficient of friction between the tyres and the road is 0.90 and it is assumed that all four tyres behave identically, the shortest distance the car would travel before coming to a stop is
 (a) 2.27 m (b) 11.35 m
 (c) 22.7 m (d) 4.54 m
69. Shots are dropping from a nozzle on a plane cushioned surface 4.05 m below. The shots fall at regular intervals of time, the first shot striking the cushion at the instant the fourth shot begins to fall. The location of the third shot from the nozzle end when the first shot hits the cushion is
 (a) 5.4 m below the nozzle (b) 0.45 m below the nozzle
 (c) 3.5 m below the nozzle (d) 5.3 m below the nozzle
70. A lead ball is dropped into a lake from the diving board 5 m above the water. It hits the water with a certain velocity and then sinks to the bottom with this same constant velocity. It reaches the bottom 5.0 s after it is dropped. The depth of the lake is
 (a) 25 m (b) 20 m
 (c) 40 m (d) 10 m



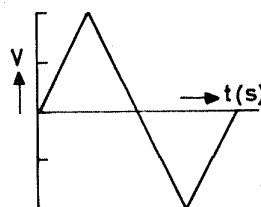
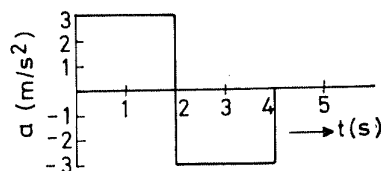
44 Problems in Physics

71. A train moves from one station to another in 2 hours, and its speed during the motion is shown in the graph. The maximum acceleration during the journey is

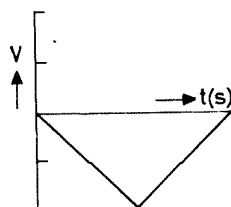
(a) 80 km/h^2
 (b) 160 km/h^2
 (c) 40 km/h^2
 (d) 60 km/h^2



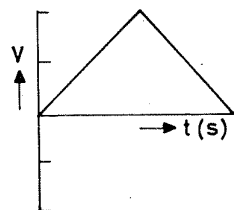
72. A particle starts from rest at time $t = 0$ and undergoes acceleration a as shown. The velocity as function of time during the interval 0 to 4 s is indicated in



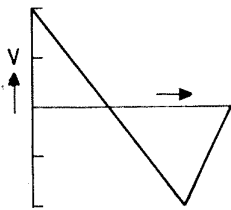
(a)



(b)



(c)



(d)

73. A body with constant acceleration travels 2 m in the first 2 seconds and 2.2 in the next 4 seconds. The velocity at the end of the seventh second from the start shall be

(a) 1 m/s
 (b) 0.1 m/s
 (c) 0.2 m/s
 (d) 0.5 m/s

74. A car covers the first half of the distance between two places at a speed of 40 km/h and the second half at 60 km/h. The average speed of the car is

(a) 50 km/h
 (b) 42 km/h
 (c) 35 km/h
 (d) 48 km/h

75. A stone is dropped from the top of a tower and one second later, a second stone is thrown vertically downward with a velocity of 20 m/s. The second stone will overtake the first one at a distance from the top that will be equal to

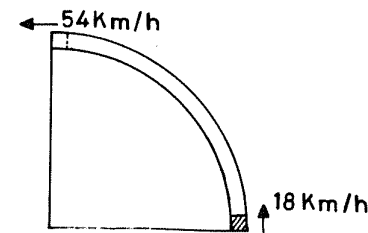
(a) 13.0 m
 (b) 31.0 m
 (c) 26.0 m
 (d) 19.5 m

76. A particle is moving along a circular path of radius 3 m in such a way that the distance travelled measured along the circumference is given by $S = (t^2/2) + (t^3/3)$. The acceleration of the particle when $t = 2 \text{ s}$ is

(a) 1.3 m/s^2
 (b) 13 m/s^2
 (c) 3 m/s^2
 (d) 10 m/s^2

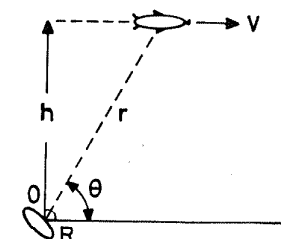
77. A car enters a curved road in the form of a quarter of a circle, the path length being 200 m. Its speed at the entrance is 18 km/h but when it leaves, it increases to 54 km/h. If the car is travelling with constant acceleration along the curve, the acceleration when the car leaves the curved road is

(a) 0.92 m/s^2
 (b) 0.54 m/s^2
 (c) 1.84 m/s^2
 (d) 2.76 m/s^2



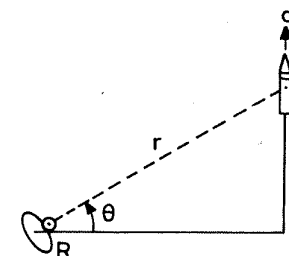
78. A jet plane flying at a constant velocity V at a height $h = 8 \text{ km}$ is being tracked by a radar R located at O directly below the line of flight. If the angle θ is decreasing at the rate of 0.025 rad/s , the velocity of the plane when $\theta = 60^\circ$ is

(a) 1440 km/h
 (b) 960 km/h
 (c) 1920 km/h
 (d) 480 km/h



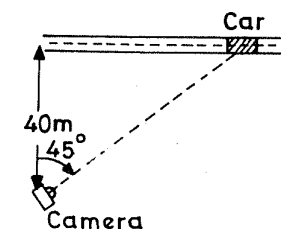
79. A rocket is fired vertically and tracked by the radar R as shown in the figure. At a particular angular position $\theta = 60^\circ$, measured parameters are $r = 9 \text{ km}$, $d^2r/dt^2 = 21 \text{ m/s}^2$ and $d\theta/dt = 0.02 \text{ rad/s}$. The acceleration of the rocket at this position is

(a) 20 m/s^2
 (b) 10 m/s^2
 (c) 2 m/s^2
 (d) 200 m/s^2



80. A racing car is travelling along a track at a constant speed of 40 m/s. A television cameraman is recording the event from a distance of 40 m directly away from the track as shown in figure. In order to keep the car under view, the camera is rotated with a certain angular velocity. This angular velocity should be

(a) 1.0 rad/s
 (b) 5 rad/s
 (c) 0.5 rad/s
 (d) 2.0 rad/s

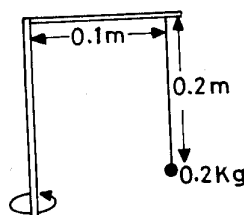


81. A particle is moving along a path given by $r = ae^{b\theta}$ where a and b are constants. The radial acceleration of the particle is

(a) $b^2\omega^2r + r\omega^2$
 (b) $r\omega^2 - b^2r$
 (c) $r\omega^2$
 (d) $b^2\omega^2r - r\omega^2$

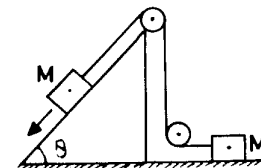
46 Problems in Physics

82. A car is moving along a path in such a way that $r = 2t^2$ and $\theta = t^2/2$. The acceleration of the car at $t = 1$ s is
 (a) 102 m/s^2 (b) 20.4 m/s^2
 (c) 5.1 m/s^2 (d) 10.2 m/s^2
83. The compass of an aircraft indicates that it is headed due north and its airspeed indicator shows that it is moving through the air at 120 miles/h. If there is a wind of 50 miles/h from west to east, the velocity of the aircraft relative to the earth is
 (a) 65 miles/h (b) 130 miles/h
 (c) 60 miles/h (d) 85 miles/h
84. A stone weighing 10 kg is dropped from a cliff in a high wind. The wind exerts a steady horizontal force of 50 N on the stone as it falls. The path that the stone follows will be
 (a) a parabola (b) an ellipse
 (c) a more complicated path (d) a straight line

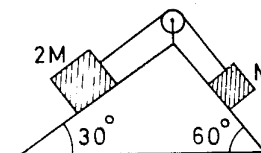
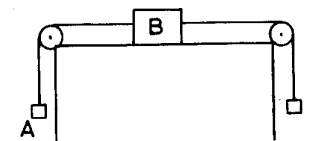


85. A rotating system as shown in the figure has a mass of 0.2 kg hanging from a cord of length 0.2 m. The length of the horizontal arm of the rotating apparatus is 0.1 m. For the cord to make an angle of 45° with the vertical, the number of revolutions per second that the system should make will be
 (a) 2.6 (b) 4
 (c) 6.68 (d) 1.33
86. A motorboat is observed to travel at 10 miles/h relative to the earth in the direction 37° north of east. If the velocity of boat due to the wind is 2 miles/h eastward and that due to the current is 4 miles/h southward, the magnitude and direction of the velocity of the boat due to its own power will be
 (a) 23.2 miles/h 36° N of E (b) 11.6 miles/h 59° N of E
 (c) 23.2 miles/h 36° E of S (d) 11.6 miles/h 59° E of S
87. If the rate of revolution of the earth were such that $g = 0$ at the equator, the length of the day would be
 (a) 1.41 h (b) 14.1 h
 (c) 7 h (d) 3.7 h
88. A man weighing 60 kg stands on a weighing machine fixed in a lift. If the reading in the machine shows 47.7 kg wt, the lift is
 (a) moving downward with a velocity of 2 m/s
 (b) moving upward with a uniform acceleration of 2 m/s^2
 (c) moving downward with a uniform acceleration of 2 m/s^2
 (d) falling freely under gravity
89. A block rests on an inclined plane that makes an angle θ with the horizontal. If the coefficient of sliding friction is 0.50 and that of static friction is 0.75, the time required to slide the block 4 m along the inclined plane is
 (a) 25 s (b) 10 s
 (c) 5 s (d) 2 s
90. A block of mass 2 kg is projected upwards along a 30° incline with an initial velocity of 22 m/s. The coefficient of friction between the block and the plane is 0.3. The block moves up the plane a distance of

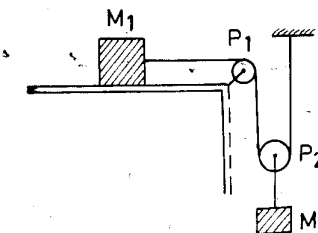
- (a) 6.8 m (b) 8.6 m
 (c) 68 m (d) 86 m
91. A 20 kg block placed on a level frictionless surface is attached to a cord which passes over a small frictionless pulley to a hanging block originally at rest 1 m above the floor. If the hanging block strikes the floor 2 s after the system is released, the weight of the hanging block is
 (a) 10.54 N (b) 5.4 N
 (c) 5.27 N (d) 3.5 N
92. Two blocks, each having a mass M , rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be



- (a) $\frac{2}{3} Mg \sin \theta$ (b) $\frac{3}{2} Mg \sin \theta$
 (c) $\frac{Mg \sin \theta}{2}$ (d) $2 Mg \sin \theta$
93. Block A has a mass of 2 kg and block B 20 kg. If the coefficient of kinetic friction between block B and the horizontal surface is 0.1, and B is accelerating towards the right with $a = 2 \text{ m/s}^2$, then the mass of the block C (see the figure) will be
 (a) 15 kg (b) 12.5 kg
 (c) 5.7 kg (d) 10.5 kg
94. Two blocks of mass M and $2M$ connected by a cord passing over a small frictionless pulley rest on frictionless planes as shown in the figure. On releasing them, the tension in the cord will be ($M = 0.8 \text{ kg}$)
 (a) 4.7 N (b) 5.5 N
 (c) 3.7 N (d) 7.3 N



95. Blocks of mass M_1 and M_2 are connected by a cord which passes over the pulleys P_1 and P_2 as shown in the figure. If there is no friction, the acceleration of the block of mass M_2 will be
 (a) $\frac{M_2 g}{(4M_1 + M_2)}$
 (b) $\frac{2M_2 g}{(4M_1 + M_2)}$
 (c) $\frac{2M_1 g}{(M_1 + 4M_2)}$
 (d) $\frac{2M_1 g}{(M_1 + M_2)}$

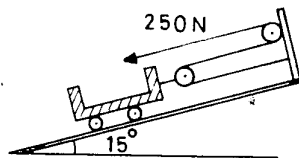


96. The velocities of a block moving on a rough inclined plane were recorded as 20 m/s and 10 m/s at points A and B respectively. If the separation between points A and B is 75 m and the angle of the incline is 15° , the coefficient of friction between the block and the incline is

(a) 0.479 (b) 0.354
(c) 0.140 (d) 0.631

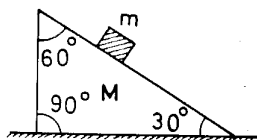
97. A trolley is being self-pulled up an incline plane by a man sitting on it. He applies a force of 250 N. If the combined mass of the man and the trolley is 100 kg, the acceleration of the trolley will be

(a) 2.48 m/s^2
(b) 9.46 m/s^2
(c) 6.94 m/s^2
(d) 4.96 m/s^2



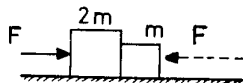
98. A triangular block of mass M rests on a smooth surface as shown in figure. A cubical block of mass m rests on the inclined surface. If all surfaces are frictionless, the force that must be applied to M so as to keep m stationary relative to M is

(a) $Mg \tan 30$ (b) $mg \tan 30$
(c) $(M + m)g \tan 30$ (d) $(M + m)g \cos 30$



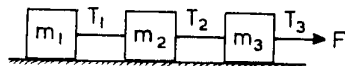
99. Two blocks are in contact on a frictionless table. One has a mass m and the other $2m$. A force F is applied on $2m$ as shown in the figure. Now the same force F is applied from the right on m . In the two cases respectively, the force of contact between the two blocks will be

(a) same (b) 1 : 2
(c) 2 : 1 (d) 1 : 3



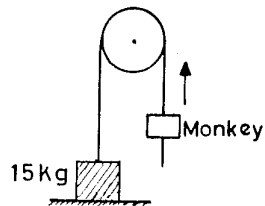
100. Three blocks of mass m_1 , m_2 and m_3 placed on a smooth table are connected with a string and pulled to the right with a force F . If $m_1 = 1 \text{ kg}$, $m_2 = 2 \text{ kg}$, and $m_3 = 3 \text{ kg}$, and $F = 9 \text{ N}$, the tension T_2 will be

(a) 1.5 N (b) 5.4 N
(c) 4.5 N (d) 6 N



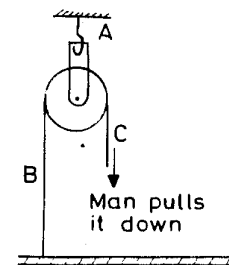
101. A monkey weighing 10 kg is climbing a massless rope and frictionless pulley attached to a 15 kg mass at the end as shown in the figure. In order to raise the 15 kg mass off the ground, the monkey must climb

(a) with a high uniform velocity
(b) with a uniform acceleration g
(c) with an acceleration greater than $g/2$
(d) with an acceleration greater than $g/4$



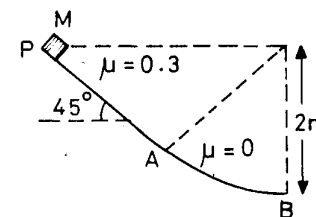
102. A man raises himself and the platform on which he is standing by pulling a rope as shown in the figure. The platform moves up with a uniform acceleration of 3 m/s^2 . The mass of the man and the platform are 70 and 30 kg respectively. The pulley is assumed to be ideal, and the mass of the rope can be neglected. Also assume that the platform does not tilt. The tensions in the ropes A, B and C respectively are

(a) 300, 600, 900 N (b) 300, 300, 1200 N
(c) 640, 640, 1,280 N (d) 320, 320, 640 N



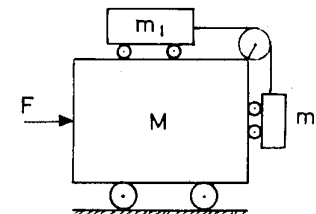
- 103*. A block of mass 20 kg slides down a 45° slide from a point P 2 m above the ground as shown in the figure. The slide has a curvature of radius 2 m starting from point A. The portion from P to A is straight and has a coefficient of friction of 0.3. Assume that there is no friction in the curved portion. The normal reaction experienced by the block when it passes point A will be

(a) 430 N (b) 150 N
(c) 230 N (d) 333 N



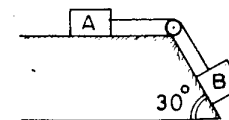
- 104*. A mass m_1 , placed on top of a trolley of mass M , is connected to another mass m_2 by means of a string passing over a smooth pulley as shown in the figure. The friction between the surfaces is negligible. In order that m_1 and m_2 do not move relative to the trolley, the horizontal force F that has to be applied should be

(a) $\frac{m_1 g (M + m_2)}{m_2}$ (b) $\frac{m_2 g (M + m_1 + m_2)}{m_1}$
(c) $\frac{m_2 g (M + m_1)}{m_1 m_2}$ (d) $\frac{m_1 g (M + m_1 + m_2)}{m_2}$



- 105*. Two blocks of identical masses 2 kg each are connected by a cord passing over a smooth pulley as shown in the figure. The coefficient of sliding friction is 0.2. The tension in the cord will be

(a) 5.2 N (b) 2.5 N
(c) 25 N (d) 2.6 N



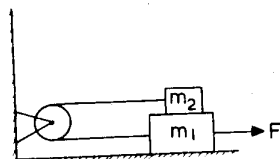
- 106*. A force F accelerates a block of mass m_1 . The coefficient of friction between the contact surfaces is μ . The acceleration of m_1 will be

(a) $\mu g - \frac{F - 2m_1g}{m_1 + m_2}$

(b) $\frac{m_1 + m_2}{F - 2\mu m_1g}$

(c) $\frac{F - 2\mu m_2g}{m_1 + m_2} - \mu g$

(d) $F - (m_1 + m_2) \mu g$



- 107*. In the given figure, $m_1 = 0.30$ kg and $m_2 = 0.50$ kg. If $F = 1.5$ N, and the friction is negligible and the pulley is ideal, then the acceleration of m_2 will be

(a) 1.64 m/s^2

(b) 0.822 m/s^2

(c) 0.411 m/s^2

(d) 3.28 m/s^2

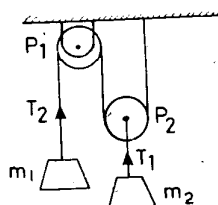
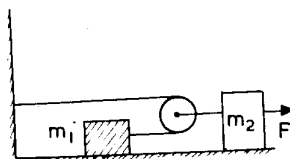
108. In the given figure, $m_1 = 20$ kg and $m_2 = 30$ kg. The pulleys are ideal and P_1 has a stationary axle but P_2 is free to move up and down. The tensions T_1 and T_2 respectively are

(a) 300 and 600 N

(b) 150 and 300 N

(c) 327 and 164 N

(d) 164 and 327 N



109. A horizontal force F is exerted on a 20 kg block to push it up an inclined plane having an incline of 30° . The frictional force retarding the motion is of 80 N. For the acceleration of the moving block to be zero, the force F must be

(a) 206 N

(b) 602 N

(c) 620 N

(d) 260 N

110. A horizontal force of 200 N is required to slide a 15 kg block up a 20° incline with an acceleration of 25 cm/s^2 . The coefficient of friction between the block and the inclined surface is

(a) 0.20

(b) 0.35

(c) 0.65

(d) 0.40

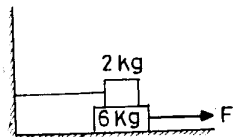
111. With reference to the figure shown, if the coefficient of friction at the surfaces is 0.42, then the force required to pull out the 6.0 kg block with an acceleration of 1.50 m/s^2 will be

(a) 36 N

(b) 24 N

(c) 84 N

(d) 51 N



112. Three blocks m_1 , m_2 and m_3 of masses 1.5 kg, 2.0 kg and 1.0 kg respectively are placed on a rough surface ($\mu = 0.20$) as shown in

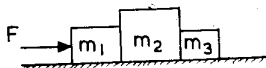


figure. If a force F is applied so as to give the blocks an acceleration of 3.0 m/s^2 , then the force that the 1.50 kg block exerts on the 2.0 kg block will be approximately

(a) 7 N

(b) 15 N

(c) 21 N

(d) 5 N

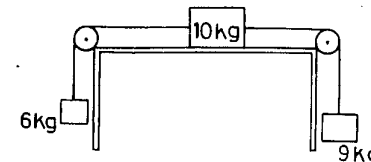
113. Three blocks of masses 6 kg, 9 kg and 10 kg are connected as shown. The coefficient of friction between the table and 10 kg block is 0.2. The tensions in the left and right cord respectively are

(a) 61 N and 85 N

(b) 20 N and 27 N

(c) 16 N and 58 N

(d) 85 N and 61 N



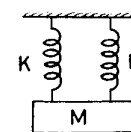
114. The free ends of two identical springs have a mass M attached to them as shown in the figure. If M is slightly displaced and released, the time period of its oscillation will be

(a) $2\pi \sqrt{\frac{M}{K}}$

(b) $2\pi \sqrt{\frac{K}{2M}}$

(c) $2\pi \sqrt{\frac{2K}{M}}$

(d) $2\pi \sqrt{\frac{M}{2K}}$



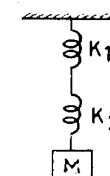
115. Two springs of force constants K_1 and K_2 are connected as shown in the figure and carry a mass M at the free end. If it is allowed to oscillate, its time period will be

(a) $2\pi \sqrt{\frac{M}{K_1 + K_2}}$

(b) $2\pi \sqrt{\frac{M(K_1 + K_2)}{K_1 K_2}}$

(c) $2\pi \sqrt{\frac{K_1 + K_2}{M K_1 K_2}}$

(d) $2\pi \sqrt{\frac{M K_1 K_2}{K_1 + K_2}}$



116. The time period of a planet round the sun is
- directly proportional to the mass of the planet and inversely proportional to the cube of its distance of separation between them
 - directly proportional to the square root of the cube of the distance of its separation and inversely proportional to the square root of the mass of the sun.
 - directly proportional to the square root of the distance of their separation and inversely proportional to the square root of the mass of the sun
 - directly proportional to the square root of the mass of the sun and inversely proportional to the square root of the cube of the distance of separation between them
117. The correct statement among the following is:

- (a) A planet will have a shorter year if it is nearer to the sun than when at a distance from it
 (b) a planet will have a shorter year if it is farther from the sun than when it is closer
 (c) The duration of a year remains unchanged but the durations of day and night change if the planet is closer to the sun
 (d) The duration of the year cannot be determined unless the mass of the planet is known

118. If the earth is at half its present distance from the sun, the number of days in a year will be around

- (a) 265 (b) 165
 (c) 230 (d) 130

119. A pendulum hanging from the ceiling of a railway carriage acts as an indicator of its acceleration. The pendulum makes an angle of 30° with the vertical when the carriage is accelerating on the straight track. If g is the acceleration due to gravity, the acceleration of the carriage is

- (a) $\left(\frac{\sqrt{3}}{2}\right)g$ (b) $\left(\frac{2}{\sqrt{3}}\right)g$
 (c) $g\sqrt{3}$ (d) $\frac{g}{\sqrt{3}}$

120. Potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as

$$u(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

where a and b are constants and x is the separation between the atoms. The value of x for which the molecule will be in stable equilibrium is

- (a) $\left(\frac{7a}{3b}\right)^{-1/6}$ (b) $\left(\frac{3a}{7b}\right)^{-1/6}$
 (c) $\left(\frac{a}{2b}\right)^{1/6}$ (d) $\left(\frac{2a}{b}\right)^{1/6}$

121. As per the data given in above problem, the force between the two atoms of the molecule will be

- (a) $\frac{6a}{x^{13}} - \frac{12b}{x^7}$ (b) $\frac{12a}{x^{13}} - \frac{6b}{x^7}$
 (c) $\frac{6a}{x^7} - \frac{12b}{x^{13}}$ (d) $\frac{12a}{x^7} - \frac{6b}{x^{13}}$

122. The potential describing the interaction between the nucleons is given by

$$u(r) = -\frac{r_0}{r} u_0 e^{-r/r_0}$$

The expression for the force will be

- (a) $-\frac{r_0 u_0}{r} e^{-r/r_0} \left(\frac{1}{r_0} - \frac{1}{r}\right)$ (b) $-u_0 \cdot e^{-r/r_0} \left(\frac{r-r_0}{r_0 r}\right)$
 (c) $-u_0 \cdot e^{-r/r_0} \left(\frac{r_0 r}{r-r_0}\right)$ (d) $-\frac{r_0 u_0}{r} e^{-r/r_0} \left(\frac{1}{r} - \frac{1}{r_0}\right)$

123. If the magnitude of the force of attraction between a particle of mass m_1 and one of mass m_2 is given by

$$F = K \frac{m_1 m_2}{x^2}$$

where K is a constant and x is the distance between the particles and $U(\infty) = 0$, the potential energy function will be

- (a) $K \frac{m_1 m_2}{x^3}$ (b) $K \frac{m_1 m_2}{x^{-3}}$
 (c) $-K \frac{m_1 m_2}{x}$ (d) $K \frac{x}{m_1 m_2}$

124. A conical pendulum of length L makes an angle θ with the vertical as shown in the figure. The time period will be



- (a) $2\pi \sqrt{\frac{L \cos \theta}{g}}$
 (b) $2\pi \sqrt{\frac{L}{g \cos \theta}}$
 (c) $2\pi \sqrt{\frac{L \tan \theta}{g}}$ (d) $2\pi \sqrt{\frac{L}{g \tan \theta}}$

125. If M is the mass of sun, r its separation from a planet and G the gravitational constant, then the time taken by the planet to make one complete revolution round the sun is

- (a) $2\pi \sqrt{\frac{r^3}{MG}}$ (b) $2\pi \sqrt{\frac{MG}{r^3}}$
 (c) $2\pi \sqrt{\frac{Mr^3}{G}}$ (d) $2\pi \sqrt{\frac{G}{Mr^3}}$

126. A particle is acted upon by two mutually perpendicular forces of 3 N and 4 N. In order that the particle remains stationary, the magnitude of the third force that should be applied should be

- (a) 12 N (b) 5 N
 (c) 8 N (d) 7 N

127. A ball is thrown in such a way that both the maximum height attained and the horizontal range are 500 m. The angle of projection with respect to the horizontal should be

- (a) $\tan^{-1} 4$ (b) $\tan^{-1} 3$
 (c) $\tan^{-1} 1/3$ (d) $\tan^{-1} 1/4$

128. Assume that an earth satellite is travelling at a height of 140 miles above the surface of the earth where $g = 30 \text{ ft/s}^2$ and the radius of the earth is 3,960 miles. The speed of the satellite shall be

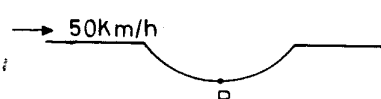
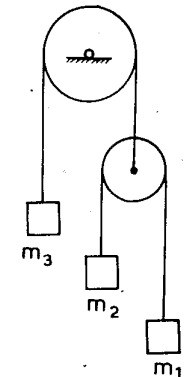
- (a) 71400 miles/h (b) 41700 miles/h
 (c) 1714 miles/h (d) 17400 miles/h

129. The motion of a particle of mass m is confined to a plane and is determined by $X = 3t^2$, $Y = 2t^3$ where X is measured in metres and t in seconds. The magnitude and direction of the force on the particle at $t = 1/2$ s is

- (a) $2\sqrt{6} \text{ m N}$ at 30° with the X-axis

- (b) $6\sqrt{2} \text{ m N}$ at 45° with the X-axis
 (c) $6\sqrt{2} \text{ m N}$ at 30° with the X-axis
 (d) $2\sqrt{6} \text{ m N}$ at 45° with the X-axis
130. Balls are thrown vertically upward in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 4.9 m, the number of balls thrown per minute will be
 (a) 60 (b) 40
 (c) 50 (d) 120
131. The reaction force between the floor and a man of mass 60 kg standing on a lift that is accelerating downwards at a uniform acceleration of 4 m/s^2 will be
 (a) 840 N (b) 360 N
 (c) 500 N (d) 300 N
132. In a tug of war game between two teams, both teams exert equal but opposite forces of 2500 N at each end, resulting in equilibrium. The tension on the rope will be
 (a) zero (b) 5000 N
 (c) 100 N (d) none of these
133. Two objects move uniformly toward each other. They get closer by 4.0 m each second but when they move uniformly in the same direction, with the same speeds, they get 4.0 m closer every 10 seconds. The speeds of the two objects are
 (a) 2.2 m/s and 1.8 m/s (b) 1.1 m/s and 1.8 m/s
 (c) 22 m/s and 18 m/s (d) 1.7 m/s and 2.1 m/s
134. A person walks up a stalled escalator in 90 s. When standing on the same escalator, now moving, he is carried in 60 s. The time it would take him to walk up the moving escalator will be
 (a) 27 s (b) 72 s
 (c) 18 s (d) 36 s
135. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string 1.2 m long. The string breaks and flies off horizontally, striking the ground 9.1 m away. The centripetal acceleration during the circular motion was
 (a) 94 m/s^2 (b) 141 m/s^2
 (c) 188 m/s^2 (d) 282 m/s^2
136. A person wants to drive on the vertical surface of a large cylindrical wooden 'well' commonly known as 'deathwell' in a circus. The radius of the well is R and the coefficient of friction between the tyres of the motorcycle and the wall of the 'well' is μ_s . The minimum speed the motorcyclist must have in order to prevent slipping should be
 (a) $\sqrt{\frac{gR}{\mu_s}}$ (b) $\sqrt{\frac{\mu_s}{gR}}$
 (c) $\sqrt{\frac{\mu_s g}{R}}$ (d) $\sqrt{\frac{R}{\mu_s g}}$
137. A motorcyclist wants to drive on the vertical surface of a wooden 'well' of radius 5 m, with a minimum speed of $5\sqrt{5} \text{ m/s}$. The minimum value of coefficient of friction between the tyres and the wall of the well must be
 (a) 0.10 (b) 0.20
 (c) 0.30 (d) 0.40
138. A circular curve of a highway is designed for traffic moving at 72 km/h. If the

radius of the curved path is 100 m, the correct angle of banking of the road should be given by

- (a) $\tan^{-1} \frac{2}{5}$ (b) $\tan^{-1} \frac{3}{5}$
 (c) $\tan^{-1} \frac{1}{5}$ (d) $\tan^{-1} \frac{1}{4}$
139. If the banking angle of a curved road is given by $\tan^{-1} (3/5)$ and the radius of curvature of the road is 6 m then the safe driving speed should not exceed
 (a) 86.4 km/h (b) 43.2 km/h
 (c) 21.6 km/h (d) 30.4 km/h
140. A car while travelling at a speed of 50 km/h passes through a curved portion of road in the form of an arc of a circle of radius 10 m. If the mass of the car is 500 kg, the reaction on the car at the lowest point P is
- 
- (a) 14.5 kN (b) 29 kN
 (c) 7.25 kN (d) 145 kN
141. A car starts from rest with a constant acceleration $a_1 = 10 \text{ m/s}^2$ which it maintains for a time $t_1 = 3 \text{ s}$. A constant deceleration that is necessary to bring the car to a stop in a distance of 110 m from the initial point should be
 (a) 6.92 m/s^2 (b) 9.62 m/s^2
 (c) 3.46 m/s^2 (d) 5.15 m/s^2
142. Three masses m_1 , m_2 and m_3 are attached to a string-pulley system as shown. All three masses are held at rest and then released. To keep m_3 at rest, the condition is
- 
- (a) $\frac{1}{m_3} = \frac{1}{m_1} + \frac{1}{m_2}$
 (b) $m_1 + m_2 = m_3$
 (c) $\frac{4}{m_3} = \frac{1}{m_1} + \frac{1}{m_2}$
 (d) $\frac{1}{m_1} + \frac{2}{m_2} = \frac{3}{m_3}$
143. If m_3 is at rest, as given in the above problem, the accelerations of m_2 and m_1 will be
 (a) $\frac{m_2 - m_1}{m_1 + m_2} g$ (b) $\frac{m_1 + m_2}{m_1 - m_2} g$
 (c) $\frac{m_1}{m_1 + m_2} g$ (d) $\frac{m_1}{m_1 - m_2} g$
144. A particle moving with uniform acceleration along a straight line covers distances a and b in successive intervals of p and q seconds. The acceleration of the particle is
 (a) $\frac{pq(p+q)}{2(bp-aq)}$ (b) $\frac{2(aq-bp)}{pq(p-q)}$

(c) $\frac{bp - aq}{pq(p - q)}$

(d) $\frac{2(bp - aq)}{pq(p + q)}$

145. A particle slides down a chord through the highest point of a vertical circle of radius a starting from the highest point; the chord is inclined at an angle θ to the vertical. The time required to descend down to the other extreme of the chord is

(a) $\sqrt{\frac{4a \cos \theta}{g}}$

(b) $\sqrt{\frac{4a}{g \cos \theta}}$

(c) $\sqrt{\frac{4a}{g}}$

(d) $\sqrt{\frac{a \tan \theta}{g}}$

Answers

- | | | | | |
|----------|----------|----------|----------|----------|
| 1. (c) | 2. (a) | 3. (b) | 4. (c) | 5. (B) |
| 6. (a) | 7. (c) | 8. (a) | 9. (c) | 10. (b) |
| 11. (c) | 12. (b) | 13. (a) | 14. (d) | 15. (b) |
| 16. (c) | 17. (a) | 18. (b) | 19. (a) | 20. (b) |
| 21. (d) | 22. (c) | 23. (b) | 24. (a) | 25. (b) |
| 26. (a) | 27. (b) | 28. (c) | 29. (c) | 30. (c) |
| 31. (c) | 32. (a) | 33. (d) | 34. (c) | 35. (a) |
| 36. (b) | 37. (c) | 38. (c) | 39. (d) | 40. (b) |
| 41. (a) | 42. (a) | 43. (c) | 44. (d) | 45. (a) |
| 46. (c) | 47. (b) | 48. (a) | 49. (b) | 50. (d) |
| 51. (c) | 52. (b) | 53. (b) | 54. (a) | 55. (d) |
| 56. (a) | 57. (c) | 58. (b) | 59. (b) | 60. (b) |
| 61. (a) | 62. (b) | 63. (a) | 64. (d) | 65. (c) |
| 66. (b) | 67. (d) | 68. (c) | 69. (b) | 70. (c) |
| 71. (b) | 72. (c) | 73. (b) | 74. (d) | 75. (a) |
| 76. (b) | 77. (c) | 78. (b) | 79. (a) | 80. (c) |
| 81. (d) | 82. (d) | 83. (b) | 84. (d) | 85. (d) |
| 86. (b) | 87. (a) | 88. (c) | 89. (d) | 90. (c) |
| 91. (a) | 92. (c) | 93. (d) | 94. (d) | 95. (a) |
| 96. (a) | 97. (d) | 98. (c) | 99. (b) | 100. (c) |
| 101. (c) | 102. (c) | 103. (d) | 104. (b) | 105. (a) |
| 106. (c) | 107. (b) | 108. (c) | 109. (a) | 110. (c) |
| 111. (d) | 112. (b) | 113. (a) | 114. (d) | 115. (b) |
| 116. (b) | 117. (a) | 118. (d) | 119. (d) | 120. (d) |
| 121. (b) | 122. (a) | 123. (c) | 124. (a) | 125. (a) |
| 126. (b) | 127. (a) | 128. (d) | 129. (b) | 130. (a) |
| 131. (b) | 132. (d) | 133. (a) | 134. (d) | 135. (c) |
| 136. (a) | 137. (d) | 138. (a) | 139. (c) | 140. (a) |
| 141. (a) | 142. (c) | 143. (a) | 144. (d) | 145. (c) |

Note: Problems marked with a star should take about 5 minutes to work out. The others should not take more than 3 minutes.

4**Work, Energy and Power****Definition of Work**

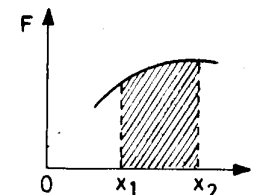
If a force $\vec{F}$ is applied on a body and it is displaced by a distance $d\vec{x}$, the work done by the force on the body

$$dW = \vec{F} \cdot d\vec{x}$$

Work done in displacing the body from a position x_1 to x_2 is

$$W = \int_{x_1}^{x_2} dW = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{x}; \text{ Units: Nm or J}$$

This integral represents the area under the F - x curve between the limits x_1 and x_2 as shown in the figure.

**Work Done by Force of Gravity**

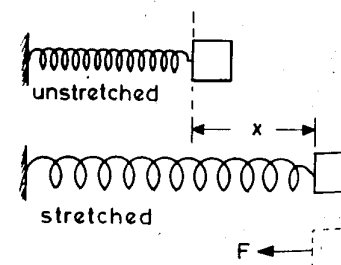
A body of mass m moves vertically down from a position y_1 to a position y_2 . Work done as the body moves from y_1 to y_2

$$W_{1-2} = \int_{y_1}^{y_2} -mg \, dy = -mg(y_2 - y_1)$$

The work done will be positive if $(y_2 - y_1)$ is negative, i.e. when the particle moves down. Notice that the work done depends only on the difference in elevations.

Work Done In Stretching a Spring

The work done in stretching a spring is an example of work done by a variable force. The force required to maintain the spring stretched by one unit of length is called the spring constant or force constant of the spring (K), and has the units N/m.



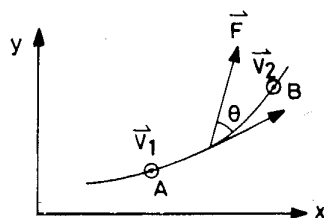
$$F = -Kx$$

Work done by the spring force as the mass moves from $x = x_1$ to $x = x_2$ is

$$W_{x_1 \rightarrow x_2} = \int_{x_1}^{x_2} -Kx \cdot dx = -\frac{1}{2} K(x_2^2 - x_1^2)$$

Work-Energy Principle

If a particle of mass m moves in a plane under the action of a resultant force $\vec{F}$, the particle will have a velocity $\vec{V}$ and acceleration $\vec{a}$. Let $\vec{V}_1$ be the velocity at a point A and $\vec{V}_2$ at point B. Path AB = S , say, then



$$\int_A^B (F \cos \theta) dS = \int_{V_1}^{V_2} mV dV = \frac{1}{2} mV_2^2 - \frac{1}{2} mV_1^2 = K_2 - K_1$$

or $W = \Delta K$ where ΔK is the change in kinetic energy.

Power

Average power is defined as

$$\text{Rate of doing work } \bar{P} = \frac{W}{t}$$

and instantaneous power delivered is

$$P = \frac{dW}{dt}$$

The unit is J/s or Nm/s and called Watt (W).

Force and Potential Energy

A mass is raised vertically up from a position y_1 to y_2 . The direction of the gravitational force mg is opposite to the upward displacement. The work done by the gravitational force is

$$W_{\text{grav}} = -mg(y_2 - y_1) = -(mgy_2 - mgy_1)$$

or $W_{\text{grav}} = -\Delta U$, change in potential energy

The relation between power and velocity is

$$P = \vec{F} \cdot \vec{V}$$

ILLUSTRATIONS

1. A truck which has a horizontal surface carries an 80 kg crate. It starts from rest and attains a speed of 70 km/h in a distance of 70 m on a level road with constant acceleration. Calculate the work done by the frictional force acting on the crate during this interval, given that $\mu_s = 0.25$ and $\mu_k = 0.20$.

Solution

Assuming the crate does not slip, the truck and the crate should have a common acceleration

$$a = \frac{V^2}{2S} = \frac{(70/3.6)^2}{2 \times 70} = 2.7 \text{ m/s}^2$$

This acceleration requires a force of friction on the crate

$$f_c = 80 \times 2.7 = 216 \text{ N}$$

If the crate does not slip, the maximum possible force of friction is

$$f_s = 0.25 \times 80 \times 10 = 200 \text{ N}$$

This frictional force for the limiting condition of the crate is less than 216 N required for no slipping. Therefore, the crate slips and frictional force is governed by the coefficient of kinetic friction.

$$\therefore f_k = 0.20 \times 80 \times 10 = 160 \text{ N}$$

Therefore, the acceleration becomes, $a = 160/80 = 2 \text{ m/s}^2$. The distance travelled by the crate and the truck are in proportion to their accelerations.

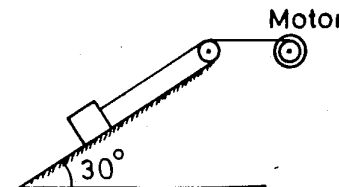
$$\therefore \text{Displacement of the crate} = \frac{2.0}{2.7} \times 70 = 52 \text{ m}$$

Work done by force of kinetic friction,

$$W_{fk} = 160 \times 52 = 8,320 \text{ J} = 8.3 \text{ kJ}$$

Note that this problem presents a situation in which the force of friction (kinetic) can do positive work when the surface which supports the body and generates the force of friction is in motion. If the supporting surface is at rest, then f_k acting on the moving body always does negative work.

2. A heavy block of mass 300 kg is being pulled up an inclined plane of angle 30° by means of a wire passing over a pulley and connected to a motor as shown in the figure. In the power output of the motor is 5 kW and the block moves up with a constant velocity of 2 m/s, find the coefficient of friction between the block and the inclined surface.



Solution

The free body diagram of the block is as shown.

$$R = Mg \cos 30 = 2550 \text{ N}$$

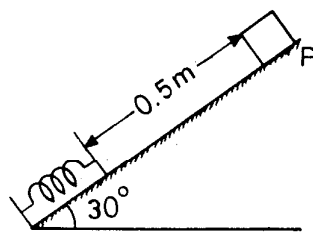
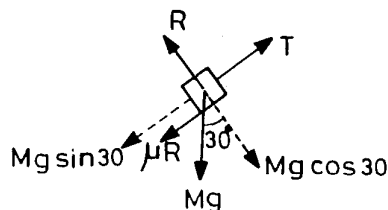
$$f = \mu \cdot R = \mu \times 2550 \text{ N}$$

$$T - 2550\mu - 300 \times 10 \times \frac{1}{2} = 0$$

$$T = \frac{P}{V} = \frac{5000}{2} = 2500 \text{ N}$$

$$2500 - 2550\mu - 1500 = 0 \Rightarrow \mu = 0.392$$

3. A block of mass 2 kg is released from rest from the position P as shown in figure. It slides down the plane and, after covering a distance of 0.5 m, strikes a spring whose force constant is 4000 N/m. If the coefficient of friction between the block and the inclined surface is 0.3, find the compression in the spring.



Solution

$$f = \mu mg \cos \theta$$

$$\text{Net downward acceleration} = g \sin \theta - \frac{f}{m} = g \sin \theta - \mu g \cos \theta$$

$$V^2 = 2g(\sin \theta - \mu \cos \theta) S = 2.5 \text{ (m/s)}^2$$

KE of block = energy of the compressed spring

$$\frac{1}{2} Kx^2 = \frac{1}{2} mV^2 \Rightarrow x = 35.3 \text{ mm}$$

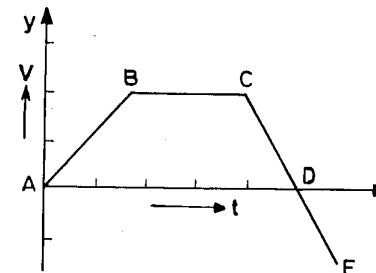
EXERCISES

1. A man pushes a block of 30 kg along a level floor at constant speed with a force directed at 45° below the horizontal. If the coefficient of friction is 0.20, the work done by the man on the block in pushing it through 20 m is
(a) 1500 J (b) 2000 J
(c) 1000 J (d) 3000 J
2. A block of mass 4 kg slides on a horizontal smooth surface of a table with a speed 1.6 m/s. It is brought to rest in compressing a spring in its path. If the spring has a force constant of 2 N/m, the spring should be compressed by
(a) 0.22 m (b) 1.4 m
(c) 2.2 m (d) 0.4 m

3. A bucket tied to a string is lowered at a constant acceleration of $g/4$. If the mass of the bucket is M and is lowered by a distance d , the work done by the string will be (assume the string to be massless)

- (a) $\frac{1}{4} Mgd$ (b) $-\frac{3}{4} Mgd$
(c) $-\frac{4}{3} Mgd$ (d) $\frac{4}{3} Mgd$

4. A plot of velocity versus time is shown in the figure. A single force acts on the body. The correct statement among the following is



- (a) in moving from A to B, work is done by the body and is negative.
(b) in moving from B to C no work is done on the body but the body does work on the system.
(c) in moving from C to D, work done by the force on the body is positive.
(d) in moving from D to E, work done by the force on the body is positive.
5. A running man has half the kinetic energy of a boy of half his mass. The man speeds up by 1.0 m/s and then has the same kinetic energy as the boy. The original speed of the boy was
(a) 2.4 m/s (b) 9.6 m/s
(c) 4.8 m/s (d) 7.2 m/s
 6. A proton starting from rest is accelerated in a cyclotron to a final speed of 3.0×10^7 m/s. The work (in electronvolts) done on the proton by the electrical force of the cyclotron is
(a) 4.7×10^6 (b) 1.47×10^7
(c) 7.41×10^7 (d) 4.7×10^{-6}
 7. A bullet of mass 50 g travelling at 500 m/s penetrates 100 cm into a wooden block. The average force exerted on the block is
(a) 6.25×10^4 N (b) 62.5×10^4 N
(c) 6.25×10^3 N (d) 2.65×10^4 N
 8. A neutron moving with a constant speed passes two points 3.6 m apart in 1.8×10^{-4} s. The kinetic energy of the neutron is
(a) 2.1×10^3 eV (b) 2.1 eV
(c) 21 eV (d) 2.1×10^{-3} eV
 9. A car starts from rest and attains a kinetic energy K by accelerating without slipping along a horizontal road in 20 seconds. If air resistance is neglected, the work done by external forces which accelerate the car will be
(a) zero (b) K
(c) $2K$ (d) $K/10$

10. A spring of mass M and length L (stiffness constant K) is compressed vertically downward against the floor so that its compressed length becomes $L/2$. On releasing it, the work done by it on the floor is

- (a) $\frac{1}{2}KL^2$
 (b) $\frac{1}{2}K\left(\frac{L}{2}\right)^2$
 (c) zero
 (d) KL^2

11. A 1.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. The work done by the force as it moves from the origin to a point $x = 10$ m is

- (a) 5 J
 (b) 30 J
 (c) -10 J
 (d) 22 J

12. In the above force-displacement graph, if the block was passing through the origin with a speed of 10 m/s, the speed of the particle at $x = 10$ m was

- (a) 5 m/s
 (c) 6.3 m/s

13. An object of mass m is tied to string of length L and a variable horizontal force is applied on it which starts at zero and gradually increases (it is pulled extremely slowly so that equilibrium exists at all times) until the string makes an angle θ with the vertical. Work done by the force F is

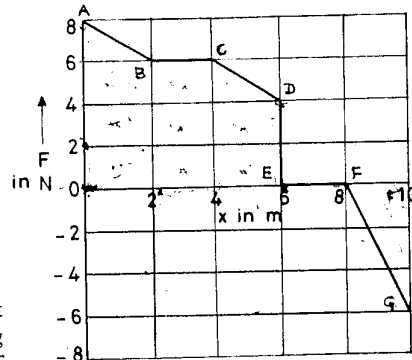
- (a) $mgL(1 - \cos \theta)$
 (b) $mgL(1 - \sin \theta)$
 (c) mgL

14. A cricketer holds a ball of mass 0.1 kg in his hand. In order to throw the ball upward, he raises his hand by 0.5 m and throws the ball vertically upward with a velocity 30 m/s. The force applied in the process is

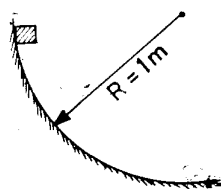
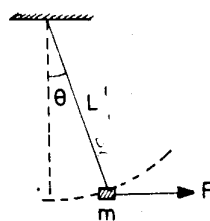
- (a) 125 N
 (b) 19 N
 (c) 82 N
 (d) 91 N

15. A block of mass 1 kg slides down a curved track that is one quadrant of a circle of radius 1 m. Its speed at the bottom is 2 m/s. The work done by frictional force is

- (a) 8 J
 (b) -8 J
 (c) 4 J
 (d) -4 J



- (b) 2.2 m/s
 (d) 1.1 m/s



16. A block of mass 1 kg falls from a height of 5 m on a vertical spring fastened to a horizontal board placed on the floor. If the spring constant of the spring is 10 N/m, the maximum compression that the spring undergoes is

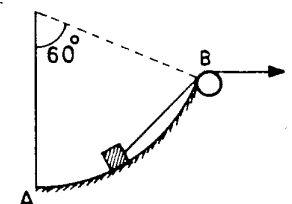
- (a) 1.7 m
 (b) 3.4 m
 (c) 4.3 m
 (d) 0.43 m

17. In the above problem, if the block and spring get fastened after contact, the height to which the block and spring would rebound is

- (a) 3.2 m
 (b) 4.3 m
 (c) 2.3 m
 (d) 1.3 m

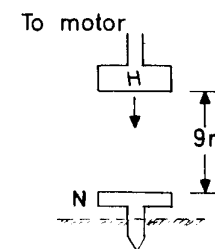
- 18.* A 10 kg block is pulled in the vertical plane along a frictionless surface in the form of an arc of a circle of radius 10 m. The applied force is of 200 N as shown in figure. If the block had started from rest at A, the velocity at B would be

- (a) 1.7 m/s
 (b) 17 m/s
 (c) 27 m/s
 (d) 34 m/s



19. A 100 kg 'drop hammer' H as shown is operated by a motor. It falls from a height of 9 m on thick nail N which is to be driven into the hard block. The hammer has to be lifted to height of 9 m in 12 s with constant velocity. If the efficiency of the motor is assumed to be 80%, the power required by the motor is approximately

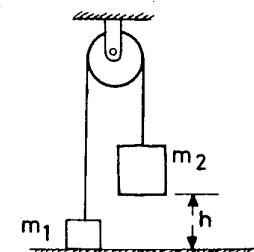
- (a) 0.49 kW
 (b) 4.9 kW
 (c) 0.94 kW
 (d) 6 kW



20. Two masses m_1 and m_2 ($m_2 > m_1$) are positioned as shown in figure, m_1 being on the ground and m_2 at a height h above the ground. When m_2 is released, the speed at which it hits the ground will be

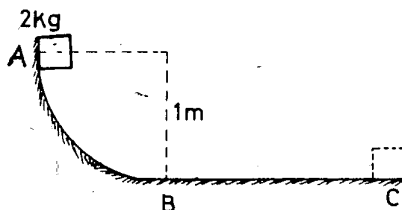
- (a) $\sqrt{\frac{2gh \cdot m_1}{m_2}}$
 (b) $\sqrt{\frac{2gh(m_1 - m_2)}{(m_1 + m_2)}}$
 (c) $\sqrt{\frac{2gh(m_1 + m_2)}{m_1 - m_2}}$

- (d) $\sqrt{\frac{2gh(m_2 - m_1)}{(m_1 + m_2)}}$

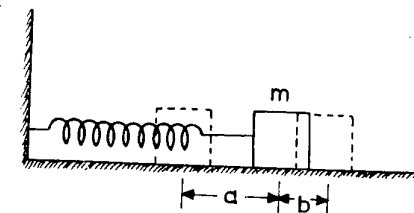
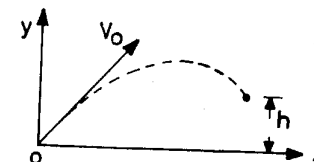


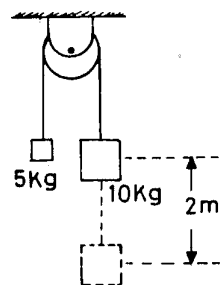
21. A man pulls a bucket of water from a depth of h from a well. If the mass of the rope and that of bucket full of water are m and M respectively, the work done by the man is

- (a) $(M + m)gh$ (b) $\left(M + \frac{m}{2}\right)gh$
 (c) $\left(\frac{M + m}{2}\right)gh$ (d) $\left(\frac{M}{2} + m\right)gh$
22. A car weighing 1000 kg is travelling on a level road with a uniform velocity of 60 km/h. The resistance offered to the motion due to air and friction is 50 N. The work done by the car in travelling a distance of 1 km is
 (a) 5×10^4 J (b) 60×10^3 J
 (c) 60×10^6 J (d) 3000 J
23. A man weighing 60 kg climbs a staircase carrying a 20 kg load, on his hand. The staircase has 20 steps and each step has a height of 20 cm. If he takes 20 seconds to climb, his power is
 (a) 160 W (b) 230 W
 (c) 320 W (d) 80 W
24. A block of mass 2 kg slides on an inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the block and the surface is $\sqrt{3}/2$. In order that the block moves down with constant speed, the force required is
 (a) 5.7 N (b) 6.2 N
 (c) 4.1 N (d) 11.4 N
25. A block of mass 2 kg is released from A on the track that is one quadrant of a circle of radius 1 m. It slides down the track and reaches B with a speed of 4 m/s and finally stops at C at a distance of 3 m from B. The work done against the force of friction is
 (a) 10 J
 (b) 20 J
 (c) 2 J
 (d) 6 J
26. An elevator of mass 1000 kg starts from rest and is being pulled upward by a motor with a constant acceleration of 4 m/s^2 . When the elevator is moving with a speed of 4 m/s, the horse power required for the motor is
 (a) 120 (b) 60
 (c) 75 (d) 300
27. A particle of mass 0.01 kg travels along a space curve with velocity given by $4\hat{i} + 16t\hat{j} \text{ m/s}$. After some time, its velocity becomes $8\hat{i} + 20\hat{j} \text{ m/s}$ due to the action of a conservative force. The work done on the particle during this interval of time is
 (a) 0.32 J (b) 6.9 J
 (c) 9.6 J (d) 0.96 J
28. A particle moves with a velocity $5\hat{i} - 3\hat{j} + 6\hat{k} \text{ m/s}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k} \text{ N}$. The instantaneous power applied to the particle is
 (a) 200 J/s (b) 40 J/s
 (c) 140 J/s (d) 170 J/s



29. The work done in moving a particle from a point (1,1) to (2,3) in a plane and in a force field with potential $u = \lambda(x + y)$ is
 (a) 3λ (b) -3λ
 (c) 0 (d) λ
30. A simple pendulum of length 2 m oscillates such that its speed at the lowest point is 4 m/s. It goes to a certain height and reverses its motion. At this position, the angle that it makes with the vertical is
 (a) 35° (b) 45°
 (c) 49° (d) 53°
31. A car seller claims that his 1000 kg car can accelerate from rest to a speed of 24 m/s in just 8.0 s. The engine of the car, on an average, should be of
 (a) 60 HP (b) 48 HP
 (c) 80 HP (d) 24 HP
32. A 0.5 HP motor is used to lift load at a constant speed of 10 cm/s. The maximum load it can lift at this speed is
 (a) 370 kg (b) 530 kg
 (c) 185 kg (d) 555 kg
33. A 30 HP engine has to pull crates along a level road at 10 m/s. If the coefficient of friction between the road and the crate is 0.2, the mass of the largest crate can be
 (a) 500 kg (b) 300 kg
 (c) 1000 kg (d) 1120 kg
34. A projectile is fired from point O with an initial speed V_0 . The speed at an altitude h from the ground is
 (a) $\sqrt{V_0^2 + 2gh}$
 (b) $\sqrt{2gh}$
 (c) $\sqrt{V_0^2 - 2gh}$ (d) $\sqrt{2gh - V_0^2}$
35. A mass-spring system as shown in the figure oscillates such that the mass moves on a rough surface having coefficient of friction μ . It is compressed by a distance a from its normal length and, on being released, it moves to a distance b from its equilibrium position. The decrease in amplitude for one half-cycle ($-a$ to b) is
 (a) $\frac{\mu mg}{K}$ (b) $\frac{2\mu mg}{K}$
 (c) $\frac{\mu g}{K}$ (d) $\frac{K}{\mu mg}$
- 36.* Two blocks of masses 10 and 5 kg are connected by a flexible but inextensible string over a shaft as shown in the figure. The system starts from rest with 10 kg mass moving downward. If a constant frictional force of 25 N acts at the shaft, the velocity when the 10 kg block has moved 2 m down is

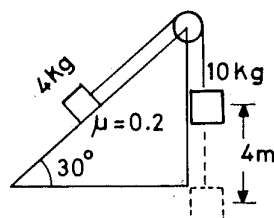




- (a) 2.6 m/s
(c) 3.2 m/s
(b) 6.2 m/s
(d) 4 m/s

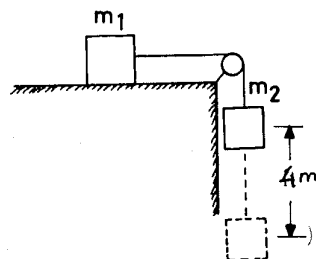
- 37.* Two blocks of masses 10 and 4 kg are connected by a flexible inextensible string as shown in the figure. The pulley is assumed to be frictionless. The coefficient of friction between the inclined surface and the 4 kg block is 0.2 and angle of inclination of the plane is 30° . Assuming that the system is released from rest, the velocity of 10 kg block when it has moved 4 m down is

- (a) 5.6 m/s
(c) 41.8 m/s
(b) 6.5 m/s
(d) 13 m/s



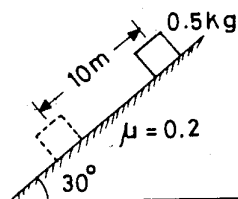
- 38.* Two masses $m_1 = 10$ kg and $m_2 = 5$ kg are connected by an ideal string as shown in the figure. The coefficient of friction between m_1 and the surface is 0.2. Assuming that the system is released from rest, the velocity when m_2 has descended by 4 m is

- (a) 17 m/s
(b) 14 m/s
(c) 4 m/s
(d) 3.5 m/s

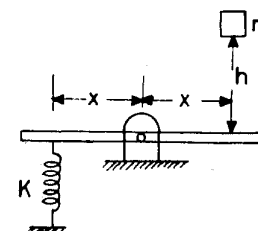


39. A block of mass 0.5 kg has an initial velocity of 10 m/s down an inclined rough plane of angle 30° , the coefficient of friction between the block and inclined surface being 0.2. The velocity of the block after it travels a distance of 10 m is

- (a) 24 m/s
(c) 8 m/s
(b) 13 m/s
(d) 17 m/s



40. A crate of mass m falls from a height h onto the end of a platform, as shown in the figure. The spring is initially unstretched and the mass of the platform can be neglected. Assuming that there is no loss of energy, the maximum elongation of the spring is



- (a) $\frac{mg + \sqrt{m^2 g^2 + 2mghK}}{K}$
(c) $\frac{K}{\sqrt{m^2 g^2 + 2mghK}}$
(b) $\frac{mg - \sqrt{m^2 g^2 + 2mghK}}{K}$
(d) $\frac{\sqrt{m^2 g^2 + 2mghK}}{K}$

41. A 60 g bullet is fired through a stack of fibre board sheets 200 mm thick. If the bullet approaches the stack with a velocity of 600 m/s and emerges out with a velocity of 300 m/s, the average resistance offered to the bullet is

- (a) 40.5 kN
(c) 20.25 kN
(b) 2 kN
(d) 10 kN

42. A pump is required to lift 800 kg of water per minute from a 10 m deep well and eject it with a speed of 20 m/s. The required horse power of the pump will be

- (a) 6
(c) 3.75
(b) 1.87
(d) 5.5

Answers

1 (a)	2 (c)	3 (b)	4 (d)	5 (c)
6 (a)	7 (c)	8 (b)	9 (a)	10 (c)
11 (b)	12 (c)	13 (a)	14 (d)	15 (b)
16 (c)	17 (c)	18 (b)	19 (c)	20 (d)
21 (b)	22 (a)	23 (a)	24 (d)	25 (b)
26 (c)	27 (d)	28 (c)	29 (b)	30 (d)
31 (b)	32 (a)	33 (d)	34 (c)	35 (b)
36 (a)	37 (b)	38 (c)	39 (b)	40 (a)
41 (a)	42 (c)			

Note: Problems marked with a star are supposed to take about 5 minutes to solve. All the rest should take about 2 to 3 minutes.

5

Impulse, Momentum and Conservation of Momentum

Newton's Second law states that

$$\vec{F} = m \frac{d\vec{V}}{dt}$$

or $\vec{F} dt = m d\vec{V}$

If V_1 is the velocity at time $t = t_1$ and V_2 at $t = t_2$.

then $\int_{t_1}^{t_2} \vec{F} dt = \int_{V_1}^{V_2} m d\vec{V}$

or Impulse = $\vec{J} = \int_{t_1}^{t_2} \vec{F} dt$ (units are N.s)

Also, one can write

$$\int_{t_1}^{t_2} \vec{F} dt = m\vec{V}_2 - m\vec{V}_1 = \vec{p}_2 - \vec{p}_1$$

$$\vec{F}_{\text{ext}} = \frac{d\vec{P}}{dt} \quad \text{and if} \quad \vec{F}_{\text{ext}} = 0, \quad \frac{d\vec{P}}{dt} = 0$$

or $\vec{P}$, i.e. total momentum of the system remains constant.

Centre of Mass

$$x_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

where x_{CM} is the centre of two masses m_1 and m_2 placed at distances x_1 and x_2 respectively from the origin O.

In general,

$$\vec{r}_{\text{cm}} = \frac{1}{M} \sum m_i \vec{r}_i$$

(for discrete distribution of masses)

$$\text{Also } \vec{r}_{\text{cm}} = \frac{1}{M} \int \vec{r} \cdot dm$$

(for a continuous distribution of mass)

Two particles of masses m_1 and m_2 move with velocities V_1 and V_2 but after collision, their velocities become

V_1' and V_2' . Then, from the principle of conservation of linear momentum,

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

During the collision of two particles, one particle is deformed due to the other. Thereafter, restitution takes place,

$$\text{Coefficient of restitution, } e = \frac{\text{impulse during restitution}}{\text{impulse during deformation}}$$

$$= \frac{\int_{t_1}^{t_2} F_r \cdot dt}{\int_{t_0}^{t_2} F_d \cdot dt}$$

where F_r and F_d are the forces of restitution and deformation respectively. The value of e is a measure of energy loss during the impact. In case of perfectly elastic collision, e is 1. In case of perfectly inelastic collision, $e = 0$.

Also,

$$e = - \frac{\text{relative velocity after impact}}{\text{relative velocity before impact}}$$

ILLUSTRATIONS

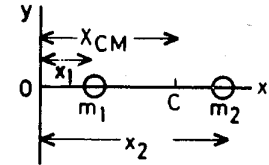
1. A ball is dropped from a height h above a mosaic floor and rebounds to a height $0.7 h$. Find the coefficient of restitution between the ball and the floor.

Solution

$$e = - \frac{\text{relative velocity after impact}}{\text{relative velocity before impact}} = - \frac{V_r}{V_i}$$

where V_r is the velocity after rebound and V_i is the velocity with which it hits the floor.

$$\text{Also } mgh = \frac{1}{2} m V_i^2 \quad \text{and} \quad mg(0.7)h = \frac{1}{2} m V_r^2$$



Let us take the downward direction to be positive. Then

$$V_i = \sqrt{2gh} \quad \text{and} \quad V_r = -\sqrt{1.4gh}$$

$$e = \sqrt{\frac{1.4}{2}} = \sqrt{0.7} = 0.84$$

2. A 4 kg block of wood rests on a table top. A 50 g bullet is shot straight up through a hole in the table beneath the block. The bullet remains embedded in the block and the block flies up a height of 50 cm. What was the speed of the bullet?

Solution

Conservation of momentum:

$$0.05V_b = (4.05)V$$

where V_b is the speed of the bullet and V is the speed of block after the bullet is embedded into it.

KE of (block + bullet) = gain in PE of (block + bullet)

$$\frac{1}{2} (4.05) V^2 = 4.05 \times 10 \times \frac{1}{2}$$

$$V^2 = 10 \rightarrow V = \sqrt{10} \text{ m/s}$$

$$\therefore V_b = \frac{4.05 \times \sqrt{10}}{0.05} = 256 \text{ m/s}$$

3. A cricket batsman straight drives on a 150 g ball with a velocity of 30 m/s. If the ball had a velocity of 20 m/s when it hit the bat, find the impulse on the ball.

Solution

$$\text{Impulse} = \vec{F} \cdot dt = d\vec{p}$$

$$= 0.015 \times 20 - (0.015)(-30)$$

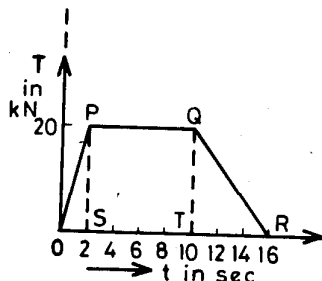
$$= 0.015 \times 50 = 0.75 \text{ N} \cdot \text{s}$$

4. The thrust vs time graph of a rocket is shown in the figure. The mass of the rocket is 1200 kg. Find the velocity of the rocket 16 s after starting from rest.

Solution

Thrust $\times$ time = change in momentum

$$\text{Area under the curve} = m(V - 0)$$



$$\text{Area (OPS + PQTS + QRT)} = mV$$

$$20 + 160 + 60 = mV$$

$$V = \frac{240}{1,200} = \frac{1}{5} = 0.2 \text{ m/s}$$

5. A boy throws a tennis ball vertically downwards. He wants the ball to rebound from the floor and just touch the ceiling of the room which is at a height of 4 m. If the coefficient of restitution is 0.8, with what velocity should the ball strike the floor?

Solution

Velocity of ball when it hits the floor = V

$\therefore$ Velocity just after rebound from the floor = $0.8V$

The kinetic energy of the ball after impact becomes the potential energy of the ball

$$\therefore \frac{1}{2} m(0.8V)^2 = m \times 10 \times 4$$

$$V^2 = \frac{10 \times 4 \times 2}{0.8 \times 0.8} = 125 \Rightarrow V = 11.2 \text{ m/s}$$

6. A particle moving in a force field $\vec{F}$ has its momentum given at any time t by

$$\vec{p} = 3e^{-t}\hat{i} - 2\cos t\hat{j} - 3\sin t\hat{k}$$

Find $\vec{F}$.

Solution

$$\vec{F} = \frac{d\vec{p}}{dt} = -3e^{-t}\hat{i} + 2\sin t\hat{j} - 3\cos t\hat{k}$$

7. Find the impulse developed by a force $\vec{F} = 4t\hat{i} - (6t^2 - 2)\hat{j} + 12t\hat{k}$ in the interval of time from $t = 0$ to $t = 2$.

Solution

$$\text{Impulse} = \int_{t_1}^{t_2} \vec{F} \cdot dt = \int_0^2 [4t\hat{i} - (6t^2 - 2)\hat{j} + 12t\hat{k}] dt$$

$$= \left[4 \cdot \frac{t^2}{2} \hat{i} - \left(6 \frac{t^3}{3} - 2t \right) \hat{j} + 12t\hat{k} \right]_0^2$$

$$= 8\hat{i} - 12\hat{j} + 24\hat{k} \text{ N} \cdot \text{s}$$

EXERCISES

- The magnitude of the impulse developed by a mass of 0.2 kg which changes its velocity from $5\hat{i} - 3\hat{j} + 7\hat{k}$ m/s to $2\hat{i} + 3\hat{j} + \hat{k}$ m/s is
 - 2.7 N · s
 - 1.8 N · s
 - 0.9 N · s
 - 3.6 N · s
- The constant force needed to give an object of 2000 kg a speed of 10 km/h in 3 minutes starting from rest will be
 - 62 N
 - 13 N
 - 21 N
 - 31 N
- A particle of mass m moves on the x -axis under the influence of a force of attraction towards the origin O given by $F = -k(1/x^2)\hat{i}$. If the particle starts from rest at $x = a$, the speed it will attain to reach the origin will be
 - $\sqrt{\frac{2k}{m}} \left(\frac{a-x}{ax}\right)^{1/2}$
 - $\sqrt{\frac{2k}{m}} \left(\frac{a+x}{ax}\right)^{1/2}$
 - $\sqrt{\frac{k}{m}} \left(\frac{ax}{a-x}\right)$
 - $\sqrt{\frac{m}{2k}} \left(\frac{a-x}{ax}\right)^{1/2}$

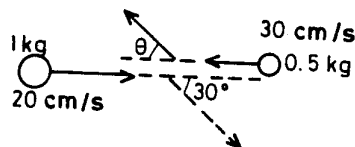
- 4.* A particle of mass m moves on the x -axis under the influence of a force of attraction towards the origin O given by

$$\vec{F} = -\left(\frac{k}{x^3}\right)\hat{i}$$

If the particle starts from rest at $x = a$, the speed it will attain to reach the origin will be

- $\sqrt{\frac{k}{m}} \left(\frac{a^2 - x^2}{x^2 a^2}\right)^{-1/2}$
 - $\sqrt{\frac{k}{m}} \left(\frac{a^2 - x^2}{x^2 a^2}\right)^{1/2}$
 - $\sqrt{\frac{k}{m}} \left(\frac{a^2 + x^2}{x^2 a^2}\right)^{1/2}$
 - $\sqrt{\frac{k}{m}} \left(\frac{x^2 - a^2}{x^2 a^2}\right)^{1/2}$
- 5.* A particle of unit mass is acted upon by a force $\vec{F} = 100te^{-2t}\hat{i}$. If, at $t = 0$, it is at rest, the change in momentum of the particle in going from time $t = 1$ to $t = 2$ will be
- $25e^2(3 - 5e^2)\hat{i}$
 - $e^{-2}(1 - 5e^{-2})\hat{i}$
 - $25e^{-2}(1 - e^{-2})\hat{i}$
 - $25e^{-2}(3 - 5e^{-2})\hat{i}$

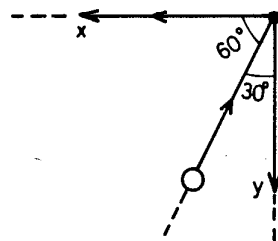
- 6.* Two balls collide and bounce off each other as shown in the figure. The 1 kg ball has a speed of 10 cm/s after collision. The velocity of the 0.5 kg ball will be
- 36 cm/s
 - 24 cm/s
 - 12 cm/s
 - 18 cm/s



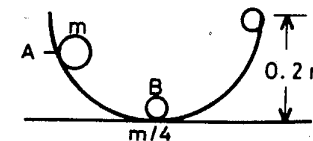
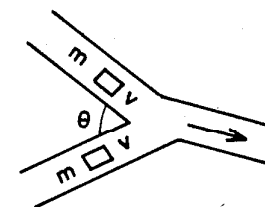
7. Referring to the figure and data of the previous problem, the angle of deflection of the 0.5 kg ball will be
- 52°
 - 59°
 - 42°
 - 36°

- In the above problem, one can conclude that
 - the collision is perfectly elastic
 - the kinetic energies before and after impact are same
 - momentum is conserved in the y -direction but not in the x -direction
 - total momentum is conserved but kinetic energy is not conserved.
- A jet of water hits a flat plate held perpendicular to the jet. The horizontal speed of water is 1 m/s and the jet ejects 50 g of water per second. Assuming that the water moves parallel to the plate after striking it, the force exerted on the stationary plate will be
 - 0.05 N
 - 0.5 N
 - 0.5 N
 - 0.05 N
- In the above problem, the conclusion that can be drawn is that
 - kinetic energy is conserved in the process
 - momentum is conserved only in the x -direction (initial direction of flow)
 - there is a force of 0.05 N of the plate on the water
 - both, kinetic energy and momentum are conserved.
- A rocket standing on its launch platform points vertically upward and the jet engines are ejecting gas at the rate of 500 kg/s. The molecules are expelled with a speed of 20 km/s. If the rocket has to slowly rise because of the thrust of the engines, the initial mass of the rocket could be
 - 10^6 kg
 - 10^5 kg
 - 10^4 kg
 - 10^7 kg
- An empty 20000 kg coal wagon is moving on a level track at 5 m/s. Suddenly 5000 kg of coal is dumped into it from directly above. The coal initially has zero horizontal velocity. The final speed of the wagon is
 - 3 m/s
 - 2 m/s
 - 8 m/s
 - 4 m/s
- A 2 kg block of wood rests on a long table top. A 5 g bullet moving horizontally with a speed of 150 m/s is shot into the block and sticks to it. The block then slides 2.7 m along the table top and comes to a stop. The force of friction between the block and the table is
 - 0.052 N
 - 0.52 N
 - 2.50 N
 - 1.04 N
- A 600 kg body travelling north at 0.5 m/s collides with a 400 kg body moving west at 1.5 m/s. The two bodies stick together after impact. The speed just after impact will be
 - 7.6 m/s
 - 0.67 m/s
 - 6.7 m/s
 - 0.335 m/s
- The average resisting force that must act on a 5 kg mass to reduce its speed from 65 to 15 cm/s in 0.2 s is
 - 12.5 N
 - 1250 N
 - 25 N
 - 125 N
- A 200 g hollow thin walled metal box suitably kept is hit by a 10 g bullet moving horizontally at 200 m/s. Just after impact the box has a horizontal speed of 150 cm/s. The speed of the bullet just after leaving the box is
 - 17 m/s
 - 340 m/s
 - 170 m/s
 - 230 m/s
- Two balls of equal mass have a head-on collision with speed 6 m/s. If the coefficient of restitution is $1/3$, the velocity of each ball after impact will be
 - 18 m/s
 - 2 m/s
 - 6 m/s
 - 6.33 m/s

18. Two balls having masses in the ratio of 1 : 3, both moving at 10 m/s, collide head-on. If the coefficient of restitution is $1/3$, their speeds after collision will be
 (a) 10 and $10/3$ m/s (b) $3/10$ and 10 m/s
 (c) $1/10$ and $10/3$ m/s (d) $1/10$ and $3/10$ m/s
19. Two balls of equal masses are moving with velocities of 10 and 30 m/s respectively and have a head-on collision. If the coefficient of restitution is $1/3$, their speed after impact will be
 (a) $10/3$ and $50/3$ m/s (b) $5/3$ and $25/3$ m/s
 (c) 10 and 50 m/s (d) $10/7$ and $50/7$ m/s
20. Two balls with masses in the ratio of 1 : 2 moving in opposite direction have a head-on elastic collision. If their velocities before impact were in the ratio of 3 : 1, then velocities after impact will have the ratio
 (a) 5 : 3 (b) 7 : 5
 (c) 4 : 5 (d) 2 : 3
21. The collision of two balls of equal mass takes place at the origin of coordinates. Before collision, the components of velocities are ($U_x = 50$ cm/s, $U_y = 0$) and ($U_x = -40$ cm/s and $U_y = 30$ cm/s). The first ball comes to rest after collision. The velocity components (V_x and V_y respectively) of the second ball are
 (a) 10 and 30 cm/s (b) 30 and 10 cm/s
 (c) 5 and 15 cm/s (d) 15 and 5 cm/s
22. A hockey player receives a corner shot at a speed of 15 m/s at an angle of 30° with the y-axis and then shoots the ball along the x-axis with a speed of 30 m/s. If the mass of the ball is 150 g and it remains in contact with the hockey stick for 0.01 s, the force imparted to the ball in the x-direction is
 (a) 281.25 N (b) 187.5 N
 (c) 562.5 N (d) 375 N
23. A projectile of mass 20 kg is fired with a velocity of 400 m/s at an angle of 60° with the horizontal. At the highest point of the trajectory the projectile explodes into two fragments of equal mass, one of which falls vertically downward with zero initial speed. The distance of the point where the other fragment falls from the point of firing is
 (a) 21208 m (b) 2120 m
 (c) 212 m (d) 4240 m
24. A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 9.22×10^{-21} kg m/s and at right angles to the direction of the electron a neutrino with momentum 5.33×10^{-21} kg m/s. The momentum of the residual nucleus shall be
 (a) 17×10^{-20} kg m/s (b) 1.07×10^{-20} kg m/s
 (c) 7.01×10^{-20} kg m/s (d) 701×10^{-20} kg m/s
25. A machine gun fires 120 shots per minute. If the mass of each bullet is 10 g and the muzzle velocity is 800 m/s, the average recoil force on the machine gun is
 (a) 120 N (b) 8 N
 (c) 16 N (d) 12 N

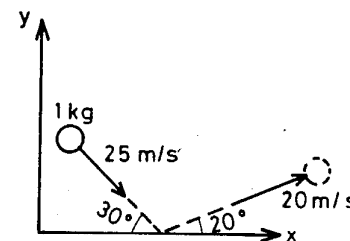


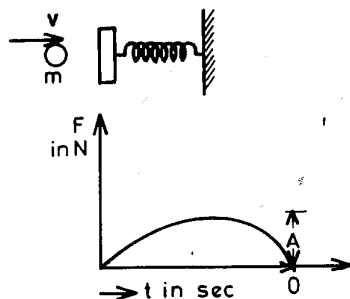
26. A car of mass 1000 kg is moving with a velocity of 70 km/h and another car of mass 1500 kg is moving 50 m ahead of the first with a velocity of 80 km/h. The velocity of the centre of mass of the two cars is
 (a) 213 m/s (b) 21.3 m/s
 (c) 321 m/s (d) 13.2 m/s
27. A uniform metal rod of length 1 m is bent at 90° so as to form two arms of equal length. The centre of mass of this bent rod is
 (a) on the bisector of the angle, 0.707 m from the vertex
 (b) on the bisector of the angle, 0.5 m from the vertex
 (c) on the bisector of the angle, 0.4 m from the vertex
 (d) on the bisector of the angle, 0.30 m from the vertex
28. A body of mass 2 kg moving with a velocity of 4 m/s collides with another body of mass 3 kg moving with a velocity of 4 m/s in the opposite direction. If the heavier body is brought to rest after collision, the velocity of the lighter body is
 (a) 6 m/s (b) 2 m/s
 (c) 12 m/s (d) 4 m/s
29. Two vehicles of equal masses are moving with same speeds V on two roads inclined at an angle θ . They collide inelastically at the junction and then move together. The speed of the combination is
 (a) $V \cos \frac{\theta}{2}$
 (b) $2V \cos \theta$
 (c) $\frac{V}{2} \cos \theta$ (d) $\frac{V}{2} \cos \frac{\theta}{2}$
30. Two carts are standing on a level frictionless track. A man on cart A throws a bag weighing 10 kg with a horizontal velocity of 4 m/s on to cart B of mass 100 kg. If the combined mass of the man and the cart A is 150 kg, the velocities of carts A and B respectively just after the bag has landed on cart B are
 (a) 0.18 and 0.24 m/s (b) 0.36 and 0.26 m/s
 (c) 0.26 and 0.36 m/s (d) 0.13 and 0.20 m/s
31. A ball of mass m is dropped from a height h on to a rigid floor. If e is the coefficient of restitution, the time required for complete bounce is
 (a) $\sqrt{\frac{2h}{g}} \left(\frac{1-e}{1+e} \right)$ (b) $\sqrt{\frac{2h}{g}} \left(\frac{1+e}{1-e} \right)$
 (c) $\sqrt{\frac{2h}{g}} (1-e)$ (d) $\sqrt{\frac{2h}{g}} (1+e)$
- 32.* A spherical ball A of mass m is released from rest on a smooth bowl 0.2 m high. The sphere slides down and collides elastically with another sphere B of mass $m/4$ placed on the bottom of the bowl. If the ball B has to just reach the top and escape the bowl, the height from where the sphere A should be released is



- (a) 0.08 m (b) 0.02 m
(c) 0.18 m (d) 0.10 m
33. A particle of mass m moves with a velocity V along a straight line and collides with another particle of mass $n.m$ which is moving with a velocity kV along the same line. If, after impact, the first particle is brought to rest, the velocity of the other particle after impact is
- (a) $\left(\frac{n}{1+nk}\right)V$ (b) $\left(\frac{1+nk}{n}\right)V$
(c) $\left(\frac{1-nk}{n}\right)V$ (d) $\left(\frac{n}{1-nk}\right)V$
- 34.* A particle of mass m is made to move with uniform speed V along the perimeter of a regular polygon of n sides, inscribed in a circle of radius a . The magnitude of impulse applied at each corner of the polygon is
- (a) $2mV \sin \frac{\pi}{n}$ (b) $mV \sin \frac{\pi}{n}$
(c) $mV \sin \frac{n}{\pi}$ (d) $2mV \sin \frac{n}{\pi}$
35. Two particles P and Q separated by a distance of 1 m have masses of 0.1 and 0.3 kg respectively and are initially at rest. A constant attractive force of 1.0×10^{-2} N acts between them. If no other external force acts between them, the distance with respect to the original position of P, where they will collide, is
- (a) 0.40 m (b) 0.75 m
(c) 0.25 m (d) 0.60 m
36. After a totally inelastic collision, two objects of the same mass and initial speed are found to move away together at half their initial speed. The angle between the initial velocities of the object is
- (a) 30° (b) 60°
(c) 120° (d) 45°
37. A gun is attached to a trolley that can move freely on a smooth level road. The total mass of the gun and the trolley is 10 kg. A bullet of mass 0.005 kg is shot horizontally to the right and is observed to have travelled a distance of 50 m in 0.2 s. In this interval of time, the trolley has moved a distance
- (a) 2.5 cm (b) -2.5 cm
(c) -4 cm (d) 4 cm
38. A body of mass M moves in outer space with velocity V . It is desired to break the body into two parts so that the mass of one part is one-tenth of the total mass. After the explosion, the heavier part comes to rest while the lighter part continues to move in the original direction of motion. The velocity of the small part will be
- (a) V (b) $\frac{V}{2}$
(c) $\left(\frac{V}{10}\right)$ (d) $10V$
39. A body A of mass 2 kg is projected upward from the surface of the ground at $t=0$ with a velocity of 20 m/s. One second later a body B, also of mass 2 kg, is dropped from a height of 20 m. If they collide elastically, then velocities just after collision are
- (a) $V_A = 5$ m/s downward, $V_B = 5$ m/s upward
(b) $V_A = 10$ m/s downward, $V_B = 5$ m/s upward

- (c) $V_A = 10$ m/s upward, $V_B = 10$ m/s downward
(d) both move downward with velocity 5 m/s
40. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to the other end of the plank. If the mass of the plank is $M/3$, the distance that the man moves relative to the ground is
- (a) $\frac{3L}{4}$ (b) $\frac{4L}{5}$
(c) $\frac{L}{4}$ (d) $\frac{L}{3}$
41. A continuous stream of particles of mass m and velocity v , is emitted from a source at a rate of n per second. The particles travel along a straight line, collide with a body of mass M and are buried in this body. If the mass M was originally at rest, its velocity when it has received N particles will be
- (a) $\frac{mun}{Nm+n}$ (b) $\frac{mvN}{Nm+M}$
(c) $\frac{mv}{Nm+M}$ (d) $\frac{Nm+M}{mv}$
42. A ballbearing strikes a steel plate at an angle α with the vertical. If the coefficient of restitution is e , the angle at which the rebound will take place is
- (a) α (b) $\tan^{-1}\left(\frac{\tan \alpha}{e}\right)$
(c) $e \tan \alpha$ (d) $\frac{e}{\tan \alpha}$
43. A ball of mass 1 kg bounces against the ground as shown in the figure. The approaching velocity is 25 m/s and the velocity after hitting the ground is 20 m/s. The impulse exerted on the ball is
- (a) 0.62 N · s
(b) 6.2 N · s
(c) 3.1 N · s
(d) 62 N · s
44. A particle of mass 15 kg has an initial velocity
- $$\vec{V}_i = 10\hat{i} - 20\hat{j} \text{ m/s}$$
- It collides with another body and the impact time is 0.1 s, resulting in a velocity $\vec{V}_f = 6\hat{i} + 4\hat{j} + 5\hat{k}$ m/s after impact. The average force of impact on the particle is
- (a) $150(-4\hat{i} + 24\hat{j} + 5\hat{k})$ N (b) $15(-4\hat{i} + 24\hat{j} + 5\hat{k})$ N
(c) $15(4\hat{i} - 24\hat{j} - 5\hat{k})$ N (d) $150(-4\hat{i} - 24\hat{j} - 5\hat{k})$ N
- 45.* A small ball of mass m and moving with a velocity V hits a spring-supported plate and rebounds. The deflection of the spring which is linear and the force on the sphere during the impact is calculated and the plot is shown to have a half-sine-curve relation with time. The impulse J will be



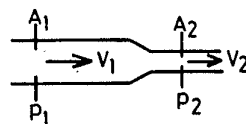


- (a) $A^2 \sin \frac{A}{5\pi}$ (b) $A \sin \pi$
 (c) $A \sin \frac{1}{5\pi}$ (d) $\frac{A}{5\pi}$

46. A machine gun fires a steady stream of bullets at the rate of n per minute into a stationary target in which the bullets get embedded. If each bullet has a mass m and arrives at the target with a velocity V , the average force on the target is

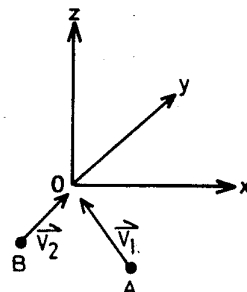
- (a) $60 mnV$ (b) $\frac{mnV}{60}$
 (c) $\frac{60V}{mn}$ (d) $\frac{mV}{60n}$

- 47.* A liquid of density ρ is flowing through a pipe whose cross-sectional area changes from A_1 to A_2 as shown in the figure. If the pressure and velocity of liquid in the wider portion are p_1 and v_1 respectively and p_2 and v_2 in the narrow portion respectively, then the force exerted by the liquid on the pipe is



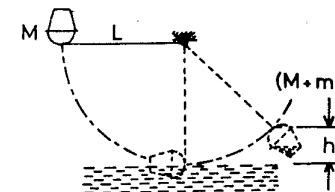
- (a) $(p_1 + \rho v_1^2) A_2 - (p_2 + \rho v_2^2) A_1$
 (b) $(p_2 + \rho v_2^2) A_2 - (p_1 + \rho v_1^2) A_1$
 (c) $(p_1 + \rho v_1^2) A_1 - (p_2 + \rho v_2^2) A_2$
 (d) $(p_1 - \rho v_1^2) A_2 - (p_2 - \rho v_2^2) A_1$
48. Two putty balls A and B of mass 2 and 3 kg respectively collide plastically at O with velocities $\vec{V}_1 = 12\hat{i} + 5\hat{j} + 3\hat{k}$ m/s and $\vec{V}_2 = 6\hat{i} + 6\hat{j}$ m/s. The velocity of the new ball is

- (a) $\frac{1}{5} (28\hat{i} + 42\hat{j} + 6\hat{k})$ (b) $\frac{1}{5} (42\hat{i} + 28\hat{j} + 6\hat{k})$
 (c) $\frac{1}{5} (6\hat{i} + 28\hat{j} - 42\hat{k})$ (d) $5(42\hat{i} - 6\hat{j} + 28\hat{k})$



49. In the above problem, the energy loss due to impact is
 (a) 62.7 J (b) 54 J
 (c) 27.6 J (d) 7.2 J

50. A small bucket of mass M kg is attached to a long inextensible cord of length L m as shown in the figure. The bucket is released from rest when the cord is in a horizontal position. At its lowest position, the bucket scoops up m kg of water and swings up to a height h . The height h in metres is



- (a) $\left(\frac{M}{M+m}\right)^2 L$ (b) $\left(\frac{M}{M+m}\right) L$
 (c) $\left(\frac{M+m}{M}\right)^2 L$ (d) $\left(\frac{M+m}{M}\right) L$

Answers

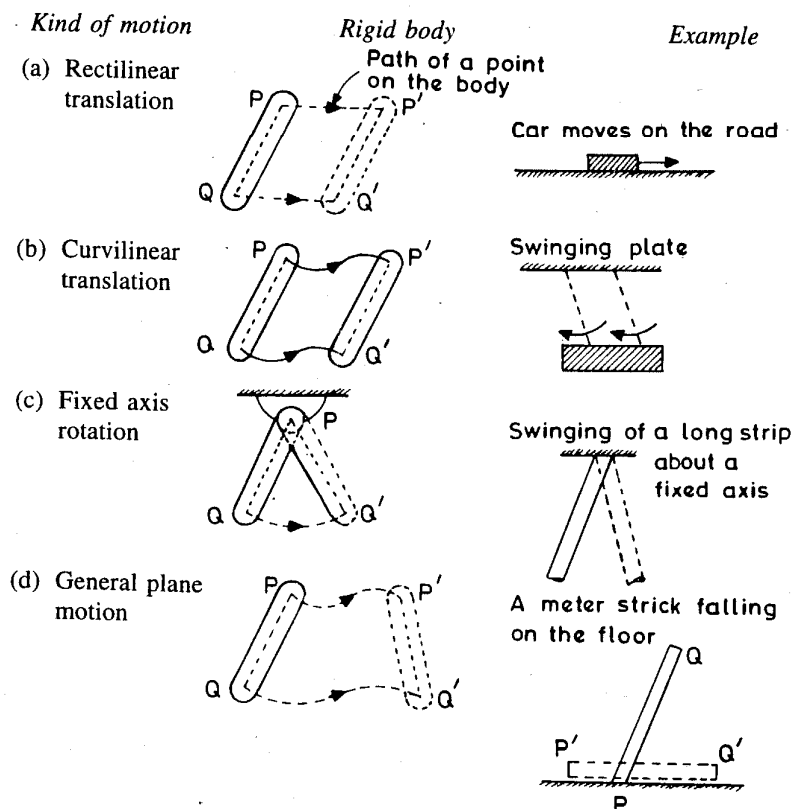
- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (a) | 4. (b) | 5. (d) |
| 6. (c) | 7. (b) | 8. (d) | 9. (a) | 10. (c) |
| 11. (a) | 12. (d) | 13. (a) | 14. (b) | 15. (a) |
| 16. (c) | 17. (b) | 18. (a) | 19. (a) | 20. (b) |
| 21. (a) | 22. (c) | 23. (a) | 24. (b) | 25. (c) |
| 26. (b) | 27. (a) | 28. (b) | 29. (a) | 30. (c) |
| 31. (d) | 32. (a) | 33. (b) | 34. (a) | 35. (b) |
| 36. (c) | 37. (b) | 38. (d) | 39. (a) | 40. (c) |
| 41. (b) | 42. (b) | 43. (b) | 44. (a) | 45. (d) |
| 46. (b) | 47. (c) | 48. (b) | 49. (c) | 50. (a) |

Note: Problems marked with a star should take about five minutes. All the rest should be solved in about three minutes each.

6

Rotational Motion of Rigid Bodies

The plane motion of a rigid body may be divided into several categories as represented in the following figures:



Translation is defined as any motion in which every line in the body remains parallel to its original position at all times. In this, there is no rotation of any line in the body. In rectilinear translation, there is no relative motion between any two points and all points move in parallel straight lines. In curvilinear

motion, all points move along congruent curves. Rotation about a fixed axis is the angular motion about the axis. It follows that particles move in circular paths about the axis of rotation and all lines in the body rotate through the same angle in the same time. General plane motion of a rigid body is a combination of translation and rotation.

Angular Displacement (θ)

Angular displacement is usually expressed in radians, in degrees or in revolutions.

$$1 \text{ rev} = 360^\circ = 2\pi \text{ rad} \quad \text{or} \quad 1 \text{ rad} = 57.3^\circ$$

In terms of arc of a circle of length s , and the radius of the circle r , θ in radians is given by

$$\theta = \frac{s}{r}$$

Angular Velocity (ω)

The angular velocity of an object is the rate at which angular displacement changes with time. If the initial angle is θ_0 and after an interval of time t , the angle becomes θ_f , then the average angular velocity $\bar{\omega}$ is

$$\bar{\omega} = \frac{\theta_f - \theta_0}{t}, \text{ given in rad/s or rpm (revolutions per minute). The frequency of rotation } \nu \text{ is given by}$$

$$\nu = \frac{\omega}{2\pi} \text{ rev/s}$$

Average angular acceleration (α) is given by

$$\bar{\alpha} = \frac{\omega_f - \omega_0}{t}$$

where initial angular velocity ω_0 changes to final angular velocity ω_f in an interval of time t . The units are rad/s^2 or rev/min^2 .

The equations of motion, similar to the equations in linear motion, are the following, with the notations having their usual meaning:

$$\bar{\omega} = \frac{1}{2} (\omega_0 + \omega_f)$$

$$\theta = \bar{\omega} \cdot t$$

$$\omega_f = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta$$

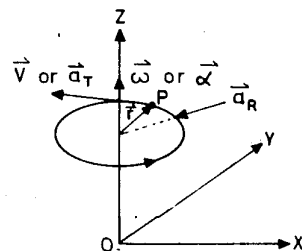
$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Vector Relations Between Variables in Linear and Rotational Motion

$$\vec{V} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{a}_T + \vec{a}_R$$

$$\vec{a}_T = \vec{\alpha} \times \vec{r} \text{ and } \vec{a}_R = \vec{\omega} \times \vec{V}$$



Torque on a Particle

If a force $\vec{F}$ acts on a single particle at a point P whose position with respect to the origin O of the inertial reference frame is given by the displacement vector $\vec{r}$, the torque $\vec{\tau}$ acting on the particle with respect to the origin O is defined as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = rF \sin \theta$$

The direction of torque shall be normal to the plane containing $\vec{r}$ and $\vec{F}$ and will be governed by the right hand rule for the vector product of two vectors. Its units are $\text{N} \cdot \text{m}$

Angular Momentum of a Particle

A particle of mass m and linear momentum $\vec{p}$ has a position vector $\vec{r}$ with respect to the origin O of an inertial frame of reference as considered in the above figure. The angular momentum $\vec{L}$ is defined as

$$\vec{L} = \vec{r} \times \vec{p} \text{ with respect to the origin O.}$$

$$|\vec{L}| = rp \sin \theta$$

Its units are $\text{kg m}^2/\text{s}$.

Relations Between Angular Momentum, Linear Momentum and Torque

$$\vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}; \quad \vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\begin{aligned} \frac{d\vec{L}}{dt} &= \frac{d}{dt} (\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \\ &= 0 + \vec{r} \times \frac{d\vec{p}}{dt} \end{aligned}$$

$$\therefore \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$$

$$\text{and } \vec{\tau} = \frac{d\vec{L}}{dt}$$

If $\vec{\tau}_{\text{ext}}$ stands for the sum of all external torques on the system, then $\vec{\tau}_{\text{ext}} = d\vec{L}/dt$.

Conservation of Angular Momentum

If $\vec{\tau}_{\text{ext}} = 0$, $\vec{L}$ is constant. This means that in the absence of external torque, angular momentum is conserved.

Kinetic Energy of Rotation and Rotational Inertia

If K is the kinetic energy of a rigid body which is rotating with an angular velocity ω about a certain axis, then

$$K = \frac{1}{2} I \cdot \omega^2$$

where I is called the moment of inertia of the body about that particular axis of rotation.

The moment of inertia or rotational inertia is given by the expression

$$I = \sum m_i r_i^2$$

The term $\sum m_i r_i^2$ is the sum of the products of masses of the particles and the squares of their respective distances from the axis of rotation. It is expressed in the units of kg m^2 .

Radius of Gyration

The radius of gyration k for an object about an axis can be expressed by the relation

$$I = Mk^2$$

where M is the total mass of the object. Hence, k is the distance a point mass M must be from the axis if the point mass is to have the same inertia as the object.

Parallel Axis Theorem

The moment of inertia I of a body about an axis parallel to an axis through the centre of mass is

$$I = I_{\text{CM}} + Mh^2$$

where I_{CM} is the MI of the body about the CM axis M is the total mass of the body and h is the perpendicular distance between the two parallel axes.

Perpendicular Axes Theorem

Consider a mass distribution in the x-y plane of an x-y-z coordinate system. Let I_x , I_y and I_z denote the moments of inertia about x-, y- and z-axis respectively. Then

$$I_z = I_x + I_y$$

Moments of Inertia of Some Common Bodies

Body	Axis	Moment of inertia
1. Hoop or hollow cylinder of radius r and mass M	Through CM and normal to the plane	Mr^2
2. Disc or cylinder of radius r and mass M	Through CM and about the axis of cylinder	$\frac{1}{2} Mr^2$
3. Uniform rod of length L and mass M	Through CM and perpendicular to the rod	$\frac{1}{12} ML^2$
4. Solid sphere of radius r and mass M	Through the diameter	$\frac{2}{5} Mr^2$
5. Hollow sphere of radius r and mass M (sphere thickness negligible)	Through the diameter	$\frac{2}{3} Mr^2$
6. Rectangular plate of sides a and b and mass M	Perpendicular to the plate through CM	$\frac{1}{12} M(a^2 + b^2)$
7. Thin rod of length L and mass M	Through one end and perpendicular to length	$\frac{ML^2}{3}$
8. Solid cylinder of mass M , radius r and length L	Through CM and perpendicular to the axis of the cylinder	$\frac{Mr^2}{4} + \frac{ML^2}{12}$
9. Annular cylinder or ring of inner radius r_1 and outer radius r_2 and mass M	Through CM and along the axis of the cylinder	$\frac{M}{2} (r_1^2 + r_2^2)$
10. Hoop of radius r and mass M	About any tangential line	$\frac{3}{2} Mr^2$

Combined Rotation and Translation

The kinetic energy of a rolling ball or other rolling object of mass M is the sum of: (i) rotational kinetic energy about an axis through its centre of mass,

and (ii) the translational kinetic energy of an equivalent point mass moving with the centre of mass.

$$K_{\text{total}} = \frac{1}{2} I \omega^2 + \frac{1}{2} M V^2$$

Note that I is the moment of inertia of the object about an axis through its centre of mass and V is the velocity of the centre of mass.

Work Done in Rotation

If on application of a torque τ on a body, it is rotated through an angular displacement θ , then the work done by the torque is

$$W = \tau \theta$$

where W is in joules and θ is in radians.

The power (P) transmitted to a body by a torque τ is given by

$$P = \tau \omega$$

$$\text{Angular Impulse } \tau \cdot t = I \omega_f - I \omega_i$$

Here $\tau \cdot t$ is the magnitude of the angular impulse where a torque τ is applied for a time t and the angular velocity undergoes a change from ω_i to ω_f in that interval of time.

ILLUSTRATIONS

1. The shaft of an electric motor starts from rest and, on the application of a torque, it gains an angular acceleration given by $\alpha = 4t - t^2$ during the first three seconds after it starts, after which $\alpha = 0$. What will be the angular velocity after 5 seconds?

Solution

$$\alpha = 4t - t^2$$

$$\frac{d\omega}{dt} = 4t - t^2 \quad \text{or} \quad \int_0^\omega d\omega = \int_{t=0}^3 (4t - t^2) dt$$

$$\text{or} \quad \omega = \frac{4t^2}{2} \Big|_{t=0}^{t=3} - \frac{t^3}{3} \Big|_0^3$$

$$\omega = (18 - 9) = 9 \text{ rad/s}$$

Since there is no angular acceleration after 3 s, the angular velocity after 5 s is the same, i.e. 9 rad/s.

2. In the above problem, what will be the total angular displacement of the shaft in 5 s?

Solution

In the first three seconds, angular displacement is

$$\int_0^{\theta_1} d\theta = \int_{t=0}^3 2t^2 dt - \int_{t=0}^3 \frac{t^3}{3} dt$$

$$\text{or } \theta_1 = \left. \frac{2t^3}{3} \right|_0^3 - \left. \frac{t^4}{12} \right|_0^3$$

$$= 11.25 \text{ rad.}$$

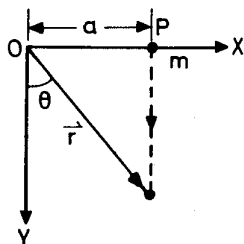
The angular displacement in the next two seconds is

$$\theta_2 = \omega \cdot t = 9 \times 2 = 18 \text{ rad.}$$

$\therefore$ Total angular displacement of the shaft = $18 + 11.25 = 29.25 \text{ rad.}$

$$\text{or } \frac{29.25}{2\pi} = 4.7 \text{ rev.}$$

3. A particle of mass m is at rest at a point P on the X-axis at a distance a from the origin. It is released to have a free fall parallel to the y-axis (shown by a dashed line). Find the torque acting on m at any instant of time, say t with respect to the origin O . Also find the angular momentum of the particle.



Solution

$$\text{Torque } \vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = rF \sin \theta$$

$$\text{Here } F = mg \quad \text{and} \quad r \sin \theta = a$$

$$\therefore \tau = mga = a \text{ constant}$$

The right-hand rule shows that τ is directed perpendicular and directed into the figure at O .

$$\text{Angular momentum, } \vec{L} = \vec{r} \times \vec{p}$$

$$|\vec{L}| = rp \sin \theta$$

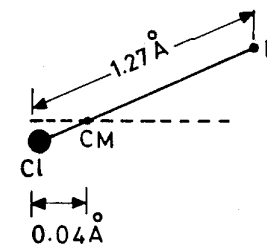
$$r \sin \theta = a \quad \text{and} \quad p = mV = m(gt)$$

$$\therefore |\vec{L}| = a \cdot m \cdot gt$$

The right-hand rule shows that the direction of the angular momentum vector is perpendicular and directed into the figure at O . We observe that $\vec{\tau}$ and $\vec{L}$ both are parallel. The magnitude of $\vec{\tau}$ remains constant with time while that of $\vec{L}$ changes with time.

Note: (i) the relation $\vec{\tau} = d\vec{L}/dt$ is satisfied in the above relations, and (ii) the values of τ and L depend on our choice of origin, that is on O . If the particle is released from the origin, then τ and L about the origin will be zero.

4. A molecule of hydrogen chloride consists of a hydrogen atom separated by $1.27 \text{ \AA}$ from a chlorine atom that has a mass 35 times greater than the mass of the hydrogen atom. The mass is concentrated in the nuclei and hence the molecule has essentially zero moment of inertia about an axis that consists of the line passing through the two atoms as indicated in the figure. Find the moment of inertia of the molecule about an axis passing through the centre of mass of the molecule and at right angles to the line joining the two atoms.



Solution

$$\text{Location of CM: } \frac{r_H}{r_{Cl}} = \frac{m_{Cl}}{m_H} \quad (i)$$

$$r_H + r_{Cl} = 1.27 \text{ \AA} \quad (ii)$$

$$\therefore r_H = 1.23 \text{ \AA} \quad \text{and} \quad r_{Cl} = 0.04 \text{ \AA}$$

$$1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$$

$$I = m_H r_H^2 + m_{Cl} r_{Cl}^2$$

$$= 1.66 \times 10^{-27} \times (1.23 \times 10^{-10})^2 + 35 \times 1.66 \times 10^{-27} \times (0.04 \times 10^{-10})^2$$

$$= (2.51 + 0.09) \times 10^{-47} \text{ kg m}^2 = 2.60 \times 10^{-47} \text{ kg m}^2$$

5. Referring to the above problem, it is found from spectroscopic data that the HCl molecule may rotate with an energy of $4.24 \times 10^{-22} \text{ J}$. What is the frequency of rotation of the molecule?

Solution

$$I = 2.60 \times 10^{-47} \text{ kg m}^2 \text{ (from the above problem)}$$

$$E_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} I (2\pi\nu)^2$$

$$\nu = \frac{1}{2\pi} \sqrt{\frac{2E_{\text{rot}}}{I}} = \frac{1}{2\pi} \sqrt{\frac{2 \times 4.24 \times 10^{-22}}{2.60 \times 10^{-47}}} = 9.09 \times 10^{11} \text{ Hz}$$

6. A mass m hangs by a massless string from a pulley of mass M and radius R . Find the rate of acceleration of the mass m as it falls. What is the angular acceleration of the pulley?



Solution

Let T be the tension in the string.

As the mass m is coming down, the equation of motion is

$$mg - T = ma$$

The tension in the string exerts a torque on the pulley

$$\tau = I\alpha$$

where $I = \frac{1}{2}MR^2$

$$TR = \tau = \left(\frac{1}{2}MR^2\right)\alpha \Rightarrow \alpha = \frac{2T}{MR}$$

$$a = R\alpha$$

$$\therefore a = R \frac{2T}{MR} = \frac{2T}{M}$$

Eliminating T , we get

$$a = \left(\frac{2m}{M+2m}\right)g$$

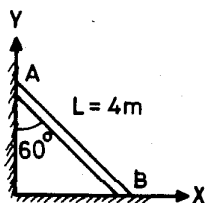
$$\tau = TR = I\alpha$$

$$\frac{a}{R} = \left(\frac{2m}{M+2m}\right)\frac{g}{R}$$

Hence Acceleration of $m = \left(\frac{2m}{M+2m}\right)g$

and Angular acceleration of pulley $= \left(\frac{2m}{M+2m}\right)\frac{g}{R}$

7. A rod AB of length L slides in the X-Y plane. The velocity and acceleration of the end A at the instant shown in the figure are respectively 4 m/s and 2 m/s² downward. If the rod makes an angle of 60° to the vertical, find the angular velocity of the rod and its angular acceleration.



Solution

From the figure, the y-coordinate of A is

$$y_A = L \cos \theta$$

$$\frac{dy_A}{dt} = -L \frac{d\theta}{dt} \sin \theta$$

$$\frac{d\theta}{dt} = \frac{-(dy_A/dt)}{L \sin \theta} = \frac{-4}{4 \times \sqrt{3}/2} = -1.15 \text{ rad/s} \quad (\text{counterclockwise})$$

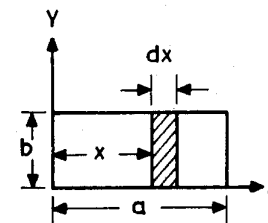
Angular acceleration,

$$\frac{d^2y}{dt^2} = -L \left(\frac{d\theta}{dt}\right)^2 \cos \theta - L \frac{d^2\theta}{dt^2} \sin \theta$$

$$\frac{d^2\theta}{dt^2} = \frac{-(d^2y/dt^2) - L(d\theta/dt)^2 \cos \theta}{L \sin \theta}$$

$$= \frac{(2) - 4 \times (1.15)^2 \times 1/2}{4 \times \sqrt{3}/2} = -0.19 \text{ rad/s}^2 \quad (\text{clockwise})$$

8. Find the moment of inertia and the radius of gyration of a rectangular plate with sides a and b about an axis passing through one of its sides.



Solution

If σ be the mass per unit area, then mass of the element (shaded part) is $= \sigma b \cdot dx$ and its moment of inertia about the y-axis is $(\sigma b dx)x^2 = \sigma b x^2 dx$

$$I = \int_{x=0}^{x=a} \sigma b x^2 dx = \frac{1}{3} \sigma b a^3$$

since mass of the plate $M = ab\sigma$, therefore, $I = \frac{1}{3} Ma^2$

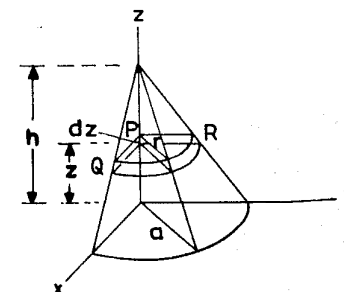
Also $I = M \cdot K^2$

$$\therefore K = \frac{a}{\sqrt{3}} \quad \text{or} \quad \frac{1}{3} a \sqrt{3}$$

9. Find the moment of inertia of a right circular cone of height h and radius a about its axis.

Solution

The moment of inertia of the circular cylindrical disc one quarter of which is represented in the figure by PQR is



$$= \frac{1}{2} (\pi r^2 \sigma dz) (r^2) = \frac{1}{2} \pi \sigma r^4 dz$$

From the figure,

$$\frac{h-z}{h} = \frac{r}{a} \quad \text{or} \quad r = a \left(\frac{h-z}{h} \right)$$

$$\therefore I = \frac{1}{2} \pi \sigma \int_{z=0}^{z=h} \left\{ a \left(\frac{h-z}{h} \right) \right\}^4 dz = \frac{1}{10} \pi a^4 \sigma h$$

$$\text{Also } M = \pi \sigma \int_{z=0}^{z=h} \left\{ a \left(\frac{h-z}{h} \right) \right\}^2 dz = \frac{1}{3} \pi a^2 h \sigma$$

$$\therefore I = \frac{3}{10} M a^2$$

10. A solid cylinder of radius R and mass M rolls without slipping down an inclined plane of angle θ . Find its acceleration.

Solution

Let N be the normal reaction of the incline on the cylinder and f be the force of static friction acting along the incline.

For motion normal to the plane:

$$N - Mg \cos \theta = 0$$

For motion along the plane

$$Mg \sin \theta - f = Ma$$

The rotational motion is governed by the relation

$$\tau = I_{CM} \alpha$$

where α is angular acceleration. Forces that pass through the centre of mass of the cylinder cannot cause any rotation about it as they have zero moment arms. The only force that causes rotation, therefore, is friction. Hence

$$fR = I_{CM} \alpha$$

$$I_{CM} = \frac{1}{2} MR^2 \text{ and } \alpha = \frac{a}{R}$$

$$\therefore f = \frac{Ma}{2}$$

$$\therefore a = \frac{2}{3} g \sin \theta$$

11. A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The

torque produces a counterclockwise angular acceleration $\alpha = 4t \text{ rad/s}^2$ where t is the time in seconds during which the torque is applied. Find the time required to reduce its clockwise angular speed to 900 rev/min. How much time is required for the flywheel to reverse its direction of rotation?

Solution

$$\text{Initial angular velocity} = \frac{-1800(2\pi)}{60} = -60\pi \text{ rad/s}$$

$$d\omega = \alpha \cdot dt \Rightarrow \int_{-60\pi}^{\omega} d\omega = \int_0^t 4t \cdot dt$$

$$\omega = -60\pi + 2t^2$$

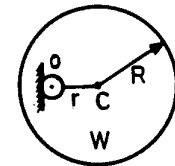
$$-900 \text{ rev/min} = -30\pi \text{ rad/s}$$

$$\therefore -30\pi = -60\pi + 2t^2 \Rightarrow t^2 = 15\pi \Rightarrow t = 6.86 \text{ s}$$

Change of direction of rotation will take place when the angular velocity becomes zero.

$$0 = -60\pi + 2t^2 \Rightarrow t^2 = 30\pi \Rightarrow t = 9.71 \text{ s}$$

12. A disc of weight W has a radius R and can rotate freely about a horizontal shaft at O which is located at a distance r from the centre of mass of the disc, C . Assume that the disc is released from the position shown in the figure, find the initial angular acceleration of the disc.



Solution

$$I_c = \frac{1}{2} \cdot \frac{W}{g} \cdot R^2 \text{ and } I = I_0 = I_c + \left(\frac{W}{g} \right) r^2$$

$$= \frac{1}{2} \cdot \frac{W}{g} \cdot R^2 + \frac{W}{g} r^2$$

$$= \frac{W}{g} \left(\frac{R^2}{2} + r^2 \right)$$

The weight of the disc acting at C causes a torque, given by

$$Wr = \frac{W}{g} \left(\frac{R^2}{2} + r^2 \right) \alpha$$

where α is angular acceleration.

$$\therefore \alpha = \frac{rg}{R^2/2 + r^2}$$

13. In the above problem, find the value of r for which the angular acceleration of the disc is maximum.

Solution

From the above problem,

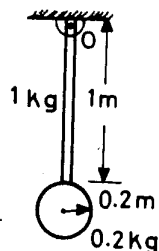
$$\alpha = \frac{rg}{(R^2/2) + r^2}$$

For a maximum, $d\alpha/dr = 0$. Hence

$$\frac{d\alpha}{dr} = \frac{g}{((R^2/2) + r^2)^2} \left(\frac{R^2}{2} + r^2 - 2r^2 \right) = 0$$

$$\text{or } r^2 = \frac{1}{2} R^2 \quad \text{or } r = \frac{R}{\sqrt{2}}$$

14. A pendulum consists of a uniform rod of mass 1 kg and length 1 m and a circular disc of mass 0.2 kg and radius 0.2 m. Given small oscillations of the pendulum, determine their frequency.



Solution

$$MI \text{ about } O = \frac{1}{3} m_R L^2 + \frac{1}{2} m_D r^2 + m_D (L + r)^2$$

(m_R and m_D are masses of the rod and disc respectively)

$$= \frac{1}{3} \times 1 \times 1^2 + \frac{1}{2} \times 0.2 \times (0.2)^2 + 0.2 (1 + 0.2)^2$$

$$= \frac{1}{3} + 0.004 + 0.288 = 0.625 \text{ kg m}^2$$

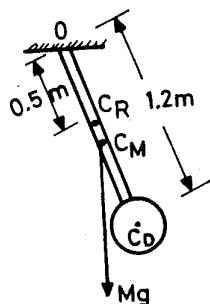
We shall have to locate the centre of mass of the rod-disc system in order to find the torque. The CM of the rod is at 0.5 m and its mass is 1 kg. The CM of the disc is at the centre and its mass is 0.2 kg.

Position of CM of the system

$$= \frac{1 \times 0.5 + 0.2 \times 1.2}{1.2}$$

$$= \frac{0.74}{1.2} = 0.62 \text{ m}$$

$$\tau = I\alpha = I \frac{d^2\theta}{dt^2} = -1.2 g (0.62) \theta$$



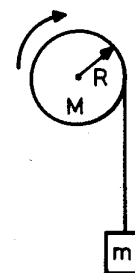
Here 1.2 is the mass, 0.62 is the effective length and θ is the angular displacement of the system. Torque on it is given by $MgL \cdot \theta$

$$\therefore \tau = -7.4 \theta \quad \text{or} \quad K = 7.4$$

$$\therefore v = \frac{1}{2\pi} \sqrt{\frac{K}{I}} = \frac{1}{2\pi} \sqrt{\frac{7.4}{0.62}} = \frac{1}{2\pi} \times 3.45$$

$$= 0.55/\text{s}$$

15. A mass m hangs from the rim of a wheel of radius R . When released from rest, the mass falls through a height h in t seconds. Find the moment of inertia of the wheel.



Solution

$$h = \frac{1}{2} at^2$$

$$\text{or } a = \frac{2h}{t^2} \Rightarrow \alpha = \frac{2h}{Rt^2}$$

$$mg - T = ma$$

$$T = m(g - a)$$

$$\tau = I\alpha \Rightarrow TR = I\alpha$$

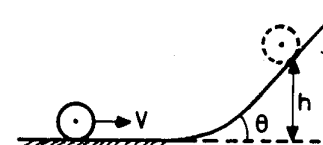
$$m(g - a)R = I\alpha \quad \text{or} \quad I = \frac{m(g - a)R}{\alpha}$$

$$\therefore I = \frac{m(g - (2h/t^2))R}{2h/Rt^2} = \frac{m(gt^2 - 2h)R^2}{2h}$$

The moment of inertia of the wheel is

$$I = \frac{m(gt^2 - 2h)R^2}{2h}$$

16. A disc of mass M and radius R rolls on a horizontal surface and then rolls up an inclined plane as shown in the figure. If the velocity of the disc is V , find the height h to which the disc will rise on the incline.



Solution

The rotational and translational kinetic energies of the disc at the bottom will be changed to gravitational potential energy when it stops. Therefore,

$$\left(\frac{1}{2} MV^2 + \frac{1}{2} I\omega^2 \right)_{\text{start}} = (Mgh)_{\text{end}}$$

$$\text{MI of disc} = \frac{1}{2} MR^2$$

$$\text{and for rolling } \omega = \frac{V}{R}$$

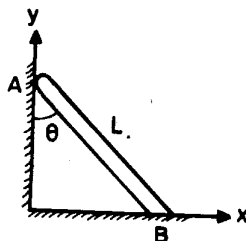
$$\frac{1}{2} MV^2 + \frac{1}{2} \left(\frac{1}{2} MR^2 \right) \left(\frac{V}{R} \right)^2 = Mgh$$

$$V^2 + \frac{1}{2} V^2 = 2gh \text{ or } h = \frac{3V^2}{4g}$$

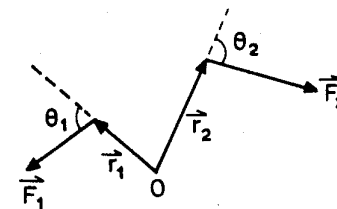
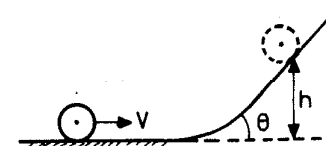
Note that the height to which the body rises is independent of its mass and the angle of the inclined plane. It is directly proportional to the square of the velocity.

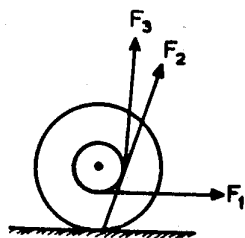
EXERCISES

- Two cones A and B are made of two different materials, the density of A being greater than that of B. The height of B is greater than that of A but their base areas and masses are the same. The correct statement about the moment of inertia of the two cones about their axis is
 - A will have larger moment of inertia than B.
 - B will have larger moment of inertia than A.
 - in such a situation, it is dependent upon the height of the cone, the mass of the cone and radius of the base.
 - the moment of inertia of the two will be the same as it is not dependent on height of the cone but depends only upon the mass and the base area.
- A rod AB of length L slides in the X-Y plane. If the rod makes an angle θ with the vertical, the angular velocity of the rod will be
 - directly proportional to the length of the rod and the linear velocity of the end A at that instant.
 - independent of the length of the rod but will depend on the angle θ .
 - independent of θ but will depend on the length of the rod and linear velocity of end A at that instant.
 - dependent upon the length of the rod, the angle θ and also on the linear velocity of the end A of the rod at that instant.
- A solid cylinder and a solid sphere, both having the same mass and radius, are released from an incline of angle θ one by one. They roll on the incline without slipping. The statement that holds good in this motion is that
 - the force of friction that acts on the two is the same.
 - the force of friction is greater in case of a sphere than for a cylinder.

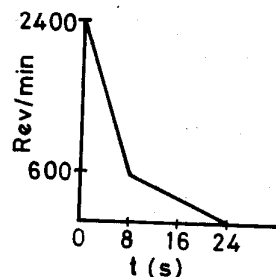
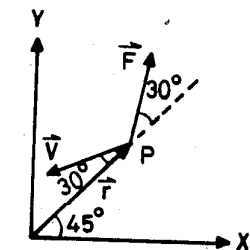


- the force of friction is greater in case of a cylinder than for a sphere.
 - the force of friction will depend on the nature of the surface of the body that is moving and that of the inclined surface, and is independent of the shape and size of the moving body.
- A sphere of mass M and radius R moves on a horizontal surface with a velocity V and then climbs up an inclined plane upto a height h where it stops. The height upto which it rises will be
 - directly proportional to the square of the velocity and inversely proportional to the angle of the inclination.
 - directly proportional to the velocity and inversely proportional to its mass.
 - directly proportional to the square of the velocity and independent of mass and the angle of the inclination.
 - directly proportional to its velocity and inversely proportional to the angle of the inclination.
 - A shaft rotating initially at 1725 rpm is brought to rest in 20 seconds. The number of revolutions that the shaft will make during this interval of time is
 - 1680
 - 840
 - 287
 - 627
 - An automobile decelerates from a speed of 60 mph to rest in a period of 15 seconds. If the radius of the wheel is 1 ft, the number of revolutions that the wheel makes during this interval is
 - 105
 - 600
 - 501
 - 510
 - Two particles, each of mass m and moving with speed V in opposite directions along parallel lines, are separated by a distance d . The vector angular momentum of this system of particles will be
 - maximum when the origin is taken beyond the two parallel lines on either side.
 - minimum when the origin is taken beyond the two parallel lines on either side.
 - maximum when the origin lies anywhere on the middle line between the two.
 - same no matter which point is taken as the origin.
 - In the figure are shown the lines of action and moment arms of two forces about the origin O. Imagining these forces to be acting on a rigid body pivoted at O, all vectors shown being in the plane of the figure, the magnitude and direction of the resultant torque will be
 - $(F_2 r_2 \sin \theta_2 - F_1 r_1 \sin \theta_1)$ out of the plane of the page.
 - $(F_1 r_1 \sin \theta_1 - F_2 r_2 \sin \theta_2)$ out of the plane of the page.
 - $(F_2 r_2 \sin \theta_2 - F_1 r_1 \sin \theta_1)$ into the plane of the page.
 - zero.
 - A yo-yo is resting on a horizontal table. Forces F_1 , F_2 and F_3 are applied separately as shown. The correct statement is

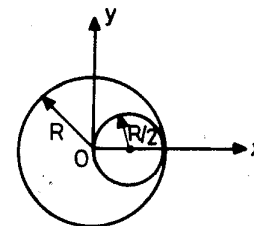




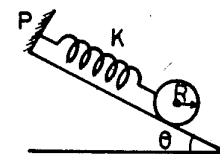
- (a) when F_3 is applied the centre of mass will move to the right.
 (b) when F_2 is applied the centre of mass will move to the left.
 (c) when F_1 is applied the centre of mass will move to the right.
 (d) when F_2 is applied the centre of mass will move to the right.
10. A particle P with a mass 2.0 kg has position vector $r = 3.0 \text{ m}$ and velocity $V = 4.0 \text{ m/s}$ as shown. It is accelerated by the force $F = 2.0 \text{ N}$. All three vectors lie in a common plane. The angular momentum vector is
- (a) $12 \text{ kg m}^2/\text{s}$ out of the plane of the figure.
 (b) $12 \text{ kg m}^2/\text{s}$ into the plane of the figure.
 (c) zero.
 (d) $24 \text{ kg m}^2/\text{s}$ out of the plane of the figure.
11. In the above problem, the torque is
- (a) 6 Nm out of the page (b) 3 Nm out of the page
 (c) 6 Nm into the page (d) 3 Nm into the page
12. A rotating wheel has a speed of 1200 rpm and then it is made to slow down at a constant rate of 4 rad/s^2 . The number of revolutions it makes before coming to rest will be
- (a) 272 (b) 314
 (c) 722 (d) 143
13. A table fan, rotating at a speed of 2400 rpm , is switched off and the resulting variation of the rpm with time is shown in the figure. The total number of revolutions of the fan before it comes to rest is
- (a) 420 (b) 280
 (c) 190 (d) 380
14. A car is decelerating uniformly from 36 km/h to 18 km/h in a distance of 50 m . If the diameter of the wheel is 600 mm , the angular acceleration of the wheel is
- (a) 2.5 rad/s^2 (b) 2 rad/s^2
 (c) 5 rad/s^2 (d) 7.5 rad/s^2



15. The moment of inertia of a solid cylinder of mass M , length L and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is
- (a) $\frac{1}{2} MR^2$ (b) $ML^2 + \frac{1}{2} MR^2$
 (c) $\frac{MR^2}{4} + \frac{ML^2}{12}$ (d) $\frac{ML^2}{4} + \frac{MR^2}{12}$
- 16.* A hole of radius $R/2$ is cut from a thin circular plate of radius R . The moment of inertia of the plate about an axis through O perpendicular to the X - Y plane (i.e. about the Z -axis) is



- (a) $\frac{5}{7} MR^2$ (b) $\frac{7}{12} MR^2$
 (c) $\frac{11}{24} MR^2$ (d) $\frac{13}{24} MR^2$
17. A rotor of mass 100 kg and radius of gyration 0.8 m is rotating at a speed of 1440 rpm . If the rotor comes to rest after 150 seconds , the frictional torque at the bearings is
- (a) 8 Nm (b) 64 Nm
 (c) 48 Nm (d) 24 Nm
18. A solid cylinder of mass M and radius R rolls from rest down a plane inclined at angle θ to the horizontal. The velocity of the centre of mass of the cylinder after it has rolled down a distance d is
- (a) $\sqrt{\frac{2}{3} gd \tan \theta}$ (b) $\sqrt{gd \tan \theta}$
 (c) $\sqrt{\frac{3}{4} gd \sin \theta}$ (d) $\sqrt{\frac{4}{3} gd \sin \theta}$
19. A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle θ to the horizontal. The cylinder is connected to a spring of spring constant K while the other end of the spring is connected to a rigid support at P . The cylinder is released when the spring is unstretched. The maximum distance that the cylinder travels is
- (a) $\frac{3}{4} \frac{Mg \sin \theta}{K}$ (b) $\frac{Mg \tan \theta}{K}$
 (c) $\frac{2Mg \sin \theta}{K}$ (d) $\frac{4}{3} \frac{Mg \sin \theta}{K}$

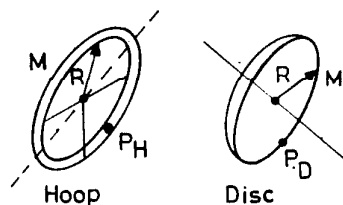


20. A uniform disc of radius 0.20 m and mass 2.5 kg is mounted on an axle which can rotate on frictionless bearings. A light string is wrapped around the rim of the disc. The tension, with which the string should be pulled down so that the angular acceleration of the disc is 20 rad/s^2 , is

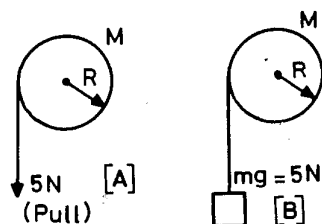
(a) 5 N (b) 10 N
(c) 50 N (d) 25 N

21. A hoop of mass M and radius R rotates about an axis passing through its centre of mass and normal to its plane (spokes shown are assumed to be massless), and a disc of same mass and radius also rotates about a similar axis as shown in the figure with the same angular momentum. The radial acceleration of a point P_H on the rim of the hoop and P_D on the rim of disc respectively will be

(a) equal (b) in the ratio of 4 : 1
(c) in the ratio of 1 : 4 (d) in the ratio of 1 : 2



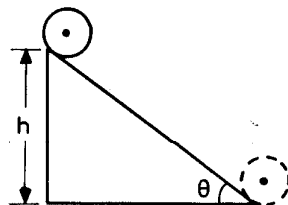
- 22.* A uniform disc of mass $M = 2.50 \text{ kg}$ and radius $R = 0.20 \text{ m}$ is mounted on an axle supported on fixed frictionless bearings. A light cord wrapped around the rim is pulled with a force 5 N. On the same system of pulley and string, instead of pulling it down, a body of weight 5 N is suspended. If the first process is termed A and the second B, the tangential acceleration will be
- (a) equal in the processes A and B
(b) greater in process A than in B
(c) greater in process B than in A
(d) independent of the two processes



23. A disc of mass 2 kg and radius 0.2 m starts rotating from rest about an axis passing through its centre and normal to its plane as in the above problem, when the string wrapped around the rim is pulled steadily with a tension of 5 N, causing a uniform angular acceleration of 10 rad/s^2 . The work done in 4.0 seconds is
- (a) 80 J (b) 40 J
(c) 50 J (d) 100 J

24. A solid cylinder of mass M and radius R rolls down an inclined plane without slipping and reaches the bottom as shown in the figure. Its velocity at the instant it reaches the bottom is

(a) $\sqrt{\frac{10}{7} gh}$
(b) $\sqrt{\frac{3}{4} gh}$



(c) $\sqrt{\frac{2}{3} gh}$

(d) $\sqrt{\frac{4}{3} gh}$

25. If, in the above problem, instead of a cylinder, a sphere of the same mass and radius is released, its velocity at the bottom will be

(a) $\sqrt{\frac{10}{7} gh}$

(b) $\sqrt{\frac{4}{3} gh}$

(c) $\sqrt{\frac{8}{7} gh}$

(d) $\sqrt{\frac{12}{7} gh}$

26. A uniform disc of radius 1 m and mass 2 kg is mounted on an axle supported on fixed frictionless bearings. A light cord is wrapped around the rim of the disc and a mass of 1 kg is tied to the free end. If it is released from rest, the tension in the cord is

(a) 10 N (b) 5 N
(c) 40 N (d) 15 N

27. In the above problem, if the applied torque acts for 4.0 seconds, the total angular displacement in this interval of time is

(a) 40 rad (b) 80 rad
(c) 20 rad (d) 10 rad

28. In the same problem, the work done by the applied torque is

(a) 400 J (b) 50 J
(c) 100 J (d) 200 J

29. In the same problem, the increase in rotational kinetic energy is

(a) 400 J (b) 100 J
(c) 200 J (d) 80 J

30. Refer to problem 22 where a disc of mass M and radius R is once pulled by a force 5 N by means of a string, and is then tied with a mass whose weight is 5 N. If the two processes are termed A and B respectively, then

(a) mechanical energy is conserved in both A and B
(b) mechanical energy is conserved in B only
(c) mechanical energy is conserved in A only
(d) mechanical energy is conserved neither in A nor in B

31. A disc of mass 5 kg and radius 1 m rotates with uniform angular speed of 300 rpm about its axis passing through its centre and normal to its plane. Its angular momentum is

(a) $78.5 \text{ kg m}^2/\text{s}$ (b) $157.0 \text{ kg m}^2/\text{s}$
(c) $12.5 \text{ kg m}^2/\text{s}$ (d) $25.0 \text{ kg m}^2/\text{s}$

32. A hoop, a solid cylinder and a solid sphere, all of the same mass and radius, start from rest and roll down the same incline. On observing their arrivals at the bottom, it is found that

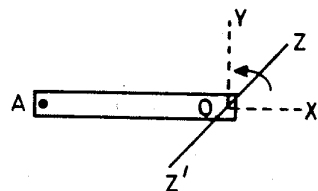
(a) the sphere reaches first
(b) the hoop reaches first
(c) the cylinder reaches first
(d) all of them reach at the same time

33. An engine develops 20 hp when rotating at a speed of 1800 rpm. The torque that it delivers is

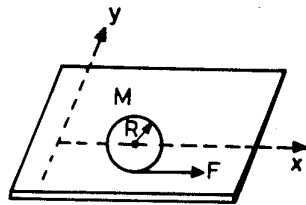
(a) $400 \text{ N} \cdot \text{m}$ (b) $60 \text{ N} \cdot \text{m}$
(c) $40 \text{ N} \cdot \text{m}$ (d) $80 \text{ N} \cdot \text{m}$

34. The angular momentum of a flywheel having a moment of inertia of 0.4 kg m^2 decreases from 3.0 to $2.0 \text{ kg m}^2/\text{s}$ in a period of 2 seconds. The average torque acting on the flywheel during this period is

- (a) $10 \text{ N} \cdot \text{m}$ (b) $2.5 \text{ N} \cdot \text{m}$
 (c) $0.5 \text{ N} \cdot \text{m}$ (d) $5 \text{ N} \cdot \text{m}$
35. In the above problem, if the angular acceleration is uniform, the number of revolutions the flywheel would have turned in that interval is
 (a) 2 (b) 4
 (c) 12 (d) 6
36. In problem 34, the work done is
 (a) $3.125 \text{ N} \cdot \text{m}$ (b) $12.5 \text{ N} \cdot \text{m}$
 (c) $62.5 \text{ N} \cdot \text{m}$ (d) $6.25 \text{ N} \cdot \text{m}$
37. A strip of length 1 m rotates about the Z-axis passing through the point O in the X-Y plane with an angular velocity of 10 rad/s in the counterclockwise direction, and O is at rest. The velocity of point A is



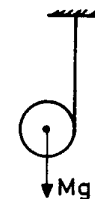
- (a) $10 \hat{k} \text{ m/s}$ (b) $-10 \hat{j} \text{ m/s}$
 (c) $+10 \hat{j} \text{ m/s}$ (d) $100 \hat{k} \text{ m/s}$
38. In the above problem, if $\omega = 10 \text{ rad/s} + (5 \text{ rad/s}^2) \times t$ in the counterclockwise direction, the velocity and acceleration respectively of point A, at $t = 0$, will be
 (a) $-10 \hat{j} \text{ m/s}$ and $(5 \hat{j} - 100 \hat{i}) \text{ m/s}^2$
 (b) $10 \hat{j} \text{ m/s}$ and $(5 \hat{j} - 100 \hat{i}) \text{ m/s}^2$
 (c) $-10 \hat{k} \text{ m/s}$ and $(5 \hat{k} - 100 \hat{j}) \text{ m/s}^2$
 (d) $-10 \hat{j} \text{ m/s}$ and $(-5 \hat{j} - 100 \hat{i}) \text{ m/s}^2$
39. A uniform rigid disc of mass M and radius R rolls without slipping on a horizontal plane. Its kinetic energy when its centre of mass has a velocity V_0 will be
 (a) $\frac{2}{5} MV_0^2$ (b) $\frac{4}{5} MV_0^2$
 (c) $\frac{2}{3} MV_0^2$ (d) $\frac{3}{4} MV_0^2$
- 40.* A uniform cylindrical disc of radius R and mass M is pulled over a horizontal frictionless surface by a constant force. The force is applied by means of a string wound around the disc as shown in the figure. If it starts from rest at $t = 0$, the linear and angular displacements respectively at time t are



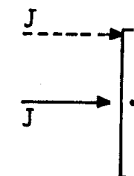
- (a) $\left(\frac{F}{M}\right)t^2, \left(\frac{F}{M}\right)t^2$ (b) $\left(\frac{F}{2M}\right)t, \left(\frac{F}{2MR}\right)t^2$

- (c) $\left(\frac{F}{2M}\right)t^2, \left(\frac{F}{MR}\right)t^2$ (d) $\left(\frac{2F}{M}\right)t^2, \left(\frac{2F}{MR}\right)t^2$

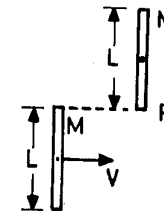
41. A string is wound around a cylinder of radius R and mass M and the end of the string is held fixed as shown in the figure. It is held at rest and then released. The correct statement about its subsequent motion is



- (a) acceleration will be uniform and its magnitude will be $2g/3$.
 (b) acceleration will be uniform and its magnitude will depend on how far below it goes from its original position.
 (c) acceleration will be a function of the distance it travels downward.
 (d) acceleration will not be uniform and its magnitude will also vary with the distance it travels.
42. A straight bar, of mass 15 kg and length 2 m, at rest on a frictionless horizontal surface, receives an instantaneous impulse of 7.5 Ns perpendicular to the bar. If the impulse is applied at the centre of mass of the bar, the energy transferred is
 (a) 3.2 J (b) 1.9 J
 (c) 3.8 J (d) 2.5 J
43. In the above problem, if the impulse is applied at the end of the bar, the angular momentum with respect to the centre of mass is
 (a) 15 Js (b) 30 Js
 (c) 7.5 Js (d) 1.9 Js
44. In problem 42, the angular velocity imparted to the bar is
 (a) 2 rad/s (b) 0.5 rad/s
 (c) 3 rad/s (d) 1.5 rad/s
45. In problem 42, the total energy imparted to the bar is
 (a) 7.5 J (b) 15 J
 (c) 3.5 J (d) 30 J

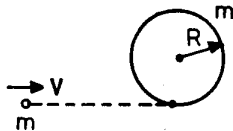


- 46.* A bar of mass M and length L is in pure translatory motion with its centre of mass velocity V . It collides with and sticks to a second identical bar which is initially at rest. (Assume that it becomes one composite bar of length $2L$). The angular velocity of the composite bar will be

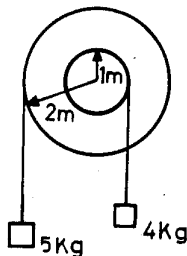


- (a) $\frac{3}{4} \frac{V}{L}$ clockwise (b) $\frac{4}{3} \frac{V}{L}$ clockwise

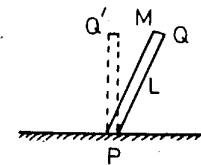
- (c) $\frac{3}{4} \frac{V}{L}$ counterclockwise (d) $\frac{V}{L}$ counterclockwise
47. A thin bar of mass M and length L is free to rotate about a fixed horizontal axis through a point at its end. The bar is brought to a horizontal position ($\theta = 90^\circ$) and then released. The angular velocity when it reaches the lowest point is
- directly proportional to its length and inversely proportional to its mass.
 - independent of mass and inversely proportional to the square root of its length.
 - dependent only upon the acceleration due to gravity and the mass of the bar.
 - directly proportional to its length and inversely proportional to the acceleration due to gravity.
- 48.* The moment of inertia of a cylinder of radius a , mass M and height h about an axis parallel to the axis of the cylinder and distant b from its centre is
- $\frac{1}{2} M(a^2 + 2b^2)$
 - $\frac{1}{2} M(2a^2 + b^2)$
 - $\frac{1}{2} M(a^2 + b^2)$
 - $\frac{1}{2} M\left(\frac{a^2}{3} + \frac{b^2}{12}\right)$
49. A solid cylindrical disc of radius a and mass M rolls along a horizontal plane with speed V . The total kinetic energy is
- MV^2
 - $\frac{3}{5} MV^2$
 - $\frac{3}{4} MV^2$
 - $\frac{2}{5} MV^2$
- 50.* A circular wooden hoop of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m , and moving with a velocity V , strikes the hoop and gets embedded in it. The thickness of the hoop is much smaller than R . The angular velocity with which the system rotates after the bullet strikes the hoop is



- $\frac{V}{4R}$
 - $\frac{V}{3R}$
 - $\frac{2V}{3R}$
 - $\frac{3V}{4R}$
51. The moment of inertia of the pulley system as shown in the figure is 4 kg m^2 . The radii of bigger and smaller pulleys are 2 m and 1 m respectively. The angular acceleration of the pulley system is
- 2.1 rad/s^2
 - 4.2 rad/s^2
 - 1.2 rad/s^2
 - 0.6 rad/s^2
52. The tensions in the two strings supporting masses of 5 and 4 kg respectively in the above problem are



- 30 and 40 N
 - 50 and 26 N
 - 40 and 30 N
 - 29 and 49 N
53. A disc of moment of inertia 0.5 kg m^2 rotates at 5 revolutions per second in a horizontal plane about a vertical axis through its centre of mass. A trickle of sand falls onto the disc at a distance of 0.8 m from the axis and builds a 0.8 m radius ring of sand on it. If the speed of the disc decreases to 4 revolutions per second, the quantity of sand that has fallen on it is
- 0.19 kg
 - 0.25 kg
 - 0.05 kg
 - 0.40 kg
54. A thin uniform rod PQ of length L and mass M is hinged at the end P to the level floor. It initially stood vertical (PQ'). If allowed to fall on the floor, the angular speed with which it will hit the floor is



- $\sqrt{\frac{3g}{4L}}$
 - $\sqrt{\frac{3g}{L}}$
 - $\sqrt{\frac{2g}{3L}}$
 - $\sqrt{\frac{g}{3L}}$
55. The radius of gyration of a solid disc of diameter 0.25 m about an axis through its centre of mass and perpendicular to its face is
- 0.36 m
 - 0.18 m
 - 0.09 m
 - 0.50 m
56. A man stands on a freely rotating platform and holds heavy weights in his hands. With his arms stretched parallel to the platform, his speed of rotation is 0.5 rev/s but when he draws them down along his body, his speed is 1.5 rev/s. The ratio of his moment of inertia in the two cases respectively is
- 3 : 1
 - 2 : 1
 - 4 : 3
 - 7.5 : 1
57. A flywheel having a radius of gyration of 2 m and mass 10 kg rotates at an angular speed of 5 rad/s about an axis perpendicular to it through its centre. The kinetic energy of rotation is
- 500 J
 - 2000 J
 - 1000 J
 - 250 J
58. The string of a simple pendulum is replaced by a uniform rod of length L and mass M while the bob has a mass m . It is allowed to make small oscillations. Its time period is
- $2\pi \sqrt{\left(\frac{2M}{3m}\right) \frac{L}{g}}$
 - $2\pi \sqrt{\frac{2(M+3m)L}{3(M+2m)g}}$
 - $2\pi \sqrt{\left(\frac{M+m}{M+3m}\right) \frac{L}{g}}$
 - $2\pi \sqrt{\left(\frac{2m+M}{3(M+2m)}\right) \frac{L}{g}}$
59. A yo-yo consists of a cylinder of mass 0.08 kg around which a string of length 0.6 m is wound. The free end of the string is held fixed and the yo-yo is allowed to fall vertically from rest. Its speed when the string gets fully unwound is

- (a) 0.7 m/s (b) 4.2 m/s
(c) 1.4 m/s (d) 2.8 m/s
60. A solid wooden door 1.0 m wide and 2.0 m high is hinged along one side and has a total mass of 50 kg. Initially, the door is open and at rest. The door is struck with a hammer at its centre and the blow provides an average force of 2000 N for a duration of 0.01 s. The angular velocity of the door after the impact is
- (a) 0.60 rad/s (b) 1.20 rad/s
(c) 1.0 rad/s (d) 1.8 rad/s

Answers

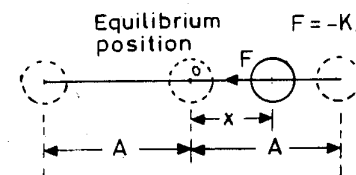
- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (d) | 3. (c) | 4. (c) | 5. (c) |
| 6. (a) | 7. (d) | 8. (c) | 9. (c) | 10. (a) |
| 11. (b) | 12. (b) | 13. (b) | 14. (a) | 15. (c) |
| 16. (d) | 17. (b) | 18. (d) | 19. (c) | 20. (a) |
| 21. (c) | 22. (b) | 23. (a) | 24. (d) | 25. (a) |
| 26. (b) | 27. (a) | 28. (d) | 29. (c) | 30. (b) |
| 31. (a) | 32. (a) | 33. (d) | 34. (c) | 35. (a) |
| 36. (d) | 37. (b) | 38. (d) | 39. (d) | 40. (c) |
| 41. (a) | 42. (b) | 43. (c) | 44. (d) | 45. (a) |
| 46. (c) | 47. (b) | 48. (a) | 49. (c) | 50. (b) |
| 51. (a) | 52. (d) | 53. (a) | 54. (b) | 55. (c) |
| 56. (a) | 57. (a) | 58. (b) | 59. (d) | 60. (a) |

Note: Questions marked with a star should take about five to six minutes while the others should take about three minutes each.

7

Simple Harmonic Motion

The figure represents a vibrating body at some instant when its displacement from the equilibrium position O is described by the coordinate X . The elastic restoring force that acts on the particle of mass m is given by $F = -kX$ or acceleration, $a = -(k/m)X$. The essential feature of simple harmonic motion is that the acceleration at each instant is proportional to the negative of the displacement at that instant.



$$U = \frac{1}{2} kX^2 \quad \text{and} \quad K = \frac{1}{2} mV^2$$

Since the elastic restoring force is conservative, we have

$$E = K + U$$

$$= \frac{1}{2} mV^2 + \frac{1}{2} kX^2 = \text{constant}$$

The maximum displacement is $\pm A$.

$$\text{Thus} \quad \frac{1}{2} mV^2 + \frac{1}{2} kX^2 = \frac{1}{2} kA^2$$

$$\therefore V = \pm \sqrt{\frac{k}{m}} \sqrt{A^2 - X^2}$$

This relation permits us to obtain the velocity for any given position.

Some useful relations are:

$$\frac{1}{2} mV_{\max}^2 = E \quad \Rightarrow V_{\max} = \sqrt{\frac{2E}{m}}$$

$$\frac{1}{2} kA^2 = \frac{1}{2} mV_{\max}^2 \quad \Rightarrow V_{\max} = \sqrt{\frac{k}{m}} A$$

$$X = A \sin \left(\sqrt{\frac{k}{m}} t + C \right)$$

where C is a constant of integration.

This shows that X is a periodic function of time and is a sinusoidal function

$$\text{Period of motion } T = 2\pi \sqrt{\frac{m}{k}}$$

and the Frequency of oscillation is given by

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

One hertz = 1 cycle per second

$$\omega = \sqrt{\frac{k}{m}}$$

where ω is angular frequency and is given by $\omega = 2\pi\nu$ and is expressed in radian per second. It follows that

$$X = A \sin(\omega t + C)$$

and $V = \omega \sqrt{A^2 - X^2}$

Note that in a spring-mass system, the frequency is independent of amplitude. Thus, an additional important characteristic of simple harmonic motion is that the frequency is *not* dependent on the amplitude of motion.

A SHM is represented by an equation $X = A \cos \omega t$, $X = A \sin \omega t$ or $X = A \cos(\omega t + \phi)$.

$$\frac{d^2 X}{dt^2} = -\omega^2 X$$

This is a differential equation and another form of it is

$$\frac{d^2 X}{dt^2} + \frac{k}{m} X = 0$$

Energy Considerations

$$U = \frac{1}{2} k A^2 \cos^2(\omega t + \phi)$$

and its maximum value is

$$\frac{1}{2} k A^2$$

$$K = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \phi) = \frac{1}{2} k A^2 \sin^2(\omega t + \phi)$$

and its maximum value is

$$\frac{1}{2} k A^2 \quad \text{or} \quad \frac{1}{2} m (\omega A)^2$$

Relation Between Circular Motion and SHM

$$X = A \cos(\omega t + \phi) \quad \text{and} \quad Y = A \sin(\omega t + \phi)$$

Both equations represent SHM but differ in phase. And if we replace ϕ by $(\phi - \pi/2)$ in one, then the two equations are identical. Thus, uniform circular motion can be described as a combination of two SHMs, occurring along perpendicular lines, which have the same amplitude and frequency but differ in phase by 90° .

Combination of Harmonic Motions

$$X = A_x \cos(\omega t + \phi_x)$$

$$Y = A_y \cos(\omega t + \phi_y)$$

But if $\phi_x = \phi_y = \phi$, the resulting motion is given by the equation $Y = (A_y/A_x)X$ (which is the equation of straight line, A_y/A_x being the slope of the line).

If the phase constants are different, the resulting motion may follow different paths. For example, when phase constants differ by $\pi/2$, the resulting motion is circular. When the amplitudes are unequal, the resulting motion is elliptical. All possible combinations of two SHMs at right angles having the same frequency correspond to elliptical paths, the circle and straight line being the special cases of an ellipse. The path traced out by a particle which is simultaneously subjected to SHMs at right angles to each other is called Lissajous's figure.

Damped Harmonic Motion

A free harmonic oscillator is one which is free from any effects that cause energy loss. An ideal oscillator, therefore, will oscillate for ever. But in practice, frictional or viscous drag forces are always present. If the retarding force is of the type $F = -bV = -b \cdot dX/dt$ where b is the damping constant, then the equation of motion is

$$m \cdot a = -kX + F$$

$$\text{or} \quad m \cdot \frac{d^2 X}{dt^2} = -kX - b \cdot \frac{dX}{dt}$$

$$\text{or} \quad m \cdot \frac{d^2 X}{dt^2} + b \cdot \frac{dX}{dt} + kX = 0$$

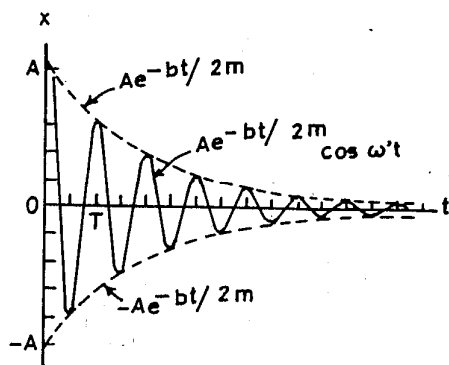
If b is small, the solution of this differential equation is

$$X = A \cdot e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\text{where} \quad \omega' = 2\pi\nu' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

A plot of the displacement X as a function of the time t of such a damped oscillator is shown in the figure. The dotted curves represent the exponential

fall in the amplitude A while the continuous curve shows the cosine wave with the amplitude term as $A \cdot e^{-bt/2m}$.



Forced Oscillations

When a body is subjected to an oscillatory external force, the oscillations that result are called forced oscillations. The forced oscillations have the frequency of the external force and not the natural frequency of the body. The response of the body depends on the relation between the forced and the natural frequency. A succession of small impulses applied at the proper frequency can produce an oscillation of large amplitude.

If external force applied is $F_m \cos \omega''t$ where F_m is the maximum value of the external force and ω'' is its angular frequency, then the equation becomes

$$m \cdot \frac{d^2 X}{dt^2} + b \cdot \frac{dX}{dt} + kX = F_m \cos \omega''t$$

The solution of this equation is

$$X = \frac{F_m}{G} \sin(\omega''t - \phi)$$

where $G = \sqrt{m^2(\omega''^2 - \omega^2)^2 + b^2\omega''^2}$ and $\phi = \cos^{-1} \frac{b\omega''}{G}$

ILLUSTRATIONS

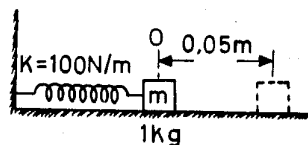
1. The force constant of a spring is 100 N/m, and it is mounted as shown in the figure. It has a block of mass 1 kg attached to it and is pulled a distance of 0.05 m and released. Find:

(a) the period and frequency of oscillation,

(b) the maximum velocity attained by the oscillating block,

(c) the maximum acceleration,

(d) the velocity and acceleration when the block has moved halfway towards the centre O from its initial position,



- (e) the time required for the block to move halfway towards the centre from its initial position, and
(f) the total energy of the oscillating system.

Solution

$$(a) \quad T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{1 \text{ kg}}{100 \text{ N/m}}} = \frac{\pi}{5} \text{ s} = 0.628 \text{ s}$$

$$v = \frac{1}{T} = \frac{5}{\pi} \text{ s}^{-1} = 1.59 \text{ Hz}$$

$$\omega = 2\pi v = \sqrt{\frac{k}{m}} = 10 \text{ s}^{-1}$$

- (b) The maximum velocity occurs at the equilibrium position, where the coordinate is zero.

$$V = \pm \omega \sqrt{A^2 - X^2} = \pm \omega A \quad \text{at } X = 0$$

$$V_{\max} = 0.05 \times 10 = 0.5 \text{ m/s}$$

$$(c) \quad a_{\max} = \pm \omega^2 A = (10)^2 \times 0.05 = 5.0 \text{ m/s}^2$$

- (d) When the block has moved halfway, it means that $X = A/2 = 0.025 \text{ m}$

$$V = -10 \sqrt{(0.05)^2 - (0.025)^2} = -0.4 \text{ m/s}$$

$$a = -\omega^2 X = -(10)^2 (0.025) = -2.5 \text{ m/s}^2$$

- (e) The position at any time is given by

$$X = A \cos \omega t$$

$$\frac{A}{2} = A \cos 10t$$

$$\cos 10t = \frac{1}{2} \quad \text{or} \quad 10t = \frac{\pi}{3} \Rightarrow t = \pi/30 \text{ s}$$

- (f) Since the total energy is conserved, we can compute it at any state of the motion

$$\text{Say, at } X = 0, E = K_{\max} = \frac{1}{2} m V_{\max}^2 = \frac{1}{2} \times 1 \times (0.5)^2 = 0.125 \text{ J}$$

At $X = A$,

$$E = \frac{1}{2} K \cdot A^2$$

$$= \frac{1}{2} \times 100 \times (0.05)^2 = 0.125 \text{ J}$$

2. A horizontal spring and block system as described in Illustration 1 has an angular frequency $\omega = 3\pi \text{ rad/s}$. If the block is displaced to a point 0.25 m from the equilibrium point and given a velocity of -1.5 m/s at $t = 0$, determine the displacement X as a function of time t .

Solution

$$X = A \sin(\omega t + \phi)$$

At $t = 0$,

$$X_0 = A \sin \phi$$

$$V_0 = A\omega \cos \phi$$

$$\tan \phi = \frac{\omega X_0}{V_0} = \frac{(3\pi)(0.25)}{(-1.5)}$$

or $\phi = \tan^{-1}(-1.571) = 122.5^\circ = 2.14 \text{ rad}$

Also $A^2 = X_0^2 + \frac{V_0^2}{\omega^2} = (0.25)^2 + \frac{(-1.5)^2}{(3\pi)^2}$

$$A = \sqrt{(0.0625) + \frac{2.25}{88.83}} = 0.30 \text{ m}$$

$$\therefore X(t) = (0.30) \sin(3\pi t + 2.14)$$

3. A block of mass 1 kg hangs without vibrating at the end of a spring whose force constant is 200 N/m and which is attached to the ceiling of an elevator. The elevator is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases.

- What is the angular frequency of the block after the acceleration ceases?
- What is the elongation in the spring during the time the elevator is accelerating?
- What is the amplitude of oscillation. (taking the upward direction to be positive)?

Solution

(a) The angular frequency under all circumstances is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{1}} = 14 \text{ rad/s}$$

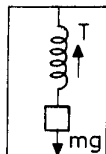
(b) When the elevator is moving up, the equation of motion is

$$T - mg = \frac{mg}{3}$$

$$\therefore T = mg + \frac{mg}{3} = \frac{4mg}{3}$$

This tension elongates the spring by X . Hence,

$$T = kX \Rightarrow X = \frac{4mg}{3k} = \frac{4 \times 1 \times 10}{3 \times 200} = \frac{40}{600} = 0.07 \text{ m}$$

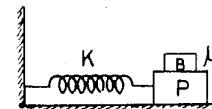


(c) Elongation is $4mg/3k$ when the elevator is moving up.

$$\text{Elongation when lift is stationary} = \frac{mg}{k}$$

$$\therefore \text{Amplitude} = \frac{4}{3} \frac{mg}{k} - \frac{mg}{k} = \frac{1}{3} \frac{mg}{k}$$

4. A flat plate P of mass M executes SHM in a horizontal plane by sliding over a frictionless surface with a frequency ν . A block B of mass m rests on the plate as shown in the figure. Coefficient of friction between the surfaces of B and P is μ_s . What is the maximum amplitude of oscillation that the plate-block system can have if the block B is not to slip on the plate?



Solution

$$f_{\max} = \mu mg$$

$$a_{\max} = \mu g$$

$$\text{If } A \text{ is the amplitude, } a_{\max} = A\omega^2 = 4\pi^2 A\nu^2 = \mu g$$

$$A = \frac{\mu g}{4\pi^2 \nu^2}$$

5. A 2 kg sphere, when falling through air, reaches a terminal velocity of 0.75 m/s. The sphere, when suspended from an ideal spring, oscillates in air with a time period of 0.30 s.

- What is the spring constant?
- What is the time interval required for the amplitude to decrease to half the value at the beginning of the interval?
- What fraction of the mechanical energy is lost during the interval when the amplitude decreases to half?

Solution

$$(a) \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow k = \frac{m \cdot 4\pi^2}{T^2} = \frac{2 \times 4 \times \pi^2}{(0.30)^2} = 877 \text{ N/m}$$

$$(b) \vec{f} = -b\vec{v} \Rightarrow bv_{\text{term}} = mg, \quad b = \frac{2 \times 10}{0.75} = \frac{20}{3} \times 4 = \frac{80}{3}$$

(v_{term} is the terminal velocity)

$$m \cdot \frac{d^2 X}{dt^2} + b \cdot \frac{dX}{dt} + m\omega^2 X = 0$$

or $\frac{d^2 X}{dt^2} + \frac{40}{3} \frac{dX}{dt} + \omega^2 X = 0$

$$m^2 + \frac{40}{3}m + \omega^2 = 0$$

$$m = \frac{(-40/3) \pm \sqrt{\frac{1600}{9} - 4\omega^2}}{2}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3} \times 10 = \frac{20\pi}{3}$$

$$X = A \cdot e^{+im}$$

$$= A_1 e^{-(20/3)t} \cos \sqrt{4\omega^2 - (1600/9)} t$$

$$\therefore e^{(-20/3)T} = \frac{1}{2} \quad \text{or} \quad \frac{20}{3}T = \ln 2 \quad \text{or} \quad T = \frac{3}{20} \ln 2$$

$$(c) \quad \langle E \rangle = \frac{A^2}{2} e^{-(40/3)t}$$

$$\frac{\langle E \rangle_{t=0}}{\langle E \rangle_{t=T}} = \frac{1}{e^{-40T/3}} = 4$$

$$\text{Energy lost} = 75\%, \quad \left[\frac{\langle E \rangle_{t=0} - \langle E \rangle_{t=T}}{\langle E \rangle_{t=0}} = \frac{3}{4} \right]$$

6. A simple pendulum of length L and mass m is suspended in a car that is travelling with a constant speed V around a circle of radius R . If the pendulum undergoes oscillations in a radial direction about its equilibrium position, what will be its frequency of oscillation?

Solution

$$v = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \quad \text{when the pendulum is stationary.}$$

It will experience a centripetal acceleration V^2/R , directed towards the centre of the circle. The resultant acceleration will therefore be

$$\sqrt{g^2 + \left(\frac{V^2}{R}\right)^2}$$

and the modified frequency of the pendulum will be

$$v = \frac{1}{2\pi} \sqrt{\frac{[g^2 + (V^2/R)^2]^{1/2}}{L}} = \frac{1}{2\pi} \sqrt{\frac{\{(gR)^2 + (V^2)^2\}^{1/2}}{RL}}$$

7. A vertical spring carries a 5 kg body and is hanging in equilibrium. An additional force is applied so that the spring is further stretched. When released from this position, it performs 50 complete oscillations in 25 seconds, with an amplitude of 5 cm.

(a) Find the value of the additional force applied.

(b) What is the force exerted by the spring on the body when it is at the lowest point, middle point and the highest point of the path?

Solution

$$(a) \quad T = \frac{25}{50} = \frac{1}{2} \text{ s}$$

$$\omega = \frac{2\pi}{T} = 4\pi \text{ rad/s}$$

$$\text{Spring constant } k = m\omega^2$$

$$= 5 \times (4\pi)^2$$

$$= 80\pi^2 \text{ N/m}$$

Force required to stretch the spring by 5 cm is

$$F = kX = 80\pi^2 \times 0.05 \text{ N} \\ = 39.5 \text{ N}$$

(b) Extension of the spring due to body of mass 5 kg is

$$X' = \frac{5g}{80\pi^2} = \frac{5 \times 10}{80\pi^2} = \frac{5}{8\pi^2} = 0.06 \text{ m}$$

$$\therefore \text{Total elongation} = 0.05 + 0.06 \text{ m} \\ = 0.11 \text{ m}$$

Therefore, force exerted by the spring on the body at the lowest point is

$$F_L = (80\pi^2) (0.11) = 789.6 \times 0.11 = 86.9 \text{ N}$$

At the middle point of the oscillation, the elongation is 0.06 m. Therefore, the force exerted by the spring is

$$F_M = 8\pi^2 \times (0.06) = 789.6 \times 0.06 = 47.4 \text{ N}$$

At the uppermost point on the path of oscillation, the elongation of the spring is

$$(0.06 - 0.05) \text{ m} = 0.01 \text{ m}$$

$$\text{Hence } F_T = 789.6 \times 0.01 = 8 \text{ N}$$

8. In Illustration 7, find the kinetic energy and potential energy of the system when the body is 2 cm below the middle of the path.

Solution

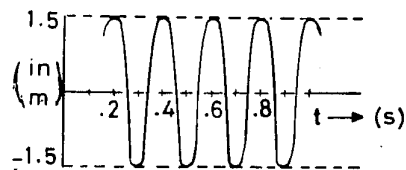
$$V^2 = \frac{k}{m} (A^2 - X^2)$$

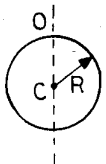
$$\text{KE} = \frac{1}{2} mV^2 = \frac{k}{2} (A^2 - X^2) = \frac{1}{2} \cdot 80\pi^2 [(0.05)^2 - (0.02)^2] \\ = 395 \times 0.0021 = 0.83 \text{ J}$$

$$\begin{aligned}\text{Potential energy stored} &= \frac{1}{2} kX^2 = \frac{1}{2} \times 80\pi^2 (0.02)^2 \\ &= 0.16 \text{ J}\end{aligned}$$

EXERCISES

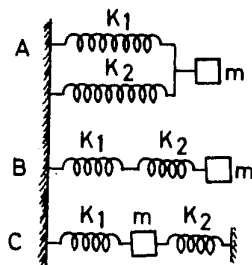
1. The motion of a vibrating system is shown in the displacement-time graph. The amplitude and frequency respectively are



- (a) 1.5 m and 2 cps
(b) 1.5 m and 5 cps
(c) 3 m and 5 cps
(d) 1.5 m and 0.2 cps
2. A uniform disc with radius R is allowed to undergo small amplitude oscillations about a point O located in its perimeter. The time period of the oscillation is
- 
- (a) $2\pi\sqrt{\frac{2R}{g}}$
(b) $2\pi\sqrt{\frac{R}{g}}$
(c) $2\pi\sqrt{\frac{3R}{2g}}$
(d) $2\pi\sqrt{\frac{2R}{3g}}$
3. A particle is attached to a vertical spring and is pulled down a distance 0.04 m below its equilibrium position and is released from rest. The initial upward acceleration of the particle is 0.30 m/s^2 . The period of the oscillation is
- (a) 2.08 s
(b) 1.92 s
(c) 2.90 s
(d) 2.29 s
4. In the above problem, the velocity of the particle when it passes through the equilibrium position is
- (a) 0.110 m/s
(b) 0.220 m/s
(c) 1.1 m/s
(d) 2.2 m/s
5. In the same problem, if the upward direction is taken to be positive, the equation of motion is
- (a) $y = 0.04 \cos 2.74t$
(b) $y = 0.04 \sin 1.2t$
(c) $y = 0.04 \cos 3t$
(d) $y = 0.04 \cos 0.3t$
6. In a spring-mass system, the length of the spring is L , and it has a mass M attached to it and oscillates with an angular frequency ω . The spring is then cut into two parts, one (A) with relaxed length fL and the other (B) with relaxed length $(1-f)L$. The force constants of the two springs A and B are

- (a) $\frac{K}{1-f}$ and $\frac{K}{f}$
(b) $\frac{K}{f}$ and $\frac{K}{1-f}$
(c) K and K
(d) K and $\frac{K}{2}$
7. In the above problem, the block is divided in the same fractions and the smaller part of the block is attached to the larger part of the spring (A), the remaining pieces are likewise joined (B), the angular frequencies of oscillation of the two systems (A and B) are
- (a) $2\pi\sqrt{\frac{K}{f(1-f)m}}$ and $2\pi\sqrt{\frac{K}{fm}}$
(b) $2\pi\sqrt{\frac{K}{fm}}$ and $2\pi\sqrt{\frac{K}{(1-f)m}}$
(c) $2\pi\sqrt{\frac{K}{(1-f)fm}}$ and $2\pi\sqrt{\frac{K}{(1-f)fm}}$
(d) $2\pi\sqrt{\frac{K}{(1-f)fm}}$ and $2\pi\sqrt{\frac{K}{fm}}$
8. A block of mass 1 kg attached to a spring executes SHM with a period of 0.50 seconds. If the block is replaced by one which has twice the mass, the period becomes 0.70 seconds. The mass of the spring is
- (a) 0.025 kg
(b) 0.0122 kg
(c) 0.250 kg
(d) 0.125 kg
9. In the above problem, the spring constant is
- (a) 165 N/m
(b) 275 N/m
(c) 561 N/m
(d) 156 N/m
10. A body performing SHM vibrates with an amplitude of 0.2 m and its frequency is 5 vib/s. The maximum value of acceleration is
- (a) 394.8 m/s^2
(b) 296.1 m/s^2
(c) 197.4 m/s^2
(d) 98.7 m/s^2
11. In the above problem, when the displacement of the body is 0.1 m, its velocity is
- (a) 4.5 m/s
(b) 5.4 m/s
(c) 2.25 m/s
(d) 10.8 m/s
12. A particle of mass 0.10 kg executes SHM with an amplitude 0.05 m and frequency 20 vib/s. Its energy while it is at the equilibrium position is
- (a) 2 J
(b) 4 J
(c) 1 J
(d) zero
13. A particle executes SHM with a frequency of 25 vib/s and has an amplitude of 0.02 m. The speed of the particle 0.30 seconds after it is in the equilibrium position is
- (a) $\pi \text{ m/s}$
(b) $-2\pi \text{ m/s}$
(c) $2\pi \text{ m/s}$
(d) $-\pi \text{ m/s}$
14. A body is describing SHM with an amplitude of 0.1 m. Its velocity while passing through the mean position is 3 m/s. Its frequency is
- (a) 15π
(b) $15/\pi$
(c) 30π
(d) $25/\pi$
15. A body executes SHM whose period is 16 seconds. Two seconds after it passes the equilibrium position, its velocity is 1 m/s. The amplitude of SHM is
- (a) 6.3 m
(b) 1.8 m
(c) 3.6 m
(d) 2.4 m

16. Two SHMs with the same time period and no phase difference between them are at right angles to each other. Their amplitudes are a and b . When superimposed, their resultant path will be
 (a) a circle (b) a straight line
 (c) an ellipse (d) a parabola
17. In the above problem, if the phase difference is $\pi/4$, the resultant path will be
 (a) a straight line (a) a circle
 (c) an ellipse (d) a parabola
18. In order that the resultant path on superimposing two mutually perpendicular SHMs be a circle, the conditions are that
 (a) the amplitudes on both SHMs should be equal and they should have a phase difference of $\pi/2$
 (b) the amplitudes should be in the ratio 1 : 2 and the phase difference should be zero
 (c) the amplitudes should be in the ratio 1 : 2 and the phase difference should be $\pi/2$
 (d) the amplitudes should be equal and the phase difference should be zero
19. The time variation of displacement of a simple harmonic oscillator is given by the equation $X = A \sin \omega t$ (ϕ is zero) and the plot is shown in the figure. The figures (A) and (B) respectively represent
 (a) the velocity vs ωt and acceleration vs ωt
 (b) acceleration vs ωt and velocity vs ωt
 (c) displacement vs ωt with a phase difference of π
 (d) both A and B represent velocity vs ωt with a phase difference of $\pi/4$
20. Two springs, with spring constants K_1 and K_2 , are connected to a block of mass m in three different ways as shown in the figure. If each system is set into oscillation, their angular frequencies respectively are

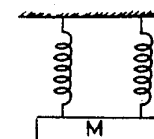


- (a) $\sqrt{\frac{m}{K_1 + K_2}}, \sqrt{\frac{m}{K_1 + K_2}}, \sqrt{\frac{m(K_1 + K_2)}{K_1 K_2}}$
 (b) $\sqrt{\frac{K_1 + K_2}{m(K_1 K_2)}}, \sqrt{\frac{K_1 + K_2}{m}}, \sqrt{\frac{K_1 + K_2}{m}}$

(c) $\sqrt{\frac{(K_1 + K_2)m}{K_1 K_2}}, \sqrt{\frac{(K_1 + K_2)m}{K_1 K_2}}, \sqrt{\frac{m}{K_1 + K_2}}$

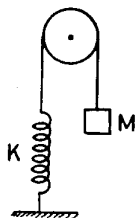
(d) $\sqrt{\frac{K_1 + K_2}{m}}, \sqrt{\frac{K_1 K_2}{(K_1 + K_2)}}, \sqrt{\frac{K_1 + K_2}{m}}$

21. A disc of radius R and mass M is pivoted at the rim and is set for small oscillations. If a simple pendulum has to have the same time period as that of the disc, the length of the pendulum should be
 (a) $\frac{5}{4} R$ (b) $\frac{2}{3} R$
 (c) $\frac{3}{4} R$ (d) $\frac{3}{2} R$
22. A massless spring of force constant 20 N/m is cut into two equal halves. The force constant of each half is
 (a) 20 N/m (b) 40 N/m
 (c) 10 N/m (d) 5 N/m
23. The two halves of the spring as in problem 22 are now suspended separately so as to support a block of mass M . If the system oscillates at a frequency of 5 Hz, the value of M is
 (a) 80 g (b) 8 g
 (c) 40 g (d) 20 g
24. A particle oscillates with SHM according to the equation $X = 5.0 \cos (4\pi t + \pi/3)$ where t is in seconds and X is in metres. Its acceleration at time $t = 4.0$ seconds is
 (a) $40\pi^2$ (b) 20π
 (c) $20\pi^2$ (d) 40π
25. A body of mass 0.20 kg executes SHM of amplitude 1.0 m and period 0.30 seconds. The value of maximum force that acts on it is
 (a) 44 N (b) 88 N
 (c) 8 N (d) 4 N
26. A block performs SHM on a horizontal floor surface with a frequency of 2 Hz. The coefficient of static friction between the surfaces of the block and floor is 0.40. If the block does not slip, the amplitude of motion is
 (a) 5.2 cm (b) 3.0 cm
 (c) 2.5 cm (d) 0.03 cm
27. A piston with a small block on top of it is undergoing vertical SHM. Its period is 2.0 seconds. The amplitude at which the block will separate from the piston is
 (a) $\frac{10}{\pi^2}$ (b) $\frac{\pi^2}{10}$
 (c) $\frac{10}{\pi}$ (d) $\frac{\pi}{10}$



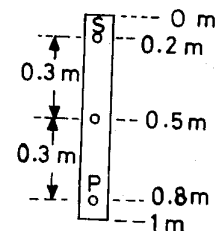
28. In the above block-piston system problem, if the piston has an amplitude of 4.0 cm, the maximum frequency for which the block and the piston will not separate is
 (a) 0.15 Hz (b) 0.5 Hz
 (c) 5.2 Hz (d) 2.5 Hz

29. A simple pendulum of length 1 m is attached to the ceiling of an elevator which is accelerating upward at the rate of 1 m/s^2 . Its frequency is
 (a) 2 Hz (b) 1.5 Hz
 (c) 5 Hz (d) 0.5 Hz
30. A metre stick is pivoted at one end ($L = 1 \text{ m}$) and is set into small oscillations. Its time period is
 (a) 1.6 s (b) 3.2 s
 (c) 1.0 s (d) 2.4 s
31. A body of mass 4 kg hangs from a spring and oscillates with a period of 0.4 seconds. The body is removed from the spring. As a result, the contraction of the spring is
 (a) 0.002 m (b) 0.02 m
 (c) 0.04 m (d) 0.08 m
- 32.* A U-tube contains 1 kg of mercury as shown and is disturbed so that it oscillates back and forth from arm to arm. If we neglect friction and one centimetre of mercury column has a mass of 20 g, the period of oscillation is
 (a) 0.5 s (b) 1.5 s
 (c) 2 s (d) 1 s
33. A U-tube contains $M \text{ kg}$ of mercury. It is disturbed so as to oscillate in the two arms of the U-tube. The frequency of oscillation is
 (a) directly proportional only to the mass of mercury contained in the U-tube
 (b) directly proportional only to the force constant of the oscillating column of mercury
 (c) inversely proportional to the square root of the mass of mercury and directly proportional to the square root of the force constant
 (d) independent of the mass of the mercury column and directly proportional to the difference in height between the mercury columns in the two arms of the U-tube.
- 34.* A mass $M = 5 \text{ kg}$ is attached to a spring as shown in the figure, and held in position so that the spring remains unstretched. The spring constant is 200 N/m . The mass M is then released and begins to undergo small oscillations. The amplitude of oscillation is
 (a) 0.5 m (b) 0.25 m
 (c) 0.2 m (d) 0.1 m

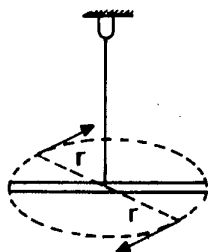


35. A toy gun uses a spring of force constant K . When charged before being triggered in the upward direction, the spring is compressed by X metres. If the mass of the shot is $m \text{ kg}$, on being triggered, it will go up to a height of
 (a) $\frac{K}{2mg} X^2$ (b) $\frac{K}{mg} X^2$
 (c) $\frac{K^2}{mg} X^2$ (d) $\frac{1}{3mg} X^2$

36. A cubical block of mass M vibrates horizontally with an amplitude of 4.0 cm and a frequency of 2.0 Hz . A small block of mass m is placed on the bigger block. In order that the smaller block does not slide on the bigger block, the minimum value of the coefficient of static friction between the two blocks is
 (a) 0.36 (b) 0.40
 (c) 0.63 (d) 0.72
37. A seconds pendulum is taken in a carriage. The period of oscillation when the carriage moves with an acceleration of 4 m/s^2 in the horizontal direction is
 (a) 2.19 s (b) 1.92 s
 (c) 1.8 s (d) 0.6 s
38. Two masses m_1 and m_2 are suspended together by a massless spring of force constant K . When the masses are in equilibrium, m_1 is removed without disturbing the system. The amplitude of oscillation is
 (a) $\frac{m_2 g}{K}$ (b) $\frac{m_1 g}{K}$
 (c) $\frac{(m_1 + m_2) g}{K}$ (d) $\frac{(m_2 - m_1) g}{K}$
39. The amplitude of a body performing SHM is 5 cm , its mass is 10 g , and the total energy of oscillation is $3.1 \times 10^{-5} \text{ J}$. If the initial oscillation phase is 60° , the time period of oscillation is
 (a) 8 s (b) 1 s
 (c) 2 s (d) 4 s
40. A material point with mass 10 g oscillates according to the equation $X = 5 \sin(\pi/5 + \pi/4) \text{ cm}$. The maximum force and the total energy respectively are
 (a) 197 N and $10 \times 10^{-6} \text{ J}$
 (b) $19.7 \times 10^{-5} \text{ N}$ and $4.93 \times 10^{-6} \text{ J}$
 (c) $7.9 \times 10^{-5} \text{ N}$ and $3.94 \times 10^{-6} \text{ J}$
 (d) $1.97 \times 10^{-5} \text{ N}$ and $3.94 \times 10^{-5} \text{ J}$
- 41.* A glass U-tube with uniform bore of 0.02 m diameter contains 10 kg of mercury. The mercury oscillates freely up and down about its position of equilibrium. The ratio of the restoring force on the column of mercury to its displacement from equilibrium and the period of oscillation respectively are
 (a) 215 N/m and 8.5 s (b) 51.2 N/m and 5.8 s
 (c) 42.7 N/m and 0.07 s (d) 85.4 N/m and 2.15 s
42. A metre stick oscillates as a compound pendulum about a horizontal axis through S . The length of an equivalent simple pendulum is
 (a) 2.3 m (b) 11.6 m
 (c) 0.58 m (d) 1.15 m
43. In the above problem, if the stick oscillates about a horizontal axis through P , its time period is
 (a) 2.4 s (b) 1.52 s
 (c) 1.94 s (d) 0.4 s
44. When the balance wheel of a watch is displaced 45° from its equilibrium position, it begins to move with an angular acceleration of 25 rad/s^2 . Its frequency of oscillation is
 (a) 0.90 Hz (b) 1.8 Hz
 (c) 0.45 Hz (d) 3.6 Hz



45. A metal rod is suspended in a horizontal position by a vertical wire attached to its centre. A horizontal couple of moment $10 \text{ N} \cdot \text{m}$ on the rod twists the wire and the rod is deflected through 18° . On releasing the rod, it oscillates as a torsion pendulum with a period of 1 seconds. The moment of inertia of the rod is



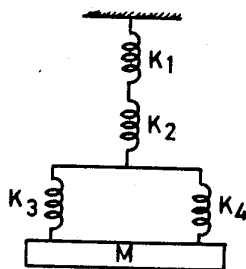
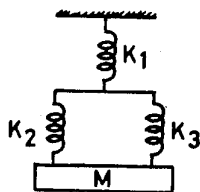
- (a) 3.2 kg m^2 (b) 1.8 kg m^2
 (c) 0.8 kg m^2 (d) 2.4 kg m^2
46. An oscillator has an acceleration of 1.25 m/s^2 at an instant when its displacement is 0.05 m . Its time period is
- (a) 2.16 s (b) 1.26 s
 (c) 0.26 s (d) 1.621 s
- 47.* A hoop of diameter 2 m oscillates as a compound pendulum about a horizontal axis at its rim and perpendicular to its plane. The equivalent length of the simple pendulum is
- (a) 2 m (b) 4 m
 (c) 1.5 m (d) 3 m

48. A system of springs and a mass is shown in the figure. It will oscillate with a frequency

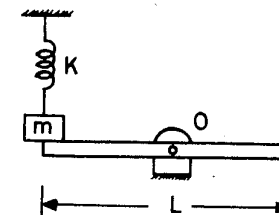
(a) $\frac{1}{2\pi} \sqrt{\frac{K_1(K_2 + K_3)}{(K_1 + K_2 + K_3)M}}$
 (b) $\frac{1}{2\pi} \sqrt{\frac{K_1^2 + K_2^2 + K_3^2}{(K_1 + K_2 + K_3)M}}$
 (c) $\frac{1}{2\pi} \sqrt{\frac{K_1 + K_2 + K_3}{(K_1 K_2 + K_2 K_3 + K_1 K_3)^{1/2} M}}$
 (d) $\frac{1}{2\pi} \sqrt{\frac{K_1 K_2 + K_1 K_3 + K_2 K_3}{(K_1 + K_2 + K_3)M}}$

49. In another system of springs and mass, as shown, the natural frequency of oscillation is

(a) $\frac{1}{2\pi} \sqrt{\frac{(K_3 + K_4)(K_2 + K_3) + K_1 K_2}{K_1 K_2 (K_3 + K_4) M}}$
 (b) $\frac{1}{2\pi} \sqrt{\frac{(K_1 + K_2)(K_2 + K_3) K_4}{K_1 K_2 (K_3 + K_4) M}}$
 (c) $\frac{1}{2\pi} \sqrt{\frac{K_1 K_2 (K_3 + K_4)}{[(K_3 + K_4)(K_2 + K_3) + K_1 K_2] M}}$
 (d) $\frac{1}{2\pi} \sqrt{\frac{K_3 K_4 (K_1 + K_2)}{[(K_1 + K_2)(K_3 + K_4) + K_2 K_3] M}}$



- 50.* A uniform rod of length L is hinged at O. The moment of inertia of the rod about O is I . A mass m , supported by a spring of force constant K , is attached to one end of the rod (as shown in the figure). The rod is horizontal in the equilibrium position. If it is given small oscillations, its frequency is



(a) $\frac{1}{2\pi} \sqrt{\frac{KL^2}{4I + mL^2}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{KL}{4I + mL^2}}$
 (c) $\frac{1}{2\pi} \sqrt{\frac{4I + mL^2}{KL^2}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{3KL^2}{2I + 4mL^2}}$

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (d) | 4. (a) | 5. (a) |
| 6. (b) | 7. (c) | 8. (d) | 9. (a) | 10. (c) |
| 11. (b) | 12. (a) | 13. (d) | 14. (b) | 15. (c) |
| 16. (b) | 17. (c) | 18. (a) | 19. (a) | 20. (d) |
| 21. (d) | 22. (b) | 23. (a) | 24. (a) | 25. (b) |
| 26. (c) | 27. (a) | 28. (d) | 29. (d) | 30. (a) |
| 31. (c) | 32. (d) | 33. (c) | 34. (b) | 35. (a) |
| 36. (c) | 37. (b) | 38. (b) | 39. (d) | 40. (b) |
| 41. (d) | 42. (c) | 43. (b) | 44. (a) | 45. (c) |
| 46. (b) | 47. (a) | 48. (a) | 49. (c) | 50. (a) |

Note: Problems marked with a star should take about five minutes. All the rest should take not more than three minutes.

8

Gravitation

Kepler's Laws of Planetary Motion

1. All planets move in elliptical orbits having the sun as one focus. This is known as the law of orbits.
2. A line joining any planet to the sun sweeps out equal areas in equal times (the law of constancy of areal velocity).
3. The square of the period of any planet about the sun is proportional to the cube of the planet's mean distance from the sun (the law of periods).

The Law of Universal Gravitation

The force between any two particles having masses m_1 and m_2 separated by a distance r is an attraction acting along the line joining the particles and has the magnitude

$$F = G \frac{m_1 m_2}{r^2}$$

where G is a universal constant having the same value for all pairs of particles.

The constant G has the dimensions $L^3 M^{-1} T^{-2}$. Its units are Nm^2/kg^2 and its value is given as $G = 6.6720 \times 10^{-11} Nm^2/kg^2$. The force of attraction between two particles m_1 and m_2 separated by a distance r_{21} is expressed in vector form as

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^3} \cdot \hat{r}_{21}$$

The acceleration due to gravity g varies with distance from the surface of earth according to the following formula

$$\frac{dg}{g} = -2 \frac{dr}{r}$$

where dg is the variation in g and dr is the variation in the distance from the surface of the earth. This relation shows that as one goes up, the value of g decreases.

Effect of Rotation of Earth on g

$$g = G \frac{M_e}{R_e^2} - a_R$$

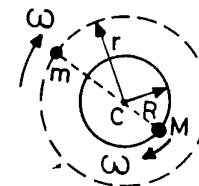
where M_e and R_e are the mass and radius of the earth and a_R is the radial acceleration.

At the poles, $a_R = 0$. Therefore,

$$g = G \frac{M_e}{R_e^2} \quad (\text{at the poles})$$

The Motion of Planets and Satellites

M and m are two spherical bodies moving in circular orbits of radii R and r respectively under the influence of each others' gravitational attraction. If ω is the angular frequency of the two, we have the relation



$$G \frac{Mm}{(R+r)^2} = m\omega^2 r$$

If R is much smaller than r , the relation becomes

$$GM_s = \omega^2 r^3$$

(as in the case of the sun and a planet where M_s is the mass of the sun)

On substituting $\omega = 2\pi/T$, the relation becomes

$$GM_s = \frac{4\pi^2 r^3}{T^2}$$

where T is the period of revolution.

This is a basic equation of planetary motion. It holds good for elliptical orbits also if r is defined to be the semi-major axis. An important consequence of the above expression is that

$$T^2 = \frac{4\pi^2 r^3}{GM_s}$$

where $4\pi^2/GM_s$ is a constant factor which is the same for all planets. Another consequence is that the time period is independent of the mass of the planet.

Gravitational Field Strength

$$\vec{g} = \frac{\vec{F}}{m}$$

Gravitational Potential Energy

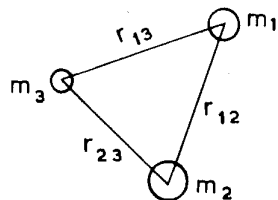
$$u(r) = -G \frac{Mm}{r}$$

The negative sign indicates that the potential energy is negative at any finite distance, that is, the potential energy is zero at infinity.

Escape Velocity The critical initial speed, such that the projectile does not return to earth, called the escape speed V_0 , is given by

$$\frac{1}{2} m V_0^2 = G \frac{M_e m}{R_e} \Rightarrow V_0 = \sqrt{2 \frac{GM_e}{R_e}} = 11.2 \text{ km/s}$$

Potential Energy of Several-Particle System Total work done in assembling the system is the total potential energy of the system.



$$U = - \left(G \frac{m_1 m_2}{r_{12}} + G \frac{m_1 m_3}{r_{13}} + G \frac{m_2 m_3}{r_{23}} \right)$$

The amount of energy required to separate these three masses into isolated masses once again is

$$+ \left(G \frac{m_1 m_2}{r_{12}} + G \frac{m_1 m_3}{r_{13}} + G \frac{m_2 m_3}{r_{23}} \right)$$

This is known as binding energy for the configuration.

Total Energy in Motion of Planets and Satellites

$$E = K + U = \frac{1}{2} G \frac{Mm}{r} - G \frac{Mm}{r} = - \frac{GMm}{2r}$$

This energy is constant and is negative.

Gravitational Force due to a Spherical Shell If M is the mass of a spherical shell and m is the mass of an external point mass, the gravitational force of attraction between them is $F = G Mm/R^2$ where R is the separation between them. Here, the spherical mass distribution behaves as a point mass, i.e. as if all its mass were concentrated at its centre. In solving problems on spherical mass distribution, the two points to be remembered are:

1. The force exerted by the shell on a particle outside is given by assuming the entire mass of the shell to be concentrated at the centre of the shell.
2. The force exerted by the shell on a particle, when it is inside the shell, is zero.

ILLUSTRATIONS

1. If the moon describes a circular orbit of radius r round the earth with a uniform angular speed ω , find an expression for ω in terms of g , radius of the orbit r , and radius of the earth R_e .

Solution

$$\text{Gravitational pull} = G \frac{mM_e}{r^2}$$

where m is the mass of the moon and M_e is the mass of the earth.

$$\text{Centripetal force} = \frac{mV^2}{r} = \frac{m(r\omega)^2}{r} = mr\omega^2$$

For motion in a circular orbit,

$$mr\omega^2 = G \frac{mM_e}{r^2} \Rightarrow \omega^2 r^3 = GM_e$$

$$\text{Also, } gR_e^2 = GM_e$$

hence,

$$\omega^2 r^3 = gR_e^2$$

$$\therefore \omega = \sqrt{\frac{gR_e^2}{r^3}}$$

2. If the radius of the orbit is 60 times that of the earth's radius and the period of revolution of the moon round the earth is 27.3 days, find the radius of the orbit of moon.

Solution

$$\frac{\omega^2}{r} = \frac{R_e^2}{r^3} \Rightarrow \frac{\omega^2}{g} = \frac{(r/60)^2}{r^3} = \frac{1}{60 \times 60 \times r} \quad (r = 60 R_e)$$

$$\therefore \frac{4\pi^2}{gT^2} = \frac{1}{60 \times 60 \times r}$$

$$r = \frac{gT^2}{60 \times 60 \times 4\pi^2}$$

$$\therefore r = \frac{10 \times (27.3 \times 24 \times 60 \times 60)^2}{60 \times 60 \times 4\pi^2} = 386 \times 10^3 \text{ km.}$$

3. Had the earth been at half its present distance from the sun, how many days would there have been in one year?

Solution

Since the earth is moving round the sun, centripetal force is provided by the gravitational attraction

$$\frac{mV^2}{R} = G \frac{mM}{R^2} \Rightarrow V^2 = \frac{GM}{R}$$

where m is the mass of the earth and M is the mass of the sun.

$$V = \frac{2\pi R}{T}$$

where T is the time taken by earth to complete one revolution round the sun.

$$\therefore \frac{4\pi^2 R^2}{T^2} = \frac{GM}{R} \Rightarrow T^2 = \frac{4\pi^2 R^3}{GM}$$

when the distance between them is reduced to half, it means R becomes $R/2$. Therefore, the new time period becomes

$$T'^2 = \frac{4\pi^2 (R/2)^3}{GM}$$

Hence,

$$T'^2 = \frac{T^2}{8} \Rightarrow T' = \frac{T}{2\sqrt{2}}$$

$$T = 365 \text{ days}$$

$$\therefore T' = \frac{365}{2\sqrt{2}} = 129 \text{ days}$$

4. An artificial satellite of mass m revolves round the earth at a height h above the surface of the earth. Find its orbital velocity and the time period of revolution.

Solution

Let V_0 be the orbital velocity and R_e be the radius of the earth. For the dynamic equilibrium of the satellite revolving in a circular orbit round the earth, we have

$$\frac{mV_0^2}{(R_e + h)} = G \frac{mM_e}{(R_e + h)^2} \Rightarrow V_0^2 = \frac{GM_e}{(R_e + h)} \quad (1)$$

Also

$$mg = G \frac{mM_e}{R_e^2} \Rightarrow GM_e = gR_e^2 \quad (2)$$

$$\therefore V_0^2 = \frac{gR_e^2}{(R_e + h)} \quad \text{or} \quad V_0 = \sqrt{\frac{g}{(R_e + h)}} R_e \quad (3)$$

$$T = \frac{2\pi(R_e + h)}{V_0}$$

where V_0 is obtained from (3).

5. An artificial satellite of mass m is revolving in a circular orbit round the earth. Find an expression for its binding energy.

Solution

Let V be the velocity of the satellite and h be the height of the satellite from the surface of the earth. If R_0 is the radius of the circular orbit and R the radius of the earth, then

$$R_0 = R + h$$

Total energy of the satellite,

$$E = \text{PE} + \text{KE}$$

$$\text{PE} = -G \frac{mM}{R_0}$$

where M is the mass of the earth

$$\text{KE} = \frac{1}{2} mV^2 = \frac{1}{2} m \frac{GM}{R_0}$$

$$\therefore E = \frac{1}{2} G \frac{mM}{R_0} - G \frac{mM}{R_0} = -\frac{1}{2} G \frac{mM}{R_0}$$

The minimum energy required to break the bond between the earth and the satellite is known as the binding energy of the satellite. Hence, the binding energy is

$$-E = G \frac{mM}{2R_0} = G \frac{mM}{2(R + h)}$$

6. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km.

(a) When S_2 is closest to S_1 find the speed of S_2 relative to S_1 .

(b) Also find the angular speed of S_2 as actually observed by an astronaut in S_1 .

Solution

$$(a) \quad \frac{mV^2}{R} = G \frac{mM}{R^2} \Rightarrow V^2 = \frac{GM}{R}$$

$$V = \frac{2\pi R}{T} \Rightarrow V^2 = \left(\frac{4\pi^2 R^2}{T^2} \right) = \frac{GM}{R}$$

$$\therefore T^2 = \frac{4\pi^2 R^3}{GM}$$

If T_1 and T_2 are the time periods for satellites S_1 and S_2 respectively,

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow R_2 = \left(\frac{T_2}{T_1}\right)^{2/3} R_1$$

$$T_1 = 1 \text{ hr}, T_2 = 8 \text{ hr} \quad \text{and} \quad R_1 = 10^4 \text{ km}$$

$$\therefore R_2 = \left(\frac{8}{1}\right)^{2/3} 10^4 \text{ km} = 4 \times 10^4 \text{ km}$$

$$V_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/hr}$$

$$V_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4 \text{ km/hr}$$

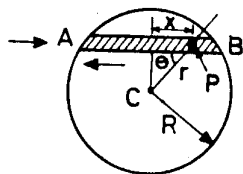
Relative velocity of S_2 with respect to S_1 is

$$V = V_2 - V_1 = (\pi \times 10^4 - 2\pi \times 10^4) \text{ km/hr}$$

$$|V| = \pi \times 10^4 \text{ km/hr}$$

$$(b) \quad \omega = \frac{|V|}{R_2 - R_1} = \frac{\pi \times 10^4}{4 \times 10^4 - 1 \times 10^4} = \frac{\pi}{3} \text{ rad/hr}$$

7. A tunnel is dug from one side of earth's surface to the other end along a chord. If a particle is dropped from one end, it oscillates within the tunnel. Find its period of oscillation. Will it attain the same maximum speed along a chord as it does along a diameter?



Solution

Let the particle of mass m be at P in the tunnel at any instant of time.

$$\text{Force on the particle at this position} = -G \frac{mMr}{R^3}$$

where r is the distance of particle from the centre of the earth and R is the radius of the earth. This force is directed towards the centre of the earth C along PC

$$\text{Force experienced by particle along chord} = F \cos \theta$$

$$= \left(-G \frac{mMr}{R^3}\right) \cdot \frac{X}{r} = -\frac{GMm}{R^3} X$$

This is of the form $F = -kX$.

Hence the force constant here is GMm/R^3 .

$$\therefore T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{R^3}{GM}}$$

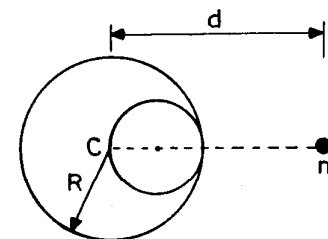
Note that the time period is independent of the mass of the particle that is dropped into the tunnel.

The maximum speed gained when the tunnel is along the chord and that when the tunnel is along the diameter will not be the same. The difference in potential energy between points B and P will be less than that between B and C. Hence, when the tunnel is along the chord, the kinetic energy gained will be less than when it is along the diameter, and the maximum speed gained will be more in the latter case.

If we substitute the values of R , M and G in the expression for time period, we get

$$T = 2\pi \sqrt{\frac{(6.4 \times 10^6)^3}{6.6 \times 10^{-11} \times 6 \times 10^{24}}} = 84.2 \text{ min}$$

8. A spherical hollow is made in a lead sphere of radius R such that its surface touches the outside surface of the lead sphere and passes through its centre. The mass of the lead sphere before hollowing was M . What is the force of attraction that this sphere would exert on a particle of mass m which lies at a distance d from the centre of the lead sphere on the straight line joining the centres of the sphere and the hollow?



Solution

$$\text{Volume of lead sphere} = \frac{4}{3} \pi R^3$$

$$\text{Volume of hollow sphere} = \frac{4}{3} \pi \left(\frac{R}{2}\right)^3 = \frac{1}{8} \left(\frac{4}{3} \pi R^3\right)$$

To calculate the force of attraction on the point mass m , we should calculate the force due to the solid sphere and subtract from this the force that the mass of the hollowed sphere would have exerted on m . The distance from the centre of the hollow sphere to the mass m is $(d - R/2)$

Hence, the net force on m is

$$F = G \frac{mM}{d^2} - \frac{Gm(M/8)}{(d - R/2)^2} = \frac{GMm}{d^2} \left[1 - \frac{1}{8(1 - R/2d)^2}\right]$$

9. A "synchronous earth satellite" is to be launched, having the same angular velocity as that of the earth, so that it always remains above the same point on the earth. What would be the radius of the orbit of such a satellite?

Solution

Kepler's third law gives a relation between the time period and the radius of the circular orbit. For some particular radius of the orbit, the time period has to be exactly equal to 24 hours. Further, if the orbit has to be stable, the centripetal force must be supplied entirely by the force of gravitation. Hence,

$$T = 24 \text{ hr} = 86400 \text{ s} \quad \text{and} \quad T = 2\pi \sqrt{\frac{r^3}{GM}}$$

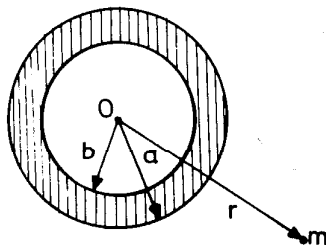
$$r = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} = \left(\frac{6.6 \times 10^{-11} \times 6 \times 10^{24} \times (86400)^2}{4\pi^2} \right)^{1/3}$$

$$= 4.2 \times 10^4 \text{ km}$$

10. A sphere of mass M and radius a has a concentric cavity of radius b , as shown in the figure.

(a) Sketch the gravitational force F exerted by the sphere on the particle of mass m , located at a distance r from the centre of the sphere as a function of r in the range $0 \leq r \leq \infty$.

(b) Sketch the corresponding curve for the potential energy $u(r)$ of the system.



Solution

The whole space can be divided into three regions:

$$(i) \quad 0 < r < b; \quad F(r) = 0$$

$$(ii) \quad b < r < a; \quad F(r) = \frac{4}{3} \pi G \rho m \left(r - \frac{b^3}{r^2} \right)$$

where ρ is the density of material of the sphere

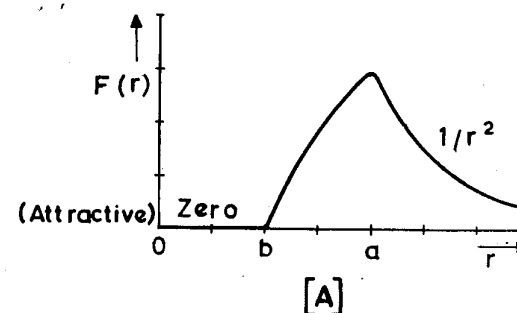
$$(iii) \quad a < r < \infty; \quad F(r) = \frac{4}{3} \pi G \rho m \left(\frac{a^3 - b^3}{r^2} \right)$$

The conclusions are as follows.

The force remains zero from 0 to b . Then it increases up to $r = a$ and decreases inversely as the square of the distance r . The curve will look as shown in Fig. (a).

$$(b) \quad (i) \quad 0 < r < b; \quad u(r) = -2\pi G \rho m (a^2 - b^2)$$

$$(ii) \quad b < r < a; \quad u(r) = \frac{-2\pi G \rho m}{3r} (3ra^2 - 2b^3 - r^3)$$

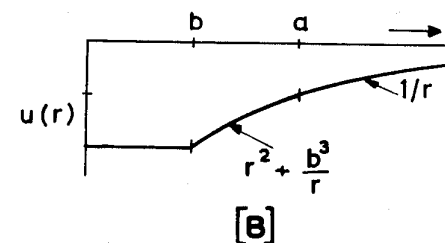


$$(iii) \quad a < r < \infty; \quad u(r) = \frac{-4\pi G \rho m}{3r^2} (a^3 - b^3)$$

Note: The force here is attractive. The conclusions are as follows.

- (i) The potential energy remains constant up to $r = b$.
- (ii) Beyond this, it increases up to $r = a$ as $[r^2 + (b^3/r)]$
- (iii) Beyond $r = a$, it increases as $1/r$.

The potential energy as a function of r is shown in Fig. (b).



EXERCISES

1. The mass of the electron is $9 \times 10^{-31} \text{ kg}$. If the two electrons are separated by a distance of 1 m, the gravitational force of attraction between them is
 - (a) $5.40 \times 10^{-51} \text{ N}$
 - (b) $5.40 \times 10^{-44} \text{ N}$
 - (c) $5.40 \times 10^{-42} \text{ N}$
 - (d) $5.40 \times 10^{-71} \text{ N}$
2. If earth is assumed to be a uniform sphere and its radius is $6.4 \times 10^6 \text{ m}$, the mass of the earth is
 - (a) $6 \times 10^{24} \text{ kg}$
 - (b) $6 \times 10^{17} \text{ kg}$
 - (c) $6 \times 10^{20} \text{ kg}$
 - (d) $6 \times 10^{12} \text{ kg}$
3. If we ignore the presence of the sun, then there exists a point on the line joining the earth and the moon where gravitational force is zero. The point is located from the moon at a distance of
 - (a) $8 \times 10^7 \text{ m}$
 - (b) $4 \times 10^{10} \text{ m}$
 - (c) $4 \times 10^7 \text{ m}$
 - (d) $8 \times 10^{10} \text{ m}$

(Given that earth is 80 times heavier than the moon and the separation between the earth and the moon is $4 \times 10^8 \text{ m}$)
4. The radius of the planet Mars is $3.40 \times 10^6 \text{ m}$ and on the surface the acceleration due to gravity is 3.7 m/s^2 . The mass of Mars is

- (a) 4.6×10^{25} kg (b) 6.43×10^{23} kg
(c) 6.43×10^{28} kg (d) 4.6×10^{27} kg
5. From the data for Mars given in the above problem, the average density is
(a) 3.90×10^3 kg/m³ (b) 9.3×10^4 kg/m³
(c) 2.0×10^5 kg/m³ (d) 2.5×10^3 kg/m³
6. Choose the correct statement from among the following
(a) a planet moves in an elliptical orbit with the centre of mass located at the intersection of major and minor axes of the ellipse
(b) the position vector for a planet, no matter from where it is measured, sweeps out equal areas in equal time intervals
(c) the time period of a planet is directly proportional to the cube root of the semimajor axis
(d) the ratio of the square of the period to the cube of the semimajor axis is approximately the same for all planets.
7. If the radius of the earth is 6.4×10^6 m, the weight of a 100 kg body, if taken to a height of 3.6×10^6 m above sea level, will be
(a) 72 kg wt (b) 60 kg wt
(c) 54 kg wt (d) 41 kg wt
8. The value of acceleration due to gravity at the bottom of a sea 30 km deep is
(a) 9.34 m/s² (b) 9.07 m/s²
(c) 9.14 m/s² (d) 9.76 m/s²
9. If the apparent weight of bodies at the equator is to be zero, then the earth should rotate on its axis faster than its present value by a factor of
(a) 17 (b) 12
(c) 34 (d) 24
10. A wrist watch working on a spring system (call it A) and a wall clock working on a pendulum system (call it B), are both set for the same timing on the surface of the earth. Thereafter, they are taken to the moon.
(a) Both A and B will give the same time as on the earth
(b) A will be faster than B
(c) B will be faster than A
(d) A and B both will give the same time but different from that on the earth.
11. A body of mass m is lifted up from the surface of earth to a height three times the radius of the earth. The change in potential energy of the body is
(a) $3 mgR$ (b) $\frac{3}{4} mgR$
(c) $\frac{1}{3} mgR$ (d) $\frac{2}{3} mgR$
12. g_e and g_p denote the acceleration due to gravity on the surface of the earth and another planet whose mass and radius are twice that of the earth. The relation that holds true is
(a) $g_p = \frac{1}{2} g_e$ (b) $g_p = \frac{g_e}{\sqrt{2}}$
(c) $g_p = 2 g_e$ (d) $g_p = \frac{\sqrt{2}}{3} g_e$
13. The distances of Neptune and Saturn from the sun are nearly 10^{13} and 10^{12} m respectively. Assuming that they move in circular orbits, their periodic times will be in the ratio
(a) 10 (b) 1000
(c) $10\sqrt{10}$ (d) 100

14. A girl is swinging on a swing in the sitting position. She now stands up and therefore the time period will be
(a) longer (b) shorter
(c) same (d) depends on the weight of the girl
15. If the radius of the earth were to shrink by 1%, its mass remaining the same, the acceleration due to gravity on the earth's surface would
(a) decrease (b) increase
(c) remain unchanged (d) become zero
16. If the change in the value of g at a height h above the surface of earth is the same as at a depth d below it (both h and d are much smaller than the radius of the earth), then
(a) $d = h$ (b) $d = 2h$
(c) $d = h/2$ (d) $d = h^2$
17. The radius of the earth is 6.6×10^6 m, its mean density is 5500 kg/m³ and $G = 6.6 \times 10^{-11}$ MKS units. The gravitational potential at a point on the surface of the earth is
(a) 0.6×10^7 J/kg (b) 13.2×10^7 J/kg
(c) 6.6×10^4 J/kg (d) 6.6×10^7 J/kg
18. In the above problem, the gravitational field at the same point is
(a) 5 N/kg (b) 20 N/kg
(c) 15.5 N/kg (d) 10 N/kg
19. * If the radius of the moon were $1/4$ th the radius of the earth and its mass $1/80$ th the mass of the earth, and escape velocity from the surface of the earth is 11.2 km/s, the velocity of escape from the surface of the moon would be
(a) $11.2/\sqrt{10}$ km/s (b) $11.2/\sqrt{5}$ km/s
(c) $11.2/\sqrt{20}$ km/s (d) $11.2/\sqrt{7}$ km/s
20. An artificial satellite is revolving round the earth at a height of 800 km above the surface of the earth. If the radius of the earth is 6.6×10^6 m, its time period is about
(a) 6040 s (b) 800 s
(c) 10000 s (d) 2180 s
21. Two point masses 100 kg and 25 kg are situated at two points 2 m apart. The gravitational potential midway between them is
(a) -258×10^{-11} J/kg (b) -25×10^{-11} J/kg
(c) -8×10^{-10} J/kg (d) -825×10^{-11} J/kg

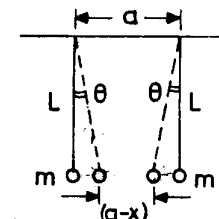
22. * Two small balls of mass m each are suspended side by side by two equal threads of length L . If the distance between the upper ends of the threads be a , the angle that the threads will make with the vertical due to attraction between the balls is

(a) $\tan^{-1} \frac{(a-X)g}{mG}$

(b) $\tan^{-1} \frac{mG}{(a-X)^2 g}$

(c) $\tan^{-1} \frac{(a-X)^2 g}{mG}$

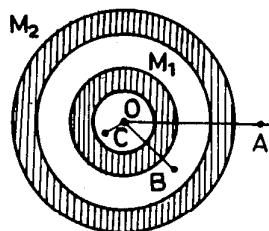
(d) $\tan^{-1} \frac{(a^2 - X^2) G}{mG}$



23. If the radius of the earth shrinks by 1%, its mass remaining same, the acceleration due to gravity on the surface of the earth will
 (a) decrease by 2% (b) decrease by 0.5%
 (c) increase by 2% (d) increase by 0.5%
24. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 2 hours and 16 hours respectively. If the radius of the orbit of S_1 is 10^4 km, the radius of the orbit of S_2 is
 (a) 8×10^4 km (b) 2×10^4 km
 (c) 16×10^4 km (d) 4×10^4 km
25. Suppose a tunnel could be dug through the earth from one side to the other along a diameter and a particle of mass M is dropped into it. If all frictional forces are neglected, the particle will
 (a) enter from one side and come out from the other with a velocity greater than that at the centre
 (b) stop at the centre of the earth as earth attracts all bodies towards its centre
 (c) undergo pendulum-like motion and never stop
 (d) take a spiral path in the tunnel till it comes out from the other end
26. In the tunnel referred to in the above problem, a sphere of mass 10 kg is dropped from one end. The time it will take to reach the other end is (density of earth is $5.5 \times 10^3 \text{ kg/m}^3$)
 (a) 84 s (b) Infinite
 (c) 84 min (d) 42 min
- 27.* A planet revolves around the sun in an elliptical orbit of eccentricity e . If T is the time period of the planet, then the time spent by the planet between the ends of the minor axis close to the sun is
 (a) $T \left(\frac{1}{2} - \frac{e}{\pi} \right)$ (b) $\frac{T \cdot e}{\pi}$
 (c) $\left(\frac{e}{\pi} - 1 \right) T$ (d) $\frac{T \pi}{e}$
28. Find the correct statement from among the following.
 (a) gravitational field strength is measured in joules per kilogram
 (b) gravitational field is an example of a scalar field
 (c) each point in a gravitational field has a vector associated with it
 (d) constantly variable distribution of matter gives rise to a stationary gravitational field
29. The length of a simple pendulum is extended to infinity and it is allowed to oscillate on the surface of the earth whose radius is R_e . Its time period is
 (a) $2\pi \sqrt{\frac{5R_e}{g}}$ (b) $2\pi \sqrt{\frac{R_e}{g}}$
 (c) $2\pi \sqrt{\frac{7R_e}{g}}$ (d) $2\pi \sqrt{\frac{9.8R_e}{g}}$
30. The longest possible period of a simple pendulum in the vicinity of the earth's surface is
 (a) 81 s (b) 120 min
 (c) 84 min (d) 3 hr
31. The binding energy of the earth-sun system (neglecting the presence of other planets or satellites) is
 (a) $5.0 \times 10^{23} \text{ J}$ (b) $-5.0 \times 10^{43} \text{ J}$
 (c) $5.0 \times 10^{30} \text{ J}$ (d) $-5.0 \times 10^{33} \text{ J}$

32. A tunnel is dug through the diameter of the moon whose mean density is $3,340 \text{ kg/m}^3$. Now a sphere of 10 kg mass is dropped from one side. The time it takes to reach the other end of the tunnel is
 (a) 54.5 min (b) 45.5 min
 (c) 84.2 min (d) 105.5 min
33. A planet of mass m is moving about the sun of mass M in a circular orbit of radius R . Assuming the sun to be at rest in an inertial reference frame, the total energy of the planet-sun system is
 (a) $G \frac{mM}{R}$ (b) $-G \frac{mM}{R}$
 (c) $-G \frac{mM}{2R}$ (d) $G \frac{mM}{2R}$
34. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C
-
- (a) A shows the kinetic energy, B the total energy and C the potential energy of the system
 (b) C shows the total energy, B the kinetic energy and A the potential energy of the system
 (c) C and A are kinetic and potential energies respectively and B is the total energy of the system
 (d) A and B are the kinetic and potential energies and C is the total energy of the system.
- 35.* Find the correct statement from among the following
 (a) the angular momenta of all planets about the sun do not necessarily remain constant
 (b) Kepler's second law is valid for any central force, that is, any force directed towards the sun
 (c) Kepler's first law requires that a force of gravitation can yield planetary orbits which are circular only with the sun at its centre
 (d) the gravitational field concept was developed and used by Newton
36. Find the correct statement from among the following
 (a) in planetary motion, total energy remains constant but total angular momentum may vary
 (b) both total energy and total angular momentum are constant in planetary motion and the total energy is negative
 (c) motion of a planet about the sun and motion of an electron about an attracting nuclear centre are governed by identical relations and the total energy is always positive in both cases
 (d) both total energy and total angular momentum are constant in planetary motion and the total energy is positive
37. The ratio of the radius of the earth to that of the moon is 10. The ratio of the acceleration due to gravity on the earth to that on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon is
 (a) 6 (b) 1.5
 (c) 12 (d) 8

38. A satellite of the earth is revolving in circular orbit with a uniform velocity V . If the gravitational force suddenly disappears, the satellite will
- continue to move with the same velocity in the same orbit
 - move tangentially to the original orbit with velocity V
 - fall down with increasing velocity
 - come to a stop somewhere in its original orbit
39. A relay satellite transmits a television programme continuously from one part of the world to another because its
- period is greater than the period of rotation of the earth about its axis
 - period is smaller than the period of rotation of the earth about its axis
 - period has no relation with the period of the earth about its axis
 - period is equal to the period of rotation of the earth about its axis
- 40.* Two concentric shells of uniform density of mass M_1 and M_2 are situated as shown in the figure. The forces experienced by a particle of mass m when placed at positions A, B and C respectively are (given $OA = p$, $OB = q$ and $OC = r$)



- zero, $G \frac{M_1 m}{q^2}$ and $G \frac{(M_1 + M_2)m}{p^2}$
 - $G \frac{(M_1 + M_2)m}{p^2}$, $G \frac{(M_1 + M_2)m}{q^2}$ and $G \frac{M_1 m}{r^2}$
 - $G \frac{M_1 m}{q^2}$, $G \frac{(M_1 + M_2)m}{p^2}$ and zero
 - $G \frac{(M_1 + M_2)m}{p^2}$, $G \frac{M_1 m}{q^2}$ and zero
41. Masses of 2 and 8 kg are placed at a separation of 0.12 m. The gravitational force on a unit mass at a point on the line joining the masses 0.04 m from the 2 kg mass is
- $8,250 \times 10^{-11}$ N
 - zero
 - 165×10^{-9} N
 - 12.4×10^{-8} N
42. In problem 41, the potential energy per unit mass at that point is
- -165×10^{-10} J/kg
 - -33×10^{-10} J/kg
 - -99×10^{-10} J/kg
 - -198×10^{-10} J/kg
- 43.* If M_e be the mass of the earth and M_m the mass of the moon ($M_e = 81 M_m$), the potential energy of a body of mass m in the gravitational field of the earth and moon which is at a distance R from the centre of the earth and a distance r from the centre of the moon, is
- $-Gm M_m \left(\frac{81}{R} + \frac{1}{r} \right)$
 - $-Gm M_e \left(\frac{81}{r} + \frac{1}{R} \right)$
 - $-Gm M_m \left(\frac{R}{81} + r \right) \cdot \frac{1}{R^2}$
 - $Gm M_m \left(\frac{81}{R} - \frac{1}{r} \right)$
44. If $M_e = 81 M_m$ as given in problem 43, the points where the gravitational field strength attributed to the earth and moon are zero, are

- only at infinity
 - exactly midway between the centres of the earth and the moon
 - nine-tenths of the way to the moon and also at infinity
 - five-sevenths of the way to the moon and also at infinity
45. The height above the surface of the earth where acceleration due to gravity is $1/64$ of its value at the surface of the earth is
- 45×10^6 m
 - 45×10^3 m
 - 54×10^5 m
 - 54×10^7 m
46. At the earth's surface, a freely suspended object is struck a horizontal blow with a hammer. The object is taken to the moon, suspended freely and struck an equally strong blow with the same hammer. The horizontal speed resulting on the moon will be
- equal to that on the earth
 - half that on the earth
 - five times that on the earth
 - one-fifth that on the earth
- 47.* The radius of the planet Jupiter is 74,000 km. A satellite completes a circular orbit around it once in every 16.7 days. The radius of the orbit of the satellite is 27 times the radius of Jupiter. The mass of Jupiter is
- 2.3×10^{42} kg
 - 2.3×10^{27} kg
 - 2.3×10^{30} kg
 - 2.3×10^{32} kg
48. The gravitational potential energy of a rocket of mass 100 kg at a distance of 10^7 m from the centre of the earth is -4.0×10^9 J. The weight of the rocket in newtons at a distance of 10^9 m from the earth's centre is
- 8.0×10^{-2}
 - 4.0×10^{-2}
 - 4.0×10^{-3}
 - 8.0×10^{-3}
49. Assuming that the moon is a sphere of the same mean density as that of the earth and one quarter of its radius, the length of a seconds pendulum on the moon (its length on the earth's surface is 99.2 cm) is
- 24.8 cm
 - 49.6 cm
 - 99.2 cm
 - $\frac{99.2}{\sqrt{2}}$ cm
50. An artificial satellite of the earth releases a package. If air resistance is neglected, the point where the package will hit (with respect to the position at the time of release) will be
- ahead
 - exactly below
 - behind
 - it will never reach the earth

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (c) | 4. (b) | 5. (a) |
| 6. (d) | 7. (d) | 8. (d) | 9. (a) | 10. (b) |
| 11. (d) | 12. (a) | 13. (c) | 14. (b) | 15. (b) |
| 16. (b) | 17. (d) | 18. (d) | 19. (c) | 20. (a) |
| 21. (d) | 22. (b) | 23. (c) | 24. (d) | 25. (c) |
| 26. (d) | 27. (a) | 28. (c) | 29. (b) | 30. (c) |
| 31. (d) | 32. (a) | 33. (c) | 34. (d) | 35. (b) |
| 36. (b) | 37. (d) | 38. (b) | 39. (d) | 40. (d) |
| 41. (b) | 42. (c) | 43. (a) | 44. (c) | 45. (a) |
| 46. (a) | 47. (b) | 48. (b) | 49. (a) | 50. (d) |

Note: Problems marked with a star should take about five to six minutes. All the rest should take not more than three minutes.

9

Fluid Statics

1. Specific Gravity and Hydrostatic Pressure

Mass Density (ρ)

$$\rho = \frac{\text{mass of body}}{\text{volume of body}} = \frac{m}{V}$$

Its SI units are kg/m^3 and the density of water is $1 \times 10^3 \text{ kg/m}^3$.

Specific Gravity

$$\text{Specific gravity} = \frac{\rho}{\rho_{\text{standard}}}$$

It is a dimensionless ratio and has the same value for all systems of units.

Pressure in a Fluid

If the pressure is the same at all points of a finite plane surface of area A , then

$$p = \frac{F}{A}$$

where F is the normal force on it.

Its units are N/m^2 or pascal.

$$1 \text{ pascal} = 1 P_a = 1 \times 10^{-5} \text{ bar}$$

$$\text{Pressure at a depth } h \text{ in an open vessel} = P_a + \rho gh$$

where P_a is the atmospheric pressure on the open surface, and ρ is the density of the liquid.

The standard atmospheric pressure,

$$1 \text{ Atm} = 1.013 \times 10^5 \text{ N/m}^2$$

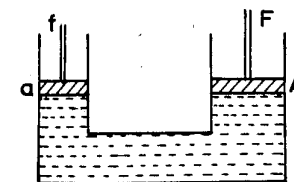
$$\text{Hydrostatic pressure } p = \rho gh$$

Pascal's Law Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel. A hydraulic

press shown in the figure here is an illustration of Pascal's law. A piston of small cross-sectional area a is used to exert a small force f directly on a liquid such as oil. The pressure $p = f/a$ is transmitted through the connecting pipe to a larger cylinder equipped with a larger piston of area A . Since the pressure is the same in both the cylinders,

$$p = \frac{f}{a} = \frac{F}{A} \Rightarrow F = \frac{A}{a} f$$

We see that the applied force f is multiplied by a factor (A/a) .



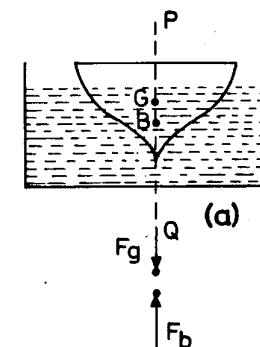
Archimedes' Principle

When a body is immersed in a fluid, the latter exerts an upward force on the body equal to the weight of the fluid that is displaced by the body.

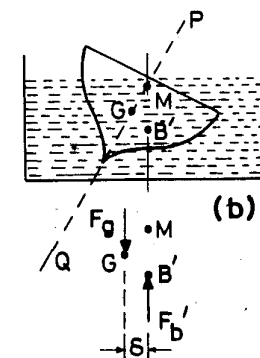
Upthrust of fluid or buoyant force = weight of displaced fluid.

Stability of Floating Bodies

A body is floating in water in an erect position. The gravitational force F_g on the body is directed vertically downwards through G and has magnitude mg where m is the mass of the body. The buoyant force F_b acts upwards and acts at B , the centre of the buoyancy (centre of the gravity of the displaced water). Points B and G do not coincide, but if the body is in equilibrium, the points must lie on the same vertical line.



Suppose the body heels slightly as shown in Fig. (b). The point G remains fixed in the body. In this condition, the shape and volume of the displaced water will be different from those when the body is erect. Then, the centre of buoyancy will shift to a new position B' . A vertical line through B' intersects the line PG at M , called the metacentre of the body, and for small angular displacements, the position of M remains essentially unchanged. If M lies above G , the torque about G produced by the new buoyant force F'_b will tend to "right" the body; thus the equilibrium is stable. The restoring torque



$$\tau = mg\delta$$

where δ is horizontal displacement between F_g and F'_b . Conversely, if M lies below G , the state of equilibrium with the centre of gravity, it results in neutral equilibrium.

ILLUSTRATIONS

1. A piece of cork (sp. gr. = 0.25) and a metallic piece (sp. gr. = 8) are tied together. If the combination neither floats nor sinks in alcohol (sp. gr. = 0.8), calculate the ratio of mass of the cork and the metal.

Solution

$$V_c = \frac{M_c}{0.25} \text{ and } V_m = \frac{M_m}{8}$$

$$\therefore \text{Volume of alcohol displaced} = \left(\frac{M_c}{0.25} + \frac{M_m}{8} \right)$$

$$\therefore \text{Mass of alcohol displaced} = \left(\frac{M_c}{0.25} + \frac{M_m}{8} \right) \times 0.8$$

From the principle of floatation,

$$M_c + M_m = \left(\frac{M_c}{0.25} + \frac{M_m}{8} \right) 0.8 \Rightarrow \frac{M_c}{M_m} = \frac{9}{22}$$

2. If W_1 and W_2 are the weights of a body when immersed in liquids of specific gravities ρ_1 and ρ_2 , find the weight of the same body in vacuum.

Solution

$$\text{Loss in weight of liquid } (\rho_1) = (W - W_1) \text{ kg}$$

If V is the volume of the body, then

$$V\rho_1 = W - W_1 \Rightarrow V = \frac{W - W_1}{\rho_1}$$

$$V\rho_2 = W - W_2 \Rightarrow V = \frac{W - W_2}{\rho_2}$$

$$\frac{W - W_1}{\rho_1} = \frac{W - W_2}{\rho_2} \Rightarrow W = \frac{W_1\rho_2 - W_2\rho_1}{\rho_2 - \rho_1}$$

3. A block of wood having density $0.5 \times 10^3 \text{ kg/m}^3$ is just balanced on an equal-arm analytical balance by brass weights of total mass 0.0500 kg. If the densities of brass and air are $8.0 \times 10^3 \text{ kg/m}^3$ and 1.3 kg/m^3 respectively, find the true mass of the wood.

Solution

The apparent weight of each body is the difference between its true weight and the buoyant force of air.

Let ρ_w , ρ_b and ρ_a be the densities of wood, brass and air respectively and V_w and V_b the volumes of wood and brass. The apparent weights, which are equal, are

$$\rho_w V_w g - \rho_a V_w g = \rho_b V_b g - \rho_a V_b g$$

The true mass of wood = $\rho_w V_w$ and the true mass of the standard is $\rho_b V_b$. Hence,

$$\begin{aligned} \text{True mass} &= \rho_w V_w = \rho_b V_b + \rho_a (V_w - V_b) \\ &= (\text{mass of standard}) + \rho_a (V_w - V_b) \end{aligned}$$

Given in the problem,

$$V_w = \frac{0.05 \text{ kg}}{0.5 \times 10^3 \text{ kg/m}^3} = 1.0 \times 10^{-4} \text{ m}^3$$

$$V_b = \frac{0.05 \text{ kg}}{8.0 \times 10^3 \text{ kg/m}^3} = 6.25 \times 10^{-6} \text{ m}^3$$

$$\rho_a = 1.3 \text{ kg/m}^3$$

Hence,

$$\begin{aligned} \rho_a (V_w - V_b) &= 1.3(1.0 \times 10^{-4} - 6.25 \times 10^{-6}) \\ &= 1.22 \times 10^{-4} \text{ kg} \end{aligned}$$

$$\begin{aligned} \therefore \text{True mass} &= 0.0500 + 0.00012 \\ &= 0.05012 \text{ kg} \end{aligned}$$

4. A cube floating on mercury has one-fourth of its volume submerged. If enough water is added to cover the cube, what fraction of its volume will remain immersed in mercury?

Solution

Let the volume of the cube be V and its density ρ .

$$\therefore \text{Weight in air} = W_a = V\rho$$

Weight of mercury displaced when $V/4$ is submerged in mercury

$$= \frac{V}{4} \rho_{\text{Hg}}$$

Weight of floating body = $V\rho$

$$\begin{aligned} \therefore V\rho &= \frac{V}{4} \rho_{\text{Hg}} \Rightarrow \rho = \frac{\rho_{\text{Hg}}}{4} = \frac{13.6 \times 10^3}{4} \text{ kg/m}^3 \\ &= 3.4 \times 10^3 \text{ kg/m}^3 \end{aligned}$$

Let fraction f of the cube be immersed in mercury when the whole cube is covered with water on top.

$$\therefore \text{Fraction immersed in water} = (1 - f)$$

Weight of the cube = weight of mercury displaced + weight of water displaced

$$V\rho = fV\rho_{Hg} + (1 - f)V\rho_w$$

$$3.4 \times 10^3 = f \times 13.6 \times 10^3 + (1 - f) \times 1 \times 10^3$$

$$f = 0.19$$

19% of the volume of the cube is immersed in mercury.

Note that it does not matter whether the shape of the floating body is cubical or irregular. For the given density of the floating body, only 19% will remain in mercury and the rest in water.

5. (a) If the density of the atmosphere is a function of altitude y , obtain an expression for the pressure of the atmosphere at a height y .

(b) What is the pressure at an altitude of 11.0 km (the density of air at sea level is 1.29 kg/m^3 for a temperature 0°C).

Solution

(a) Let us take the upward direction to be positive.

The force equation is

$$pA - (p + dp)A - \rho(y)gA dy = 0$$

so that

$$dp = -\rho(y)g dy$$

Let us assume that the atmosphere is at a constant temperature so that the density varies in direct proportion to the pressure (actually, temperature varies with altitude, hence the result we obtain will be only approximately correct); that is

$$\rho(y) = \rho_0 \frac{p}{p_0}$$

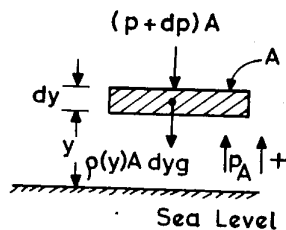
where $\rho(0) = \rho_0$ and $p(0) = p_0$

$$dp = -\rho_0 \frac{p}{p_0} g dy \Rightarrow \frac{dp}{p} = -\frac{\rho_0}{p_0} g dy$$

$$\text{or } \int_{p_0}^p \frac{dp}{p} = \frac{-g\rho_0}{p_0} \int_0^y dy \Rightarrow \ln \frac{p}{p_0} = -\frac{g\rho_0}{p_0} y$$

$$\text{or } p = p_0 e^{(-g\rho_0/p_0)y} \Rightarrow p = p_0 e^{-\alpha y}$$

$$\text{where } \alpha = \frac{g\rho_0}{p_0}$$



Thus, in this approximation, the atmospheric pressure decreases exponentially with altitude.

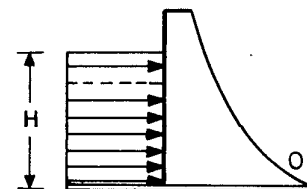
$$(b) \quad \text{Here } \alpha = \frac{g\rho_0}{p_0} = \frac{(9.8 \text{ m/s}^2) \times (1.29 \text{ kg/m}^3)}{1.013 \times 10^5 \text{ Pa}} \\ = 1.25 \times 10^{-4} \text{ m}^{-1}$$

$$p(y) = p_0 e^{-\alpha y}$$

$$p(11.0 \text{ km}) = 1 \text{ atm } e^{-(1.25 \times 10^{-4} \text{ m}^{-1})(11.0 \times 10^3 \text{ m})} \\ = 0.253 \text{ atm}$$

Note: If we take into account the variation of temperature with altitude, the pressure at this height should come to 0.224 atm., about 10% smaller than the value we have obtained above.

6. Water stands at a depth H behind the vertical upstream face of a dam and exerts a certain resultant horizontal force on the dam tending to slide it along its foundation and a certain torque tending to overturn the dam about the point O. If the total width of the dam is L , find the total horizontal force and the total torque about O.



Solution

Pressure at an elevation y is

$$p = \rho g(H - y)$$

The force dF against the shaded strip is

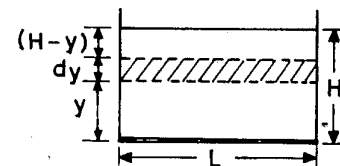
$$dF = p dA = \rho g(H - y)L dy$$

$$\therefore F = \int dF = \int_0^H \rho gL(H - y) dy = \frac{1}{2} \rho gLH^2$$

The moment of the force dF about an axis through O is

$$d\tau = y dF = \rho gLy(H - y) dy$$

$$\text{Total torque } \tau = \int d\tau = \int_0^H \rho gLy(H - y) dy \\ = \frac{1}{6} \rho gLH^3$$



7. The bag of a balloon is a sphere of diameter 50 m filled with hydrogen of density 0.090 kg/m^3 . What is the total mass of fabric, car and contents that can be lifted by this balloon in air of density 1.29 kg/m^3 .

Solution

The buoyant force ($V \rho_{\text{air}} g$) must lift the entire weight of the fabric, car and contents. Let the total weight of all this be W . Therefore,

$$V \rho_{\text{air}} g = W + V \rho_{\text{Hyd}} g$$

$$V (\rho_{\text{air}} - \rho_{\text{Hyd}}) g = W$$

$$\therefore \text{Total mass} = V (\rho_{\text{air}} - \rho_{\text{Hyd}}) = \frac{4}{3} \pi (25)^3 (1.29 - 0.090) \\ = 78540 \text{ kg}$$

8. A column of mercury is placed at the middle of a very long glass capillary tube and both its ends are closed when the tube is horizontal and the pressure of the enclosed air is 75 cm of mercury. When the tube is placed in a vertical position, it is found that the length of the tube occupied by the air above the mercury is three times as great as that occupied by the air below the mercury. What is the length of the mercury column.

Solution

Let the mercury column be of length x . In the vertical position, the lengths of air columns below and above the mercury column will be y and $3y$ respectively. Consequently, in the horizontal position, when the mercury column is in the middle, the air columns will be of length $2y$ on each side. Applying Boyle's law in the vertical position,

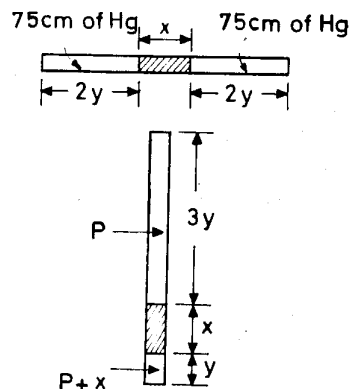
$$P \times 3y \times A = 75 \times 2y \times A$$

(where A is area of cross section of the tube)

$$P = 50 \text{ cm of mercury}$$

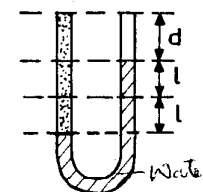
The pressure of air enclosed below mercury $= P + x = 50 + x$

$$(50 + x) \times y \times A = 75 \times 2y \times A \Rightarrow x = 100 \text{ cm}$$



EXERCISES

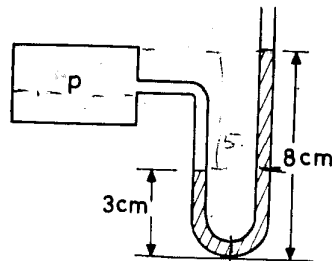
- (a) 3.09 litre
(c) 1.03 litre
- (b) 4.12 litre
(d) 2.06 litre
2. A U-tube is partly filled with water. Another liquid, which does not mix with water, is poured into one side until it stands a distance d above the water level on the other side, which has meanwhile risen a distance l . The density of liquid relative to that of water is



- (a) $\frac{2l}{[l + d]}$
(c) $\frac{2l}{[2l + d]}$
- (b) $\frac{l}{[l + d]}$
(d) $\frac{l}{[l + 2d]}$
3. Two hollow bodies of equal weight and volume and having the same shape except that one has an opening at the bottom and the other is sealed, are immersed to the same depth in water.
- (a) The work done in both the cases will be the same.
(b) The work done in case of sealed body will be more than in case of the other.
(c) The work done in case of sealed body will be less than in case of the other.
(d) No work will be done in case of the body with an opening while work will be done in case of the sealed body.
4. A ball floats on the surface of water in a container exposed to the atmosphere. Find the correct statement among the following.
- (a) The ball remains immersed a little more than its former depth when the container is covered and the air is removed.
(b) The ball sinks when the container is covered and the air is removed.
(c) The ball rises a little when the container is covered and the air is compressed.
(d) The ball remains at its former depth regardless of whether it is covered and the air is removed or whether the air is compressed.
5. A spherical bob made of cork floats half submerged in a tub of water at rest on the earth. Find the correct statement among the following.
- (a) It will float in the same way if the tub is placed in the spaceship coasting in free space.
(b) It will sink in the tub if placed in the spaceship coasting in free space.
(c) It will not behave the same way on the surface of Jupiter as on the surface of the earth.
(d) It will not float on the surface of Jupiter.
6. A patient is unable to lift a limb because of an injury but when his body is immersed in water, he can move the limb with much less effort. The reason is
- (a) that water acts as a lubricating material and makes the movement smooth
(b) the viscosity of water helps in movement
(c) the surface tension of water helps in movement
(d) that the weightlessness is felt due to buoyancy effect.
7. A sensitive analytical balance is used for very precise weight determination. A block of wood whose density is $0.5 \times 10^3 \text{ kg/m}^3$ is just balanced on an equal-arm balance by a brass weight of total mass 0.0500 kg. The true mass of wood would be

- (a) exactly equal to 0.0500 kg as the balance is, absolutely accurate
 (b) greater than 0.0500 kg because of buoyancy effects
 (c) less than 0.0500 kg because of buoyancy effects
 (d) less than 0.0500 kg because of air resistance.

8. An open mercury manometer connected to a vessel containing gas at pressure p is shown in the figure. The pressure at the bottom of the U-tube is (given that atmospheric pressure is 0.970×10^5 Pa)



- (a) 1.076×10^5 N/m²
 (b) 1.637×10^5 N/m²
 (c) 0.970×10^5 N/m²
 (d) 0.107×10^5 N/m²

9. In the above problem, the absolute pressure in the open tube at a depth 5 cm below the free surface is

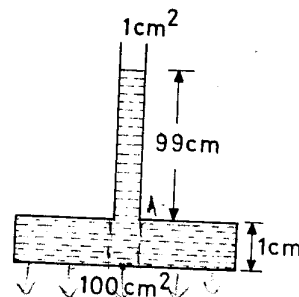
- (a) 1.637×10^5 N/m² (b) 0.107×10^5 N/m²
 (c) 1.037×10^5 N/m² (d) 1.076×10^5 N/m²

10. A diving bell is designed to withstand the pressure of water at a depth of 800 m. If sea water has a density of 1.03×10^3 kg/m³, the force that a circular glass window of diameter 1 m will experience is (atm. pr. = 1×10^5 Pa)

- (a) 2.5×10^6 N (b) 4.5×10^6 N
 (c) 9.5×10^6 N (d) 6.5×10^6 N

11. A tube 1 cm² in cross-section is attached to the top of a vessel 1 cm high and of cross-section 100 cm². Water fills the system, up to a height of 100 cm from the bottom of the vessel. The force at the bottom of the vessel is

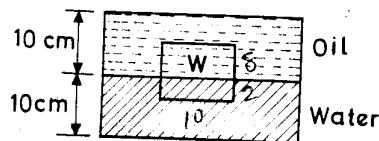
- (a) 195 N
 (b) 98 N
 (c) 19.5 N
 (d) 1.95 N



12. A block of ice of total area A and thickness 0.5 m is floating in fresh water. In order to just support a man of 100 kg, the area A should be (the specific gravity of ice is 0.917)

- (a) 1.24 m² (b) 4.21 m²
 (c) 2.41 m² (d) 7.23 m²

13. A cubical block of wood W of side 10 cm floats at the interface between oil and water as shown in the figure. The lower surface is 2 cm below the interface. If the density of the oil is 0.6 g/cc, then the pressure above that of the atmosphere at the lower face of the block is



- (a) 200 N/m² (b) 680 N/m²
 (c) 400 N/m² (d) 800 N/m²

14. A block of wax is 0.4 m long, 0.1 m wide and 0.01 m thick and its specific gravity is 0.6. A block made of lead of volume V is fastened underneath to make the wax block sink in calm water so that its top is just even with the water level. If the specific gravity of lead is 11, the volume V is

- (a) 1.6 cc (b) 64 cc
 (c) 16 cc (d) 8 cc

15. A piece of gold ($\rho = 19.3$ g/cc) has a cavity within it. It weighs 38.20 g in air and 36.20 g in water. The volume of the cavity in gold is

- (a) 0.2 cc (b) 0.04 cc
 (c) 0.02 cc (d) 0.01 cc

16. A balloon of mass 8 kg and volume V is filled with helium ($\rho_{He} = 0.178$ kg/m³). If it has to lift a 42 kg load, V should be (density of air = 1.29 kg/m³)

- (a) 45 m³ (b) 4.5 m³
 (c) 90 m³ (d) 135 m³

17. A cubical block of steel of side 0.2 m ($\rho = 8$ g/cc) floats on mercury ($\rho = 13.6$ g/cc). If water is poured on the mercury surface so that the water surface just rises to the top of the steel block, the depth of the water layer is

- (a) 0.04 m (b) 0.02 m
 (c) 0.06 m (d) 0.08 m

18.* A piece of alloy consisting of gold and silver weighs 20 g in air and 18.5 g in water. If the specific gravities of gold and silver are 19.3 and 10.5 respectively, the quantity of gold in the alloy is

- (a) 9.3 g (b) 13.9 g
 (c) 4.65 g (d) 5.53 g

19.* When equal volumes of two substances are mixed, the specific gravity of the mixture is 4; when equal weights of the same substances are mixed, the specific gravity of the mixture is 3. The specific gravities of the two substances could be

- (a) 6 and 2 or 2 and 6 (b) 3 and 4 or 4 and 3
 (c) 2.5 and 3.5 or 3.5 and 2.5 (d) 5 and 3 or 3 and 5

20. A balloon has volume of 1000 m³. It is filled with hydrogen ($\rho = 0.09$ g/litre). If the density of air is 1.29 g/litre, it can lift a total weight of

- (a) 600 kg (b) 1200 kg
 (c) 300 kg (d) 1800 kg

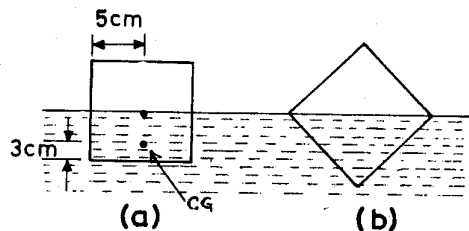
21. The upper and lower parts of a 15 cm-long cylinder are of two different metals whose specific gravities are 9.6 and 21.6. If the cylinder floats completely immersed in mercury (specific gravity 13.6), the lengths of two parts of the cylinder are

- (a) 8 cm and 7 cm (b) 4 cm and 11 cm
 (c) 5 cm and 10 cm (d) 3 cm and 12 cm

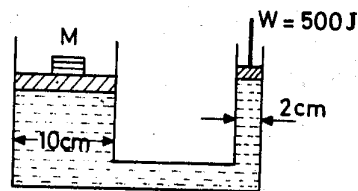
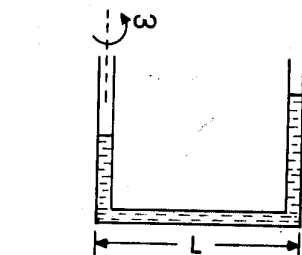
22. When a life preserver having a volume of 0.03 m³ is immersed in sea water (specific gravity 1.03), it will just support a man (specific gravity 1.2) weighing 80 kg with 1.5 of his volume above water. The density of the material of preserver is

- (a) 0.194 g/cc (b) 0.380 g/cc
 (c) 0.291 g/cc (d) 0.582 g/cc

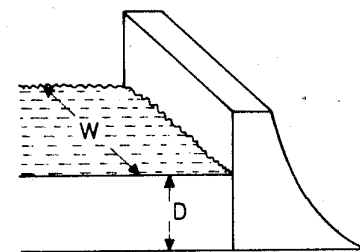
23. A cubical block of wood ($\rho = 0.6 \times 10^3$ kg/m³) of side 10 cm is weighted so that its centre of gravity is at a point shown in Fig. (a) and it floats in water with one half of its volume submerged. The restoring torque when the block is "heeled" at an angle of 45° as in Fig. (b) is

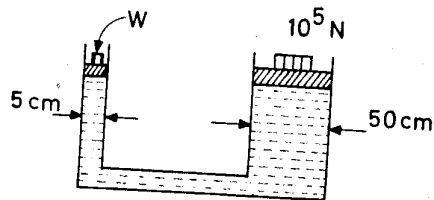


- (a) 0.17 Nm (b) 0.086 Nm
(c) 0.043 Nm (d) 0.022 Nm
24. A U-tube of length L contains a liquid. It is mounted on a horizontal turnable rotating with an angular velocity ω with one of the vertical arms on the axis of rotation. The difference in height between the liquid columns in the vertical arms will be
- (a) $\frac{\omega^2 L^2}{2g}$ (b) $\frac{L\omega}{2g}$
(c) $\frac{\omega^2}{2g}$ (d) $\frac{\omega^2 g}{2L}$
25. The upper edge of a vertical gate in a dam lies along the water surface. The gate is 2 m wide and is hinged along the bottom edge which is 3 m below the water surface. The torque about the hinge is
- (a) 4.84×10^4 Nm (b) 2.21×10^4 Nm
(c) 3.30×10^4 Nm (d) 8.84×10^4 Nm
26. The percentage of the iceberg that is visible outside above the sea is ($\rho_{\text{seawater}} = 1.028 \times 10^3 \text{ kg/m}^3$ and $\rho_{\text{ice}} = 0.917 \times 10^3 \text{ kg/m}^3$)
- (a) 33 (b) 11
(c) 22 (d) 27
27. The hydraulic press shown in the figure is used to raise the mass M through a height of 5.0 mm by performing 500 J of work at the small piston. The diameter of the large piston is 10 cm while that of the smaller one is 2 cm. The mass M is of
- (a) 10^4 kg (b) 10^3 kg
(c) 100 kg (d) 10^5 kg
28. A wooden cylinder of mass m and base area A floats in water with its axis vertical. If it is given a small vertical displacement, it undergoes SHM. The frequency of its motion is



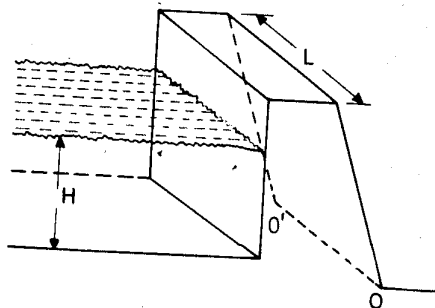
- (a) $\frac{1}{2\pi} \sqrt{\frac{A\rho_w g}{m}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{\rho_w}{Agm}}$
(c) $\frac{1}{2\pi} \sqrt{\frac{A\rho_w}{m}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{\rho_w g}{m}}$
29. A solid wooden cube 30 cm on each edge, can be totally submerged in water if it is pushed downward with a force of 54 N. The density of wood is
- (a) 800 kg/m^3 (b) 400 kg/m^3
(c) 300 kg/m^3 (d) 500 kg/m^3
30. The hydrostatic difference in blood pressure in a person of height 1.83 m between his brain and foot, assuming that the density of blood is $1.06 \times 10^3 \text{ kg/m}^3$, is
- (a) $1.90 \times 10^3 \text{ Pa}$ (b) $1.90 \times 10^4 \text{ Pa}$
(c) $19.0 \times 10^4 \text{ Pa}$ (d) $1.90 \times 10^{-4} \text{ Pa}$
31. Water stands at a depth D behind the vertical upstream face of a dam as shown in the figure. If W be the width of the stream, the force exerted by the water on the dam is
- (a) $\frac{1}{3} \rho g W D^2$ (b) $\frac{1}{2} \rho g W^2 D$
(c) $\frac{1}{3} \rho g W^2 D$ (d) $\frac{1}{2} \rho g W D^2$
32. In the above problem the torque exerted about O by the above force is
- (a) $\frac{1}{3} \rho g D^3 W$ (b) $\frac{1}{6} \rho g D^3 W$
(c) $\frac{1}{6} \rho g W^3 D$ (d) $\frac{1}{3} \rho g W^3 D$
33. A tank has the dimensions $10 \text{ m} \times 8 \text{ m} \times 5 \text{ m}$. When filled with water, the force on the bottom, on the ends and on the sides are respectively
- (a) $[4, 1, 3] \times 10^6 \text{ N}$ (b) $[3, 1, 4] \times 10^6 \text{ N}$
(c) $[4, 1, 1.25] \times 10^6 \text{ N}$ (d) $[1.25, 1, 4] \times 10^6 \text{ N}$
34. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ . The area of either base is A but in one vessel the liquid height is h_1 and in the other h_2 . If the two vessels are connected, the work done by gravity in equalising the levels is
- (a) $\frac{1}{2} (h_2 - h_1)^2 A \rho g$ (b) $\frac{1}{2} \left(\frac{h_1}{h_2} \right) A \rho g$
(c) $\frac{1}{2} (h_1 h_2)^2 A \rho g$ (d) $\frac{1}{4} (h_2 - h_1)^2 A \rho g$
35. A piston of small cross-sectional area has a diameter of 5 cm, while that of the larger cross-sectional area has a diameter of 50 cm. The weight on the small piston that will support a 10^5 N weight on the large piston is





- (a) $1 \times 10^3 \text{ N}$ (b) $1 \times 10^4 \text{ N}$
 (c) $2 \times 10^3 \text{ N}$ (d) $1 \times 10^5 \text{ N}$

36. The water behind a dam has a depth of 20 m and the width of the dam $L = 100 \text{ m}$. The upstream face of the dam is vertical. The torque exerted by the water behind the dam about the toe O is



- (a) $1.31 \times 10^9 \text{ Nm}$ (b) $3.1 \times 10^7 \text{ Nm}$
 (c) $1.31 \times 10^7 \text{ Nm}$ (d) $3.1 \times 10^9 \text{ Nm}$

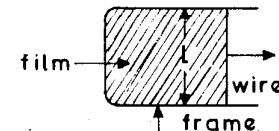
37. A cylinder filled with air at atmospheric pressure is lowered in water with its mouth downwards till it is one-third full of water. It is further lowered slowly to a depth d when it is two-thirds full of water. If the atmospheric pressure is 76 cm of mercury and the density of mercury is 13.6 g/cc, the value of d is
 (a) 5.5 m (b) 1.5 m
 (c) 15.5 m (d) 3.0 m
38. The pressure of a water pipe at the basement of a house is 34 lb. wt/sq. inch whereas at the roof it is only 20 lb. wt/sq. inch. If one cubic foot of water weighs 62 lb, the height of the house is
 (a) 23.0 ft (b) 16.25 ft
 (c) 15.5 ft (d) 32.5 ft
39. A water column of height 48 inches is required to support a man on a hydraulic lift. If the surface area of the lift is 144 sq. inches and 1 cubic ft of water weighs 62.5 lb, the weight of the man is
 (a) 288 lb (b) 180 lb
 (c) 125 lb (d) 250 lb
40. A man 1.6 m tall changes from the vertical position to the horizontal position. If the density of blood is 1.03 g/cc, the change in his blood pressure is
 (a) $16.16 \times 10^3 \text{ N/m}^2$ (b) $16.16 \times 10^2 \text{ N/m}^2$
 (c) $1.6 \times 10^5 \text{ N/m}^2$ (d) 0.16 N/m^2

2. Surface Tension

A molecule at the surface of a liquid has potential energy greater than that of a molecule in the interior by an amount equal to the work that has to be done in bringing the molecule from the interior to the surface. A liquid thus has a surface potential energy proportional to its surface area. To increase the surface area requires work equal to the increase in surface energy as more molecules are brought from the interior to surface positions. The surface tends, like a stretched membrane, to assume a shape of minimum area, because this shape is the one having minimum potential energy (a condition for stable equilibrium). Thus, a water droplet in the air assumes a spherical form, since a sphere has the minimum surface area for a given volume. In fact, the surface of a liquid behaves in all respects like a membrane stretched with constant tension per unit width.

The surface tension, σ of a liquid is defined as the surface potential energy per unit area of surface, which is the same as the work done by unit increase in surface area.

The behaviour of a liquid surface can be demonstrated by means of a wire frame equipped with a sliding crosswire as shown in figure. The frame is dipped into a soap solution and a film is formed as indicated. Since the film tends to have minimum surface area, it tends to pull the crosswire to the left. In order to keep the crosswire at rest, it is necessary to exert a force F to the right.



If the film is to be stretched a distance ΔX to the right, work $= F\Delta X$ has to be done. This work results in an increase of surface area of film $= 2l\Delta X$ (there are two surfaces of the film). The surface tension σ is given by

$$\sigma = \frac{(F\Delta X)}{(2l\Delta X)} = \frac{F}{2l}, \text{ hence } F = 2l\sigma$$

F is proportional to l as σ is constant under given conditions.

The surface tension σ thus can be considered as force per unit width as well as work per unit area; it can be measured in J/m^2 or in N/m , which are actually identical units.

Pressure Difference in Case of a Spherical Droplet

The pressure within a spherical droplet is greater than the external pressure by $2\sigma/r$ or

$$P - P_0 = \frac{2\sigma}{r}$$

where P is the pressure inside the droplet and P_0 is the pressure outside. This pressure difference can become very large for small values of r . Surface tension

of water at 20°C is 0.0728 N/m while that of mercury is 0.465 N/m. For a soap bubble, $P - P_0 = 4\sigma/r$ as it has two surfaces.

Contact Angle and Capillarity

If σ_{SV} and σ_{SL} are the surface tensions of solid-vapour and solid-liquid films respectively, the curvature of the surface of a liquid near a solid wall depends upon the difference between σ_{SV} and σ_{SL} .

When σ_{SV} is greater than σ_{SL} , the line along which the three films meet is pulled upward as shown in Fig. (a). Angle β is called the contact angle.

When σ_{SV} is greater than σ_{SL} , the angle of contact is between zero and 90° [Fig. (a) (Example: glass and methyl iodide)].

When σ_{SV} is less than σ_{SL} , the line of intersection is pulled downward and the contact angle is between 90° and 180° [Fig. (b) (example: glass and mercury)].

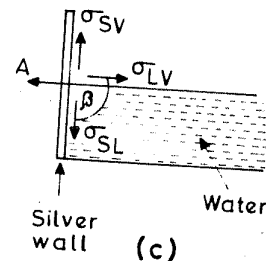
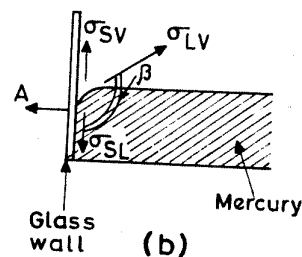
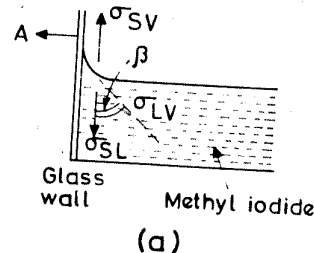
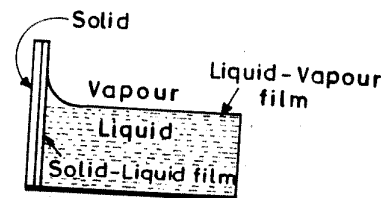
When σ_{SV} and σ_{SL} are very nearly equal, the contact angle is 90° [Fig. (c) (example: silver and water)].

Effect of Impurities on Surface Tension

Impurities and adulterants present in or added to a liquid may alter the contact angle considerably. Wetting agents or detergents change the contact angle from a large value, greater than 90°, to a value much smaller than 90°. On the other hand, waterproofing agents applied to cloth cause the contact angle of water in contact with the cloth to be larger than 90°.

Capillarity

In case of a wetting liquid for which $\beta < \pi/2$, there is a rise of liquid in a capillary (a small bore tube) as shown in Fig. (a). If the liquid does not wet the tube, $\beta > \pi/2$ and the liquid in the tube is depressed [Fig. (b)]. If R is the radius of the capillary, β is the angle of contact between glass and liquid, ρ



the density of the liquid, and σ the surface tension of the liquid, then the rise or the depression as the case may be, h , is given by

$$h = \frac{2\sigma \cos \beta}{\rho g R}$$

SI Units: The SI unit of pressure is the pascal (Pa); 1 Pa = 1 N/m² and 1 Atm = 1.013 × 10⁵ Pa; 1 bar = 10⁵ Pa

ILLUSTRATIONS

1. What is the gauge pressure (in atmospheres) inside a soap bubble with a radius of 1.5 cm and with $\sigma = 30 \times 10^{-3}$ N/m.

Solution

$$p - p_0 = \frac{4\sigma}{R} = \frac{4 \times 30 \times 10^{-3}}{0.015} = \frac{120}{15} = 8 \text{ Pa} = 8 \times 10^{-5} \text{ Atm.}$$

2. A capillary tube with an inside diameter of 0.1 mm can support a 10 cm column of a liquid that has a density of 0.93×10^3 kg/m³. The observed contact angle is 15°. What is the surface tension of the liquid?

Solution

$$\begin{aligned} h &= \frac{2\sigma \cos \beta}{\rho g R} \Rightarrow \sigma = \frac{h \rho g R}{2 \cos \beta} \\ &= \frac{0.1 \times 0.93 \times 10^3 \times 9.81 \times 0.05 \times 10^{-3}}{2 \times \cos 15^\circ} \\ &= \frac{0.0456}{1.93} = 0.0236 \\ &= 23.6 \times 10^{-3} \text{ N/m} \end{aligned}$$

3. Two concentric tubes are dipped vertically into an unknown liquid. The larger tube has an inside diameter of 6 mm and the smaller tube has an outside diameter of 4 mm. The liquid rises to a height of 2.0 cm in the inner tube and to a height of 1.5 cm in the region between the tubes. The density of the liquid is 0.92×10^3 kg/m³ and the contact angle is 20°. What is the surface tension of the liquid. What is the thickness of the wall of the smaller tube?

Solution

$$\begin{aligned} h &= \frac{2\sigma \cos \beta}{\rho g R} \Rightarrow \sigma = \frac{h \rho g R}{2 \cos \beta} \\ &= \frac{1.5 \times 10^{-2} \times 0.92 \times 10^3 \times 9.81 \times 1 \times 10^{-3}}{2 \times 0.94} \\ &= 72 \times 10^{-3} \text{ N/m} \end{aligned}$$

$$\begin{aligned} \text{Radius of the inner tube} &= \left(\frac{4 \text{ mm} - 2t}{2} \right) \quad (t \text{ is wall thickness in mm}) \\ &= (2 - t) \times 10^{-3} \text{ m} \end{aligned}$$

$$h = 2 \times 10^{-2} \text{ m}$$

$$\therefore R = \frac{2\sigma \cos \beta}{h\rho g}$$

$$\text{or } (2 - t) \times 10^{-3} = \frac{2 \times 72 \times 10^{-3} \times 0.94}{2 \times 10^{-2} \times 0.92 \times 10^3 \times 9.81}$$

$$t = 1.25 \text{ mm}$$

4. A capillary tube is dipped in water with its lower end 10 cm below the water surface. Water rises in the tube to a height of 4 cm above that of the surrounding liquid and the angle of contact is zero. What gauge pressure is required to blow a hemispherical bubble at the lower end of the tube?

Solution

$$p_{\text{inside}} = \rho gh + \frac{2\sigma}{r} \quad (h \text{ is the depth upto which capillary is immersed in water})$$

$$= (10^3 \times 10 \times 0.1) + \frac{2\sigma}{r}$$

We have another relation from the rise of water in a capillary
 $\pi r^2 \rho gh' = 2\pi r \sigma$ (h' is the height up to which water rises)

$$\text{or } \rho gh' = \frac{\sigma}{r}$$

$$\therefore p_{\text{inside}} = 10^3 + 2\rho gh' = 10^3 + 2 \times 10^3 \times 10 \times 0.04$$

$$= 10^3 + 0.8 \times 10^3$$

$$= 1.8 \times 10^3 \text{ N/m}^2$$

5. 1000 droplets of water each of 10^{-8} m diameter coalesce to form one large spherical drop. If the surface tension of water is 0.075 N/m, find the energy that will be released in this process.

Solution

$$\text{Radius of each droplet} = 5 \times 10^{-9} \text{ m}$$

If R is the radius of the large drop and r that of the droplet, we have

$$\frac{4}{3} \pi R^3 = 1000 \times \frac{4}{3} \pi r^3 \Rightarrow R^3 = 10^3 r^3$$

$$\therefore R = 10r$$

$$\text{or } R = 10 \times 5 \times 10^{-9} = 5 \times 10^{-8} \text{ m}$$

$$\text{Decrease in surface area} = 4\pi(1000r^2 - R^2)$$

$$= 4\pi[10^3 \times (5 \times 10^{-9})^2 - (5 \times 10^{-8})^2]$$

$$= 900\pi \times 10^{-16} \text{ m}^2$$

$$\therefore \text{Amount of energy liberated} = 900\pi \times 10^{-16} \times 0.075$$

$$= 212 \times 10^{-16} \text{ J} = 2.12 \times 10^{-14} \text{ J}$$

6. (a) Suppose the xylem tubes in the actively growing outer layer of a tree are uniform cylinders, and that the rising of the sap is due entirely to capillarity, with a contact angle of 45° and surface tension of 0.05 N/m. What is the maximum radius of the tubes for a tree 20 m tall (Assume the density of sap to be the same as that of water).

Solution

Given that $\beta = 45^\circ$, $h = 20$ m, $\sigma = 0.05$ N/m, $\rho = 1 \times 10^3$ kg/m³.
 Let us find the value of the radius.

$$R = \frac{2\sigma \cos \beta}{\rho gh} = \frac{2 \times 0.05 \times 1/\sqrt{2}}{1 \times 10^3 \times 9.81 \times 20}$$

$$= 3.6 \times 10^{-4} \text{ mm}$$

(b) In the above problem, let the tree be 50 m tall and the radius of the sap-carrying pipes be 2×10^{-4} mm. What is the minimum pressure that must be developed in the roots for it to be possible for the sap to reach the top of the tree, if the pressure at the top is one atmosphere?

$$p = \rho gh - \frac{2\sigma \cos \beta}{r}$$

$$= (10^3 \times 10 \times 50) - \frac{2 \times 0.05 \cos 45}{2 \times 10^{-4} \times 10^{-3}}$$

$$= 5 \times 10^5 - 3.6 \times 10^5 = 1.4 \times 10^5 \text{ N/m}^2$$

EXERCISES

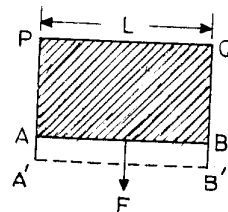
- If a greased needle is placed on a piece of blotting paper and the paper is lightly placed on a water surface, the blotting paper will soon sink to the bottom but the needle will remain floating on the surface. The reason is that
 - the weight of the needle is balanced by the upthrust of water
 - the needle is too light to sink in water
 - the needle is supported by the surface tension of water
 - the grease reduces the density of the needle-grease system.
- When a camel-hair paintbrush is immersed in water, its hair spreads out to give a bushy appearance but the moment it is withdrawn, they all come close together in a more or less compact mass. This behaviour is because of
 - force of attraction between the hair
 - surface tension in the membrane that is formed
 - electrostatic force between the hair and water molecules
 - viscosity of water.
- Small pieces of camphor are joined together to form the shape of a scorpion.

When this is placed on the surface of water, it starts moving rapidly. This movement is on account of

- reduction of surface tension due to dissolution of camphor in water as compared to surrounding uncontaminated water
 - increase of surface tension due to dissolution of camphor in water as compared to surrounding uncontaminated water
 - unbalanced electrostatic forces on the molecules of camphor
 - camphor evaporation giving it a jet action.
4. Find the correct statement among the following.
- potential energy of the molecules on the surface is smaller than those inside the liquid.
 - the entire kinetic energy of molecules when within the liquid is converted into potential energy when they come to the surface.
 - the potential energy of molecules in the surface-film is greater than that of those inside the liquid.
 - the total energy per unit area of the surface-film is called its surface energy.

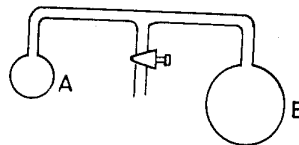
5. A soap film is formed on a wire frame with horizontal wire AB free to move up and down. The wire AB is pulled down to a position A'B' by a distance x . If σ is the surface tension of the soap solution, the work done is

- $Lx\sigma$
- $2Lx\sigma$
- σx
- $2\sigma x$

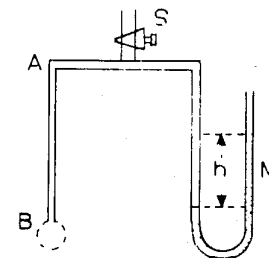


6. When the film gets stretched in the above problem,
- it gets cooled and therefore takes heat from the atmosphere
 - it gets heated up and therefore delivers heat to the atmosphere
 - it remains at the same temperature but mechanical work is done in creating the new surface
 - the potential energy per unit area is equal to the sum of the surface energy and the heat energy absorbed from the atmosphere.

7. Soap bubbles of unequal size are formed as shown in the figure. They are then put in communication with each other.

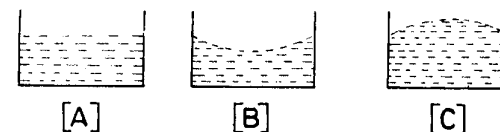


- air will pass from B to A till they are of equal size.
 - the bubble A will go on increasing in volume and B will shrink to a hemisphere.
 - since the surface tension for soap solution is the same in A and B, their sizes will not change.
 - air will pass from A to B so that A goes on shrinking and B continues to swell until A is reduced to a hemisphere.
8. A tube of fine bore AB is connected to a manometer M as shown. The stop cock S controls the flow of air. AB is dipped into a liquid whose surface tension is σ . On opening the stop cock for a while, a bubble is formed at B and the manometer level is recorded, showing a difference h in the levels in the two arms. If ρ be the density of manometer liquid and r the radius of curvature of the bubble, then the surface tension σ of the liquid is given by



- $rhrg$
- $2rhgr$
- $4rhrg$
- $\frac{rh\rho g}{4}$

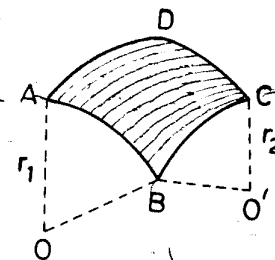
9. Three surfaces of liquids are shown here. Find the correct statement:



- the surface tension of liquid A is infinite and such a surface is not possible practically.
 - in case of liquid B, the resultant force on a molecule on the surface would be downward, and the surface is concave.
 - in case of liquid C, the resultant force due to surface tension is directed downwards
 - the surfaces of all three liquids A, B and C are in equilibrium and so the net force due to surface tension is the same.
10. The work done in blowing a soap bubble of radius r and surface tension σ is
- $2\pi r^2\sigma$
 - $4\pi r^2\sigma$
 - $16\pi r^2\sigma$
 - $8\pi r^2\sigma$

11. A curved surface ABCD of a liquid of surface tension σ is shown here. The radii of the two curvatures are r_1 and r_2 respectively. The total difference of pressure on the two sides of the surface is

- $\sigma(r_1 - r_2)$
- $4\pi[r_1^2 - r_2^2]$
- $\sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$
- $\sigma\pi(r_1^2 + r_2^2)$



12. In the above problem where AB and BC both are concave, if BC becomes convex, the total difference of pressure on the two sides of the surface will be
- $\sigma(r_1 - r_2)$
 - $\sigma 4\pi[r_1^2 - r_2^2]$
 - $\sigma\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$
 - $\sigma\pi(r_1^2 - r_2^2)$
13. When two glass plates are placed one over the other, there is no difficulty in

separating them but if a drop of liquid, say water, is placed between them and squeezed into a thin layer, it requires a considerable force to pull them apart. The reason for this is that

- (a) the pressure inside the film is less than the outside atmospheric pressure
 - (b) the thin layer of water produces a large resistive force on account of viscosity
 - (c) there is internal friction between the microlayers of the film
 - (d) electrostatic charges of opposite nature are produced between the plates.
- 14.* A sphere of water of radius 1 mm is sprayed into a million droplets of equal size. The work needed to do so is [$\sigma_w = 72 \times 10^{-3}$ N/m]
- (a) 1.24×10^{-6} J
 - (b) 2.48×10^{-6} J
 - (c) 89.5×10^{-6} J
 - (d) 8.9×10^{-6} J
- 15.* A glass plate of length 0.1 m, breadth 15×10^{-3} m and thickness 2×10^{-3} m weighs 8×10^{-3} kg in air. It is held vertically with its longer side horizontal and its lower half immersed in water. If the surface tension of water is 72×10^{-3} N/m, the apparent weight of the plate will be
- (a) 97.4×10^{-3} N
 - (b) 36.1×10^{-3} N
 - (c) 72.2×10^{-3} N
 - (d) 79.4×10^{-3} N
16. A glass plate 0.1 m in length, 0.01 m in width and 0.001 m in thickness is held very close to the surface of water. If the surface tension of water is 72×10^{-3} N/m, the force due to surface tension on the plate will be
- (a) 14.54×10^{-4} N
 - (b) 15.44×10^{-3} N
 - (c) 14.54×10^{-3} N
 - (d) 15.44×10^{-4} N
17. The pressure of air in a soap bubble of diameter 0.008 m is 8 mm of water above atmospheric pressure. The surface tension of the soap solution is
- (a) 60×10^{-3} N/m
 - (b) 80×10^{-3} N/m
 - (c) 40×10^{-3} N/m
 - (d) 70×10^{-3} N/m
18. The surface tension of rain water is 72×10^{-3} N/m. If the diameter of a drop of rain is 2×10^{-4} m, the difference between the pressures inside and outside the drop is
- (a) 72 N/m²
 - (b) 144.0 N/m²
 - (c) 14.0 N/m²
 - (d) 1440 N/m²
- 19.* An oil bath (density of oil = 0.85×10^3 kg/m³) has a spherical cavity of diameter 26×10^{-6} m at a depth of 0.2 m. If the surface tension of oil is 26×10^{-3} N/m and the pressure of air over the surface of oil is 76 cm of mercury, the pressure inside the cavity will be
- (a) 1.03×10^5 N/m²
 - (b) 1.07×10^5 N/m²
 - (c) 1.17×10^5 N/m²
 - (d) 3.07×10^5 N/m²
20. A capillary tube of bore 0.5 mm stands vertically in a wide vessel containing a liquid whose surface tension is 30×10^{-3} N/m. The liquid wets the tube and has a specific gravity of 0.8. The rise of liquid in the tube is (take $\beta = 0$)
- (a) 0.03 m
 - (b) 0.30 m
 - (c) 0.06 m
 - (d) 0.001 m
21. Water rises to a height of 0.05 m in a certain capillary tube. In the same tube, the level of mercury surface is depressed by 0.015 m. If the angles of contact of water and mercury are 0° and 130° respectively, the ratio of the surface tension of mercury to that of water is
- (a) 2.3
 - (b) 3.2
 - (c) 4.6
 - (d) 6.4
22. The work done in blowing a soap bubble of radius 0.1 m (the surface tension of the soap solution is 30×10^{-3} N/m) is

- (a) 7.5×10^{-4} J
 - (b) 15×10^{-4} J
 - (c) 75×10^{-4} J
 - (d) 75×10^{-3} J
23. The pressure inside a soap bubble of radius 0.01 m balances an oil column of height 1.4×10^{-3} m. If the specific gravity of oil is 0.80, the surface tension of the soap solution is
- (a) 28×10^{-3} N/m
 - (b) 58×10^{-3} N/m
 - (c) 14×10^{-3} N/m
 - (d) 21×10^{-3} N/m
24. A mercury barometer tube is 0.003 m in diameter. If the surface tension of mercury is 465×10^{-3} N/m and the contact angle is 128° , the error introduced in the reading due to surface tension is
- (a) -0.0014 m
 - (b) 0.0021 m
 - (c) -0.0028 m
 - (d) 0.0012 m
25. A needle is 5 cm long. Assuming that it is not wetted in water, the maximum mass of the needle that could be so as to still float on water, will be ($\sigma = 74 \times 10^{-3}$ N/m)
- (a) 7.3×10^{-4} kg
 - (b) 3.7×10^{-3} kg
 - (c) 3.7×10^{-4} kg
 - (d) 7.3×10^{-3} kg
26. The work that has to be done to break a mercury droplet of diameter 2 mm into eight droplets of equal size is (σ for mercury = 470×10^{-3} N/m)
- (a) 60×10^{-5} J
 - (b) 120×10^{-7} J
 - (c) 30×10^{-7} J
 - (d) 60×10^{-7} J
27. A drop of water 4 mm in diameter is formed by combining one million droplets of equal size. If surface tension of water is 0.07 N/m, the change in energy that takes place in the process is
- (a) 3.5×10^{-4} J
 - (b) 4.88×10^{-4} J
 - (c) 3.5×10^{-5} J
 - (d) 7.0×10^{-5} J
28. A glass capillary tube of radius 1 mm dips into a liquid of density 0.8×10^3 kg/m³. The liquid rises to a height of 3 cm and the angle of contact is 15° . The surface tension of the liquid is
- (a) 1.2×10^{-4} N/m
 - (b) 0.12 N/m
 - (c) 1.2×10^{-3} N/m
 - (d) 0.24 N/m
29. The pressure inside a spherical drop of water is equal to 2 atmospheres. The pressure outside is 1 atmosphere. If the surface tension of water is 74×10^{-3} N/m, the diameter of the bubble is
- (a) 0.003 mm
 - (b) 0.06 mm
 - (c) 0.5 mm
 - (d) 0.05 mm
30. A square loop of wire 6 cm on a side is suspended from a thread by fastening the thread at the middle of one side. The loop is then lowered into water. It is subsequently raised slowly so as to form a film of water. If the surface tension of water is 75×10^{-3} N/m, the upward force required to break the film is
- (a) 1.8×10^{-2} N
 - (b) 9×10^{-3} N
 - (c) 9×10^{-4} N
 - (d) 6×10^{-3} N
31. A soap bubble 2 cm in diameter is formed on a pipe attached to a water manometer. The differential pressure is observed as 1.20 mm of water. The surface tension of the soap solution is
- (a) 0.030 N/m
 - (b) 0.060 N/m
 - (c) 0.015 N/m
 - (d) 0.070 N/m
32. The pressure inside a small air bubble of radius 10^{-4} m situated just below the surface of water will be (given $\sigma = 0.072$ N/m and atmospheric pressure = 1×10^5 N/m²)

- (a) $5.024 \times 10^5 \text{ N/m}^2$ (b) $1.0144 \times 10^5 \text{ N/m}^2$
 (c) $3.010 \times 10^5 \text{ N/m}^2$ (d) $1 \times 10^4 \text{ N/m}^2$
33. A thin metal ring of internal radius 8 cm and external radius 9 cm is supported horizontally from the pan of a balance so that it comes in contact with water in a glass vessel. It is found that an extra weight of 7.48 g is required to pull the ring out of water. The surface tension of water is
 (a) $80 \times 10^{-3} \text{ N/m}$ (b) $75 \times 10^{-3} \text{ N/m}$
 (c) $65 \times 10^{-3} \text{ N/m}$ (d) $70 \times 10^{-3} \text{ N/m}$
34. Water rises to a height of 16.3 cm in a capillary of height 18 cm above the water level. If the capillary is cut at a height of 10 cm ($\sigma = 70 \times 10^{-3} \text{ N/m}$),
 (a) water will come out as a fountain from the capillary tube
 (b) water will stay at height of 10 cm in the capillary tube
 (c) water will flow down slowly through the sides
 (d) water will stay at a height of 8.4 cm.
35. A drop of water is placed between two parallel glass plates which are then pressed together until the water film is 0.01 cm thick. If the area in contact with either plate is 10 sq. cm, the force due to surface tension drawing the plates together is ($\sigma = 70 \times 10^{-3} \text{ N/m}$)
 (a) 140 N (b) 70 N
 (c) 280 N (d) 105 N
36. Air is pushed into a soap bubble of radius R , until this radius is doubled. If the surface tension of the soap solution is σ , the work done in the process is
 (a) $8 \pi R^2 \sigma$ (b) $4 \pi R^2 \sigma$
 (c) $16 \pi R^2 \sigma$ (d) $24 \pi R^2 \sigma$
37. Two capillary tubes A and B dip into a liquid which rises in them by 2.8 cm and 4.2 cm respectively. If the diameter of A is 0.12 cm, the diameter of B will be
 (a) 0.08 cm (b) 0.04 cm
 (c) 0.02 cm (d) 0.03 cm
38. A spherical soap bubble of radius 1.0 cm is formed inside another of radius 2 cm. If a single soap bubble is formed which maintains the same pressure difference as inside the smaller and outside the larger bubble, the radius of this bubble is
 (a) 0.005 m (b) 0.05 m
 (c) 0.0067 m (d) 0.067 m
39. The difference of pressure between the inside and outside of a spherical drop of water of radius 1 mm is (σ for water is 0.073 N/m)
 (a) 14.6 N/m^2 (b) 219 N/m^2
 (c) 292 N/m^2 (d) 146 N/m^2
40. A small hollow sphere which has a small hole in it is immersed in water to a depth of 0.5 m before any drop penetrates into it. If σ for water is 0.073 N/m, the radius of the hole is
 (a) 0.03 mm (b) 0.06 mm
 (c) 0.09 mm (d) 0.15 mm

Answers

1. Specific Gravity and Hydrostatic Pressure

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (b) | 4. (d) | 5. (a) |
| 6. (d) | 7. (b) | 8. (a) | 9. (c) | 10. (d) |
| 11. (b) | 12. (c) | 13. (d) | 14. (c) | 15. (c) |
| 16. (a) | 17. (d) | 18. (a) | 19. (a) | 20. (b) |
| 21. (c) | 22. (a) | 23. (b) | 24. (a) | 25. (d) |
| 26. (b) | 27. (a) | 28. (a) | 29. (a) | 30. (b) |
| 31. (d) | 32. (b) | 33. (c) | 34. (d) | 35. (a) |
| 36. (a) | 37. (c) | 38. (d) | 39. (d) | 40. (a) |

2. Surface Tension

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (b) | 3. (a) | 4. (c) | 5. (b) |
| 6. (a) | 7. (d) | 8. (d) | 9. (c) | 10. (d) |
| 11. (c) | 12. (c) | 13. (a) | 14. (b) | 15. (d) |
| 16. (c) | 17. (b) | 18. (d) | 19. (b) | 20. (a) |
| 21. (d) | 22. (c) | 23. (a) | 24. (c) | 25. (c) |
| 26. (d) | 27. (a) | 28. (b) | 29. (a) | 30. (b) |
| 31. (a) | 32. (b) | 33. (d) | 34. (b) | 35. (a) |
| 36. (d) | 37. (a) | 38. (c) | 39. (d) | 40. (a) |

Note: Problems marked with a star should take about five minutes. All the rest should be solved in about three minutes each.

10

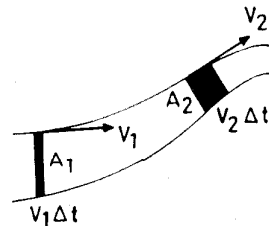
Fluid Dynamics

The Equation of Continuity

Suppose an incompressible fluid fills a pipe and has a steady flow through it. A_1 is the area of cross-section of the pipe at one point and A_2 that at another. Since the flow through A_1 must equal the flow through A_2 , one has

$$Q = A_1 V_1 = A_2 V_2 = \text{constant}$$

Here Q is the flow discharge and V_1 and V_2 are the average fluid velocities over A_1 and A_2 respectively.



Bernoulli's Equation

Consider two different points along the stream path. Let one point be at a height h_1 and let V_1 , ρ_1 and p_1 be the fluid velocity, density and pressure at that point. Similarly, at another point at a different height h_2 , let the velocity, density and pressure be V_2 , ρ_2 and p_2 . If the fluid is incompressible and has negligible viscosity, then

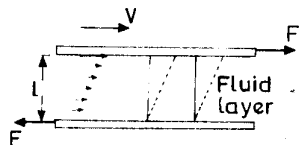
$$p_1 + \frac{1}{2} \rho V_1^2 + h_1 \rho g = p_2 + \frac{1}{2} \rho V_2^2 + h_2 \rho g$$

($\rho_1 = \rho_2 = \rho$ as it is a homogeneous fluid of density ρ .)

The *shear rate* of a fluid is the rate at which shear strain within the fluid is changing.

The viscosity (η) of a fluid is a measure of how large a shear stress is required to produce unit shear rate.

Let us take a plate (bottom) which is stationary while the other plate (top) moves with a constant speed V . It is found that the fluid in contact with each surface has the same speed as that of the surface. As a consequence, different layers of fluid slide over one another.



$$\text{Rate of change of shear strain} = \frac{V}{l}$$

The coefficient of viscosity of the fluid, η , is defined as the ratio of the shear stress, F/A , to the rate of change of shear strain

$$\eta = \frac{\text{shear stress}}{\text{rate of change of shear strain}} = \frac{F/A}{V/l}$$

$$\text{or } F = \eta A \frac{V}{l}$$

The units of η are Ns/m^2 or kg/m.s , called the poiseuille (Pl). The other unit is the poise (P) where $1 \text{ P} = 0.1 \text{ Pl}$.

Poiseuille's Law

The fluid flow through a cylindrical pipe of length L and cross-sectional radius r is given by

$$Q = \frac{\pi r^4 (p_1 - p_2)}{8 \eta L}$$

where $p_1 - p_2$ is the pressure difference between the two ends of the pipe. Here Q is the total volume of flow per unit time. We can make the following inferences.

1. The volume rate of flow is inversely proportional to viscosity.
2. It is proportional to the pressure gradient along the pipe.
3. It varies as the fourth power of the radius of the cross-section of the pipe.

Stokes's Law

The viscous force F is given by the relation

$$F = 6\pi\eta rV$$

where r is the radius of the sphere around which the fluid flows, V is the velocity of the sphere relative to the fluid and η is the viscosity.

Terminal Velocity

A sphere falling in a viscous fluid reaches a terminal velocity V_T at which the viscous retarding force plus the buoyant force equals the weight of the sphere. If ρ and ρ' are the densities of the sphere and fluid respectively then

$$V_T = \frac{2}{9} \frac{r^2 g}{\eta} (\rho - \rho')$$

Reynolds Number

Combination of four factors determines whether the flow of a fluid through a tube or pipe is laminar or turbulent. This combination is known as the Reynolds Number N_R and is given by

$$N_R = \frac{\rho V D}{\eta}$$

where ρ is the density of the fluid, V the average forward velocity, η the viscosity and D the diameter of the tube.

Reynolds number is a dimensionless quantity and has the same numerical value in any consistent system of units. It has been found that when N_R is less than about 2000, the flow is laminar whereas above about 3000, the flow is turbulent.

ILLUSTRATIONS

1. A garden hose having an internal diameter of 2 cm is connected to a lawn sprinkler that consists of an enclosure with 24 holes, each of diameter 0.15 cm. If water in the hose has a speed of 1 m/s, at what speed does it leave the sprinkler's holes?

Solution

$$\text{Area of the hose } (A_1) = \pi R^2 = \left(\frac{\pi}{4}\right) D^2 = \frac{\pi}{4} \times \left(\frac{2}{100}\right)^2 \text{ m}^2$$

$$\begin{aligned} \text{Total area of the holes } (A_2) &= 24 \times \pi r^2 \\ &= 24 \times \frac{\pi}{4} \times d^2 = 24 \times \frac{\pi}{4} \times \left(\frac{0.15}{100}\right)^2 \text{ m}^2 \end{aligned}$$

Speed of water in hose = 1 m/s (say V_1)

Speed of water through sprinkler's holes = V_2

Since the flow rate is constant,

$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi}{4} \left(\frac{2}{100}\right)^2 \times 1 = 24 \times \frac{\pi}{4} \times \left(\frac{0.15}{100}\right)^2 \times V_2$$

$$V_2 = 7.4 \text{ m/s}$$

2. In a horizontal pipeline of constant cross-sectional area, the pressure difference between two points 10 m apart is 3×10^4 Pa. What is the energy loss per cubic metre of oil per unit distance.

Solution

$\Delta p = 3 \times 10^4$ Pa, and let A be the area of cross-section of the pipe

$$\begin{aligned} \text{Work done against viscous force} &= W = (\Delta p) \times (\text{volume}) \\ &= \Delta p \times AL \\ &= 3 \times 10^4 \times A \times L \\ &= (3 \times 10^4 \times 10A) \text{ J} \end{aligned}$$

$$(A \times L = \text{Volume of oil} = 10 \times A \text{ m}^3)$$

(Energy loss)/(cubic metre of oil per unit distance)

$$= \frac{3 \times 10^4 \times 10 A J}{10A \times 10} = 3 \times 10^3 \text{ J/m}^3 \text{ per m}$$

3. How much water will flow in one minute through 25 cm of a capillary tube of internal diameter 2 mm if the pressure difference across the ends of the tube is 5 cm of mercury, given that η for water is 0.80 centipoise.

Solution

We shall make use of Poiseuille's law with

$$\begin{aligned} p_1 - p_2 &= \rho gh = (13.6 \times 10^3 \text{ kg/m}^3) \times (9.8 \text{ m/s}^2) \times (0.05 \text{ m}) \\ &= 6660 \text{ N/m}^2 \end{aligned}$$

$$\eta = 0.80 \text{ centipoise} = 0.80 \times 10^{-3} \text{ kg/ms} = 8 \times 10^{-4} \text{ kg/ms}$$

$$Q = \frac{\pi r^4 (p_1 - p_2)}{8 \eta L} = \frac{\pi \times (1 \times 10^{-3})^4 (6660)}{8 \times 8 \times 10^{-4} \times 0.25} \text{ m}^3/\text{s}$$

$$= \frac{\pi \times 666 \times 10^{-5}}{64 \times 25} = 1.3 \times 10^{-5} \text{ m}^3/\text{s}$$

$$= 13 \text{ mL/s}$$

In one minute, the quantity of water that would flow out of the tube is $13 \times 60 = 780 \text{ mL}$.

4. The bottom of a cylindrical vessel has a circular hole of diameter d while the diameter of the vessel is D . Find the relation between the velocity V with which the water level in the vessel drops and the height h of this level.

Solution

Cross-sectional area of vessel = A_1

Velocity with which water level drops = V_1

Cross-sectional area of hole = A_2

Velocity of water that flows out of the hole = V_2

According to Bernoulli's theorem,

$$\frac{\rho V_1^2}{2} + \rho gh = \frac{\rho V_2^2}{2}$$

$$\text{or } V_1^2 + 2gh = V_2^2 \quad (1)$$

From the equation of continuity,

$$V_1 A_1 = V_2 A_2 \quad \text{or} \quad V_2 = \frac{V_1 A_1}{A_2} \quad (2)$$

$$V_1 = \frac{A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{d^2 \sqrt{2gh}}{\sqrt{D^4 - d^4}} \quad (3)$$

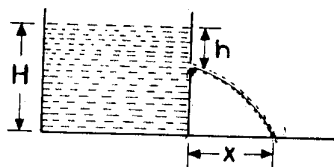
If $d^4 \ll D^4$, then

$$V_1 = \frac{d^2}{D^2} \sqrt{2gh} \quad (4)$$

5. A tank is filled with water to a height H . A hole is punched in one of the walls at a depth h below the water surface.

(a) Find the distance x from the foot of the wall at which the stream strikes the floor.

(b) Could a hole be punched at another depth so that this second stream has the same range? Find the position of this hole.



Solution

The speed of efflux in the horizontal direction is $V = \sqrt{2gh}$ and it will cover a distance x in time t , hence $x = \sqrt{2gh} \cdot t$.

The vertical height $(H - h)$ is traversed in the same time t since the initial vertical component of velocity is zero.

$$\therefore (H - h) = 0 + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2(H - h)}{g}}$$

$$\therefore x = \sqrt{2gh} \cdot \sqrt{\frac{2(H - h)}{g}} = 2\sqrt{h(H - h)}$$

(b) Let the second hole be punched at some other depth, say h' .

$$\therefore V = \sqrt{2gh'} \quad \text{and} \quad x = 2\sqrt{h'(H - h')}$$

$$\therefore 2\sqrt{h'(H - h')} = 2\sqrt{h(H - h)}$$

$$\text{or} \quad h'(H - h') = h(H - h)$$

$$h'H - h'^2 = hH - h^2$$

$$h^2 - h'^2 + H(h' - h) = 0$$

$$\text{or} \quad (h' - h)(H - h' - h) = 0$$

There are two solutions:

(i) $h' = h$ (already obtained)

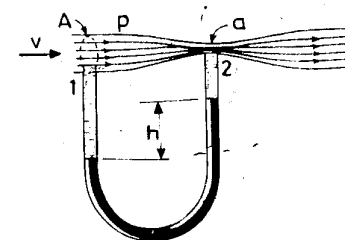
(ii) $h' = H - h$

This means that another hole at a height h above the bottom will give the same range as a hole at a height h from the top.

6. An instrument called the venturimeter is employed to measure the flow speed of a liquid. This is shown in the figure here. Show that the speed of flow at the entrance is

$$V = a \sqrt{\frac{2(\rho' - \rho)gh}{\rho(A^2 - a^2)}}$$

where ρ' is the density of mercury in the manometer and ρ is the density of the liquid. A and a are the areas of cross-section of the pipe and the throat.



Solution

Apply Bernoulli's equation

$$p + \frac{1}{2}\rho V^2 + \rho gy = \text{constant}$$

to two points (1) and (2) shown in the figure.

As the centres of cross-sectional areas A and a are at the same horizontal level, the potential energy term ρgy is ignored.

$$\therefore \Delta p = p_1 - p_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

The speed at (1) is $V_1 = V$, and so we have

$$VA = V_2a$$

$$\Delta p = \frac{1}{2}\rho \left(\frac{V^2 A^2}{a^2} - V^2 \right) = \frac{1}{2}\frac{\rho V^2}{a^2}(A^2 - a^2)$$

But the pressure difference shown in the manometer is

$$\Delta p = hg\rho' - hg\rho = hg(\rho' - \rho)$$

$$\therefore hg(\rho' - \rho) = \frac{1}{2}\frac{\rho V^2}{a^2}(A^2 - a^2)$$

$$\text{or} \quad V = a \sqrt{\frac{2(\rho' - \rho)gh}{\rho(A^2 - a^2)}}$$

7. A venturimeter has a pipe diameter of 28 cm and a throat diameter of 14 cm. If the water pressure in the pipe is 55000 Pa and that in the throat is 40000 Pa, determine the rate of flow of water in cubic metres per second.

Solution

$$\begin{aligned} \Delta p &= \frac{1}{2}\rho V^2 \left(\frac{A^2}{a^2} - 1 \right) = \frac{1}{2}\rho V^2 \left(\frac{(\pi/4)d_1^2}{(\pi/4)d_2^2} - 1 \right) \\ &= \frac{1}{2}\rho V^2 \left(\frac{d_1^2}{d_2^2} - 1 \right) = \frac{1}{2}\rho V^2 (4 - 1) \end{aligned}$$

$$\text{or } 15000 = \frac{3}{2} \rho V^2$$

$$15000 = 1 \times 10^3 \times \frac{3}{2} \times V^2$$

$$\text{or } V = \sqrt{10} \text{ m/s}$$

$$\begin{aligned} \text{Rate of flow of water} &= VA = \sqrt{10} \times (\pi \times (0.14)^2) \\ (\text{volume flux}) &= 0.194 \text{ m}^3/\text{s} \end{aligned}$$

8. What will be the power output of the heart if, in each heartbeat, it pumps 80 mL of blood at an average pressure of 100 mm of Hg. (Assume 60 heartbeats per minute.)

Solution

$$\text{Work done by the heart} = p dV$$

$$\begin{aligned} dV &= (60) \times (80 \times 10^{-6} \text{ m}^3) \quad \text{and} \quad p = (100 \text{ mm Hg}) \times \frac{1.01 \times 10^5 \text{ Pa}}{760 \text{ mm Hg}} \\ &= 1.33 \times 10^4 \text{ Pa} \end{aligned}$$

$$\begin{aligned} \text{Power} &= \frac{\text{work done}}{\text{time}} = \frac{1.33 \times 10^4 \text{ Pa} \times 60 \times 80 \times 10^{-6} \text{ m}^3}{60 \text{ s}} \\ &= 1.06 \text{ W} \end{aligned}$$

9. Find the volume of water that will drain out per minute from an open-top tank through an opening 2 cm in diameter that is 4 m below the water level in the tank.

Solution

Let the top level be represented by t and the hole level by h . Then $p_t = p_h$ and $H_t = 4 \text{ m}$ and $H_h = 0$.

Using Bernoulli's equation,

$$\begin{aligned} p_t + \frac{1}{2} \rho V_t^2 + H_t \rho g &= p_h + \frac{1}{2} \rho V_h^2 + H_h \rho g \\ \frac{1}{2} \rho V_t^2 + H_t \rho g &= \frac{1}{2} \rho V_h^2 + H_h \rho g \end{aligned}$$

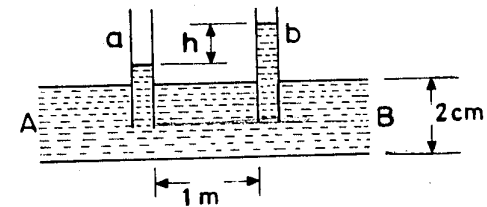
If the tank is large, V_t can be approximated to zero.

$$\begin{aligned} \therefore V_h &= \sqrt{2g(H_t - H_h)} = \sqrt{2 \times 9.81 \times (4 - 0)} \\ &= 8.86 \text{ m/s} \end{aligned}$$

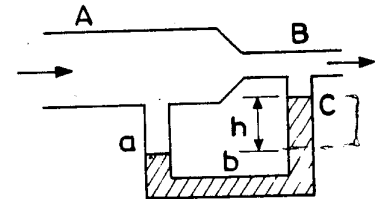
$$\begin{aligned} \text{Flow } Q &= V_h A_h = (8.86)(\pi)(0.01)^2 = 0.00278 \text{ m}^3/\text{s} \\ &= 0.167 \text{ m}^3/\text{minute} \end{aligned}$$

EXERCISES

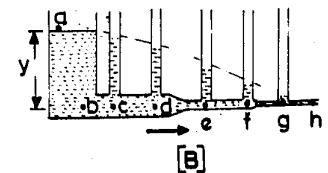
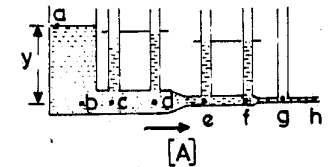
- The pressure that will be built up by a compressor in a paint gun if a stream of liquid paint flows out of it with a velocity of 25 m/s (density of paint is 0.8 g/cc) is
 (a) $2.5 \times 10^3 \text{ N/m}^2$ (b) $2.5 \times 10^2 \text{ N/m}^2$
 (c) $5 \times 10^5 \text{ N/m}^2$ (d) $2.5 \times 10^5 \text{ N/m}^2$
- A liquid flows along a horizontal pipe AB. The difference between the levels of the liquid in tubes a and b is 10 cm. The diameters of tubes a and b are the same. The velocity of the liquid flowing along the pipe AB is (viscosity of liquid = 1.0 N s/m^2 and its density = $1.1 \times 10^3 \text{ kg/m}^3$)



- 0.7 cm/s
 - 4.1 cm/s
 - 1.4 cm/s
 - 2.8 cm/s
- Air is blown through a pipe AB at a rate of 15 litres per minute. The cross-sectional area of the broad portion of pipe AB is 2 cm^2 and of the narrow portion and the tube abc is 0.5 cm^2 . The difference in the levels (h) of the water poured into tube abc is (density of air = 1.32 kg/m^3).
 (a) 16 mm
 (b) 1.6 mm
 (c) 10 mm
 (d) 3.2 mm

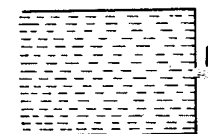


- Figures a and b here show two identical liquid reservoirs connected to a broad pipe bcd and to a narrow pipe ef and then to a very narrow pipe gh . The two reservoirs are filled with two different liquids up to the same height. The rise of liquid in five identical vertical tubes in Fig. (a) and Fig. (b) is shown. The following conclusions can be drawn about the two liquids.



- The surface tension of liquid in (a) is more than the liquid in (b).
- The density of the liquid in (a) is less than that of the liquid in (b).
- The liquid in (a) has higher viscosity than the liquid in (b).
- The liquid in (a) has no viscosity while (b) contains a viscous liquid.

5. A sphere is falling in a viscous fluid. Its velocity will
 (a) increase as it goes down and will come to a stop after attaining the maximum velocity
 (b) decrease as it goes down as viscous forces cause retardation
 (c) attain a velocity after which it becomes constant
 (d) remain constant throughout.
6. When a sphere falling in a viscous fluid attains the terminal velocity, then
 (a) the net force on the sphere acts downwards
 (b) the viscous retarding force plus the buoyant force equals the weight of the sphere
 (c) the viscous retarding force plus the weight of the sphere equals the buoyant force
 (d) the buoyant force is zero and the viscous force equals the weight of the sphere.
7. Two spheres of the same material but of radii 0.01 m and 0.02 m are dropped one by one in the same viscous fluid. Their terminal velocities respectively will be
 (a) the same for the two spheres
 (b) in the ratio of 1 to 2
 (c) in the ratio of 1 to 4
 (d) in the ratio of 4 to 1.
8. The correct statement among the following is:
 (a) Millikan used the terminal velocity principle in the determination of the radii of very small electrically charged oil drops.
 (b) If the spheres are not charged, then measuring the terminal velocity will not help in the determination of their radii.
 (c) If the fluid is nonviscous, the terminal velocity will be very small.
 (d) The terminal velocity of a sphere depends only on the density of the fluid and not on the density of the material of the sphere.
9. If the Reynolds number N_R for water at 20°C in CGS units is 1000, its value in SI units shall be
 (a) 1000×10^5 (b) 10^5
 (c) 1000 (d) 1000×10^3
10. A combination of four factors— ρ , the density of the fluid, V , the average forward velocity, D , the diameter of the tube or pipe, and η , the viscosity of fluid—in the following relationship decide whether the flow of a fluid through a tube or pipe is laminar or turbulent.
- (a) $\frac{VD}{\eta\rho}$ (b) $\eta \frac{\rho V}{D}$
 (c) $\eta \frac{D}{\rho V}$ (d) $\frac{\rho VD}{\eta}$
11. Air is flowing in a tube of diameter 1 cm with an average velocity of 30 cm/s. If the density of air is 0.0013 g/cc and its viscosity is 181×10^{-6} dynes.s.cm⁻², the Reynolds number will be
 (a) 215 (b) 1000
 (c) 2150 (d) 100
12. A capillary tube has a radius of 0.5 mm and a length of 30 cm. A pressure difference of 6500 N/m² is maintained between its two ends and water ($\eta = 8 \times 10^{-4}$ kg/ms) is allowed to flow through it. The quantity of water that would flow out in one minute is
 (a) 0.66 mL (b) 80 mL
 (c) 40 mL (d) 200 mL
13. An artery in a certain person has been widened to twice its original diameter. If the pressure differential across the artery is maintained constant, the blood flow through the artery will be increased by a factor of
 (a) 4 (b) 16
 (c) 64 (d) 8
14. A heart pumps out 60 mL of blood at an average pressure of 80 mm of Hg and makes 70 heartbeats per minute. Its pumping power is
 (a) 0.74 W (b) 0.47 W
 (c) 1.47 W (d) 1.74 W
15. A large tank filled with water has a circular hole at a depth of 3 m from the top level of water. If the rate of flow of water is 14.4×10^{-4} m³/min, the area of cross-section of the hole is
 (a) 6 mm² (b) 3 mm²
 (c) 0.3 mm² (d) 0.6 mm²
16. A water tank springs a leak at position P as shown in the figure. The water pressure at P is 5×10^5 N/m². If the density of water is 1000 kg/m³, the velocity with which it gushes out is
 (a) 16 m/s (b) 320 m/s
 (c) 32 m/s (d) 3.2 m/s
17. 4 m³ of water is to be pumped to a height of 20 m and forced into a reservoir at a pressure of 2×10^5 N/m². The work done by the motor is
 (a) 8×10^5 J (b) 16×10^5 J
 (c) 1×10^5 J (d) 32×10^5 J
18. The velocity of an engine oil in a 5-cm internal diameter pipe is 0.63 m/s. The velocity in a 3-cm internal diameter pipe that connects the first one is
 (a) 1.75 m/s (b) 1.50 m/s
 (c) 3.0 m/s (d) 9.0 m/s
- 19.* Water flows in a 20-cm-long tube whose internal diameter is 4 mm. The pressure difference across the tube is 5000 N/m². If the viscosity of water is 0.80×10^{-3} kg/m.s, the time required by 1 litre of water to flow through the pipe is
 (a) 1.5 s (b) 2.50 s
 (c) 10.2 s (d) 5.1 s
- 20.* A molten polyester flows out of a tube 10 cm in length at a rate of 15 cc/min when the pressure difference between the ends of the tube is 20 cm of mercury. The internal diameter of the tube is 1.20 mm. The viscosity of the molten polyester is
 (a) 0.094 kg/m.s (b) 0.025 kg/m.s
 (c) 0.054 kg/m.s (d) 0.0054 kg/m.s
- 21.* The needle of a syringe is 4 cm in length and has an internal diameter of 0.40 mm. It is used to draw blood ($\eta = 4 \times 10^{-3}$ kg/m.s) when the pressure differential across the needle is maintained at 80 cm of Hg. The time it will take to draw 10 mL of blood is
 (a) 35.7 s (b) 16.8 s
 (c) 26.8 s (d) 23.8 s
22. In a blood-transfusion operation, the blood bottle is hanging at a height of 1 m.



- from the vein. The blood flows from the bottle at atmospheric pressure. The vein is at a pressure 25 mm of Hg higher than the atmosphere. The needle piercing the vein has a length of 3 cm and an internal diameter of 0.40 mm. If the viscosity and density of blood are respectively 8×10^{-3} kg/m.s and 1.005×10^3 kg/m³, the quantity of blood that flows into the vein in one minute is
- 1.05 cc
 - 10.5 cc
 - 2.10 cc
 - 3.15 cc
23. A drop of water of radius 0.01 cm is falling through the atmosphere ($\rho = 1.20$ kg/m³ and $\eta = 1.8 \times 10^{-5}$ N.s/m²). If the density of water is 1×10^3 kg/m³, the terminal velocity of the water drop is
- 1.2 m/s
 - 2.1 m/s
 - 1.2×10^{-2} m/s
 - 2.1×10^{-2} m/s
24. The end area of the piston of a hydraulic press is 0.80 cm² and the pressure in the fluid is 1×10^5 N/m². If the stroke is of 5 cm, the work done by the system is
- 4 J
 - 4×10^{-3} J
 - 0.4 J
 - 4×10^{-2} J
25. A glass plate of area 20 cm² is separated from another larger plate of glass by a 1-mm-thick layer of viscous fluid whose viscosity is 20 poise. If the smaller plate is to be moved over the larger one with a velocity of 2 cm/s, the force required to keep it moving is
- 4×10^{-4} N
 - 4×10^{-3} N
 - 0.4 N
 - 0.08 N
26. A raindrop of radius 1 mm is falling through air ($\eta = 1.8 \times 10^{-5}$ N.s/m² and $\rho = 1.3$ kg/m³). If the flow around is close to turbulent, i.e. if the Reynolds number is close to 10, the maximum velocity the falling drop can have is
- 6.9 cm/s
 - 9.6 cm/s
 - 3.45 cm/s
 - 6.9 m/s
27. Capillaries of lengths L , $2L$ and $L/2$ are connected in series. Their radii are r , $r/2$ and $r/3$ respectively. If the pressure across the first capillary is p_1 and the flow of liquid is maintained streamlined, the pressures across the second and third capillaries are
- $16p_1$ and $20.5p_1$
 - $16p_1$ and $40.5p_1$
 - $32p_1$ and $40.5p_1$
 - $40.5p_1$ and $32p_1$
28. A tube of radius R and length L is connected in series with another tube of radius $R/2$ and length $L/4$. If the pressure across the two tubes taken together is P , the pressures across the two tubes separately are
- $\frac{P}{5}$ and $\frac{4}{5}P$
 - $\frac{P}{3}$ and $\frac{2}{3}P$
 - $\frac{P}{4}$ and $\frac{3}{4}P$
 - $\frac{P}{2}$ and $\frac{P}{2}$
29. The terminal velocity of a raindrop is 30 cm/s. If the viscosity of water is 1.8×10^{-5} N.s/m², the radius of the rain drop is
- 0.01×10^{-4} m
 - 0.05×10^{-2} m
 - 1×10^{-3} m
 - 0.05×10^{-3} m
30. A constant waterhead of 5 cm is maintained across a capillary of radius 1 mm and length 30 cm through which 50 cc of a fluid flows in a minute. The coefficient of viscosity of the fluid is
- 7×10^{-2} N.s/m²
 - 7.7×10^{-4} N.s/m²
 - 0.77×10^{-4} N.s/m²
 - 7.7×10^{-3} N.s/m²
31. Water flows along a horizontal pipe of which the cross-section is not constant. The pressure is 1 cm of Hg where the velocity is 35 cm/s. At a point where the velocity is 65 cm/s, the pressure will be
- 0.89 cm of Hg
 - 8.9 cm of Hg
 - 0.5 cm of Hg
 - 1 cm of Hg
32. The area of cross-section at two points in a pipe are A_1 and A_2 where a venturimeter (to show pressure difference) is connected, indicating Δp as the difference in pressure at two points. If the density of fluid flowing through the pipe is ρ , the quantity of fluid flowing in one second is
- $\sqrt{\frac{2\Delta p}{\rho(A_1^2 - A_2^2)}}$
 - $A_1 A_2 \sqrt{\frac{2\Delta p}{\rho(A_1^2 - A_2^2)}}$
 - $\frac{A_1 A_2}{\rho} \sqrt{\frac{\Delta p}{(A_1^2 - A_2^2)}}$
 - $\frac{2A_1 A_2}{\rho} \sqrt{\frac{\Delta p}{A_1^2 - A_2^2}}$
33. Three capillaries of the same length but of internal radii $3r$, $4r$ and $5r$ are connected in series and a liquid flows through them in streamline conditions. If the pressure difference across the third capillary is 8 mm, the pressure difference across the first capillary is
- 26 mm
 - 48 mm
 - 84 mm
 - 62 mm
34. Water flows in a tube of diameter 2 cm ($\eta = 1 \times 10^{-3}$ N.s/m²) so that the flow is streamline. If the Reynolds number of the streamline flow is 2000, the maximum average velocity of water is
- 0.1 m/s
 - 1 m/s
 - 0.5 m/s
 - 0.2 m/s
35. Water is to be pushed through a horizontal tube of diameter 8 cm and length 4 km at the rate of 20 litres per second. If atmospheric pressure is 1.013×10^5 N/m², $\eta = 0.001$ N.s/m² and only viscous resistance is operative, the pressure required to maintain the flow is
- 0.8013×10^5 N/m²
 - 1.8013×10^4 N/m²
 - 1.8013×10^5 N/m²
 - 8.013×10^5 N/m²
36. A tank containing water has an orifice on one vertical side. If the centre of the orifice is 4.9 m below the surface level on the tank, the velocity of the discharge of water is (assume no wastage of energy)
- 19.6 m/s
 - 14.7 m/s
 - 9.8 m/s
 - 4.9 m/s
37. Water flows through a pipe of radius 1.0 cm. The viscosity of water is 1 centipoise (10^{-3} kg/m.s). If the velocity at the centre is 10 cm/s, the pressure drop along a 2-m section of the pipe due to viscosity is
- 8 N/m²
 - 0.008 N/m²
 - 0.08 N/m²
 - 0.8 N/m²
38. A steel ball of radius 1 mm falls in a tank of glycerine. If the densities of steel and glycerine are 8.5 and 1.32 g/cc respectively and the viscosity of glycerine is 8.3 poise, the terminal velocity of the ball is
- 18 cm/s
 - 8.1 cm/s
 - 1.8 cm/s
 - 3.6 cm/s
- 39.* A capillary tube 1.0 mm in diameter and 25 cm in length is fitted horizontally to a vessel full of alcohol whose density and viscosity are 8×10^2 kg/m³ and 0.0012 kg/m.s respectively. The depth of the centre of the capillary tube from the surface of the alcohol is 50 cm. The quantity of alcohol (in grams) that will flow out in 5 minutes is

- (a) 58.7 (b) 85.7
(c) 57.8 (d) 78.5
40. Two tubes that carry water have equal input pressure applied and both exhaust in air. Tube A has a diameter of 4 mm and length 10 m and tube B has a diameter of 6 mm and length 20 m. The ratio Q_B/Q_A of the discharge rates from B and A is
- (a) 27/32 (b) 81/32
(c) 32/27 (d) 32/81

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (b) | 4. (d) | 5. (c) |
| 6. (b) | 7. (c) | 8. (a) | 9. (c) | 10. (d) |
| 11. (a) | 12. (c) | 13. (b) | 14. (a) | 15. (b) |
| 16. (c) | 17. (b) | 18. (a) | 19. (d) | 20. (c) |
| 21. (d) | 22. (a) | 23. (a) | 24. (c) | 25. (d) |
| 26. (a) | 27. (c) | 28. (a) | 29. (d) | 30. (b) |
| 31. (a) | 32. (b) | 33. (d) | 34. (a) | 35. (c) |
| 36. (c) | 37. (a) | 38. (c) | 39. (d) | 40. (b) |

Note: Problems marked with an asterisk should take about five minutes. All the rest should be solved in about three minutes each.

11

Elasticity

Stress

It is a measure of the strength of the agent that is causing a deformation. If a force F is applied to a surface of area A , then

$$\text{Stress} = \frac{\text{force}}{\text{surface area on which the force acts}} = \left(\frac{F}{A}\right) \text{N/m}^2$$

Strain

It is the fractional deformation resulting from a stress. It is measured as a ratio of the change in some dimension of a body to the original dimension in which the change takes place.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Hooke's Law

According to this law, strain is proportional to the stress producing it within the elastic limits.

$$\frac{\text{Stress}}{\text{Strain}} = \text{constant, called modulus of elasticity}$$

Modulus of elasticity has the same units as stress because strain has no units.

Young's Modulus or Tensile Modulus, Y

If a wire or rod of original length L and cross-sectional area A is elongated by an amount ΔL under a stretching force F applied to its end, then

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

Bulk Modulus or Volume Elasticity, B

Bulk modulus describes the volume elasticity of a material. If the force is applied normally and uniformly to the surface of the body, it undergoes a

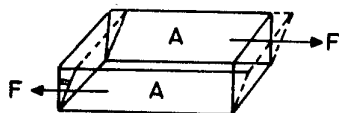
change in volume, its shape remaining unchanged. Such a force per unit area is called normal stress and the change in volume per unit volume is called the volume strain. The ratio of normal stress to the volume strain is called bulk modulus of elasticity.

$$\text{Bulk modulus, } B = \frac{F/A}{v/V} = \frac{FV}{Av} = \left(\frac{PV}{v} \right)$$

Its units are N/m^2 .

Shear Modulus or Modulus of Rigidity, η

This describes the shape elasticity of a material. Equal and opposite tangential forces F act on a rectangular block shown in the figure. These shearing forces distort the block, but its volume remains unchanged.



$$\text{Shearing stress} = \frac{\text{tangential force acting}}{\text{area of surface being sheared}} = \frac{F}{A}$$

$$\text{Shearing strain} = \frac{\text{distance sheared}}{\text{distance between surfaces}} = \frac{\Delta L}{L}$$

$$\eta = \frac{FL}{A\Delta L}$$

but ΔL is usually very small and the ratio $\Delta L/L$ is approximately equal to the shear angle ϕ in radians, and so

$$\eta = (F/A \cdot \phi)$$

Its units are N/m^2

Poisson's Ratio

The ratio of the lateral strain to the longitudinal strain is constant for a material (within the elastic limits) of the body and is known as Poisson's ratio (σ).

If longitudinal strain per unit stress is denoted by α and the lateral strain per unit stress by β , then

$$\sigma = \frac{\beta}{\alpha}$$

If a wire of length L and diameter D is subjected to a tensile stress, the length of the wire increases but the diameter decreases. Let the increased length become $(L + l)$ and the new diameter be decreased to d . Then,

$$\text{Longitudinal strain} = \frac{l}{L} \text{ and lateral strain} = \frac{D-d}{D}$$

$$\therefore \sigma = \frac{(D-d)/D}{l/L} = \frac{L(D-d)}{Dl}$$

Relations Between Different Elastic Moduli

$$B = \frac{1}{3(\alpha - 2\beta)}, \text{ where } B \text{ is the bulk modulus, } \alpha \text{ the longitudinal strain and } \beta \text{ the lateral strain.}$$

$$B = \frac{Y}{3(1 - 2\sigma)}, \text{ where } Y \text{ is the Young's modulus and } \sigma \text{ is the Poisson's ratio}$$

$$\eta = \frac{1}{2(\alpha + \beta)}, \text{ where } \eta \text{ is the coefficient of rigidity}$$

$$\eta = \frac{Y}{2(1 + \sigma)}, Y = \frac{9\eta B}{3B + \eta} \text{ and } \sigma = \frac{3B - 2\eta}{2(\eta + 3B)}$$

Time Period of a Torsion Pendulum

This is given by

$$T = 2\pi \sqrt{\frac{I}{C}}$$

where I is the moment of inertia of the oscillating disc about the axis of the wire and C is the torsional rigidity of the material of the wire.

Work Done During three Cases of Strain

In order to deform a body, work must be done by the applied force. The energy so spent is stored up in the body and is called the energy of strain. When the applied forces are withdrawn, the stress disappears and energy of strain appears as heat.

Elongation Strain

$$\text{Work done per unit volume} = 1/2 \text{ stress} \times \text{strain}$$

$$= \frac{1}{2} \frac{F}{A} \cdot \frac{l}{L}$$

Volume Strain

$$W = \frac{1}{2} P \frac{v}{V}$$

Shearing Strain

$$W = \frac{1}{2} \frac{F}{A} \frac{x}{L}$$

where x is the linear shift in the top surface of the body on which the tangential force is applied.

Thus, we see that in any kind of strain, work done per unit volume is equal to $(1/2) \times \text{stress} \times \text{strain}$.

ILLUSTRATIONS

1. Calculate the work done in stretching a wire of cross-section 1 mm^2 and length 2 m through 0.1 mm , given that the Y for the material is $2 \times 10^{11} \text{ N/m}^2$.

Solution

$$\begin{aligned} W &= \frac{1}{2} \times \frac{YAl}{L} \times l = \frac{1}{2} \frac{YAl^2}{L} \\ &= \frac{1}{2} \times \frac{(2 \times 10^{11}) \text{ N/m}^2 \times (1 \times 10^{-6}) \text{ m}^2 \times (0.1 \times 10^{-3})^2 \text{ m}^2}{2} \\ &= 5 \times 10^{-4} \text{ J} \end{aligned}$$

2. A 0.4 m metal bar of square cross-section of 0.02 m side length, weighing 0.80 kg , is suspended by a wire passing through the centre of gravity, and undergoes oscillations in the horizontal plane. It is observed to go through 60 complete oscillations in one minute. Find the twisting couple per unit deflection.

Solution

If I is the moment of inertia of the bar about the suspension axis through the wire, then the time period is given by

$$T = 2\pi \sqrt{\frac{I}{C}}$$

where C is the couple per unit deflection

$$\therefore C = \frac{4\pi^2 I}{T^2} \quad \left\{ \begin{array}{l} \text{here } I = M \left(\frac{L^2 + B^2}{12} \right) \\ = 0.8 \left(\frac{0.4^2 + 0.02^2}{12} \right) \\ = 0.01 \text{ kg.m}^2 \end{array} \right.$$

$$\begin{aligned} &= \frac{4\pi^2}{1} \times 0.01 \\ &= 0.4 \text{ N.m} \end{aligned}$$

3. The breaking stress of aluminium is $7.5 \times 10^7 \text{ N/m}^2$. If the density of aluminium is 2.7 g/cc , find the maximum length of the aluminium wire that could hang vertically without breaking.

Solution

The breaking stress must be equal to the maximum stress that the wire can withstand.

Let A be the area of cross-section of the wire and L the length that could hang without breaking. Therefore, the weight of such a wire is

$$(L \times A \times 2.7 \times 10^3 \times g) \text{ N}$$

Breaking stress $= 7.5 \times 10^7 \text{ N/m}^2$ and the force that can be applied on the wire is $(7.5 \times 10^7 \times A) \text{ N}$

Hence,

$$L \times A \times 2.7 \times 10^3 \times 10 = 7.5 \times 10^7 \times A$$

$$\text{or } L = \frac{7.5 \times 10^7}{2.7 \times 10^4} = 2.7 \text{ km}$$

4. A copper wire of length 3 m and diameter 1 mm is hanging vertically where, at the lower end, a weight of 10 kg is attached. Find the extension produced, calculate the lateral strain and also find the lateral compression (Y for copper is $12 \times 10^{10} \text{ N/m}^2$ and Poisson's ratio for copper is 0.25).

Solution

$$l = \frac{FL}{AY} = \frac{10 \times 10 \times 3}{\pi (0.5 \times 10^{-3})^2 \times 12 \times 10^{10}} = 0.003 \text{ m}$$

$$0.25 = \frac{\text{lateral strain}}{l/L} \Rightarrow \text{lateral strain} = 2.5 \times 10^{-4}$$

If d is the decrease in diameter and D is the original diameter, we have

$$\frac{d}{D} = 2.5 \times 10^{-4} \Rightarrow d = 2.5 \times 10^{-7} \text{ m}$$

$$\therefore \text{Lateral compression} = 2.5 \times 10^{-7} \text{ m}$$

5. A steel wire of diameter 2 mm is just stretched between two fixed points at a temperature of 20°C . What will be the tension in the wire if the temperature is reduced to 10°C ? (The coefficient of linear expansion of steel is $11 \times 10^{-6}/^\circ\text{C}$ and $Y = 2 \times 10^{11} \text{ N/m}^2$.)

Solution

$$\begin{aligned} L_t &= L_0 (1 + \alpha t) \text{ or decrease in length} = L_0 \alpha t \\ &= L_0 \times 11 \times 10^{-6} \times 10 \\ &= 11 \times 10^{-5} L_0 \end{aligned}$$

$$\text{Strain produced} = \frac{11 \times 10^{-5} L_0}{L_0} = 11 \times 10^{-5}$$

Let T newtons be the tension on the wire.

$$\therefore \text{Stress} = \frac{T}{\pi r^2} = \frac{T}{\pi (1 \times 10^{-3})^2} \text{ N/m}^2$$

$$\begin{aligned} Y = \frac{\text{stress}}{\text{strain}} &= \frac{T/\pi (1 \times 10^{-3})^2}{11 \times 10^{-5}} \Rightarrow T = 2 \times 10^{11} \times 11 \times 10^{-5} \times \pi \times 10^{-6} \\ &= 22\pi \text{ N} \end{aligned}$$

6. What is the couple that has to be applied to a wire 1 m long and 1 mm in diameter in order to twist it through 90° , the other end remaining fixed ($\eta = 2.8 \times 10^{10} \text{ N/m}^2$)?

Solution

The couple required is given by

$$\begin{aligned}\text{Couple} &= \eta \pi r^4 (\Delta\theta) / 2L \\ &= \frac{2.8 \times 10^{10} \times \pi \times (0.5 \times 10^{-3})^4 \times \pi / 2}{2} \\ &= 43.17 \times 10^{-4} \text{ N.m}\end{aligned}$$

7. A steel wire has the following properties:

$L = 5 \text{ m}$, area of cross-section = 0.05 cm^2 , $Y = 1.8 \times 10^{11} \text{ N/m}^2$, η (shear modulus) = $0.6 \times 10^{11} \text{ N/m}^2$, proportional limit = $3.6 \times 10^8 \text{ N/m}^2$, breaking stress = $7.2 \times 10^8 \text{ N/m}^2$. The wire is fastened at its upper end and hangs vertically.

(a) How great a load can it support without exceeding the proportional limit?

(b) How much will the wire stretch under this load?

(c) What is the maximum load that it can support?

Solution

(a) 'Proportional limit' means the maximum stress under the limit of elasticity. Hence the load under this stress is

$$\begin{aligned}Mg &= \text{stress} \times \text{area of cross-section} \\ &= 3.6 \times 10^8 \text{ N/m}^2 \times 0.05 \times 10^{-4} \text{ m}^2\end{aligned}$$

$$\therefore Mg = 1800 \text{ N or load } M = 180 \text{ kg}$$

$$\begin{aligned}\text{(b) } Y &= \frac{MgL}{A \cdot l} \text{ or } l = \frac{MgL}{AY} = \frac{1800 \times 5}{0.05 \times 10^{-4} \times 1.8 \times 10^{11}} = \frac{18}{1.8} \times 10^{-3} \text{ m} \\ &= 10 \text{ mm}\end{aligned}$$

$$\begin{aligned}\text{(c) Breaking stress} &= 7.2 \times 10^8 \text{ N/m}^2 = \frac{Mg}{A} \Rightarrow Mg = 7.2 \times 0.05 \times 10^4 \\ &= 3600 \text{ N or } M = 360 \text{ kg}\end{aligned}$$

8. A 20 kg mass, fastened to the end of a steel wire of an unstretched length of 1 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The area of cross-section of the wire is 0.02 cm^2 . Calculate the elongation of the wire when the weight is at the lowest point of its path ($Y = 2.0 \times 10^{11} \text{ N/m}^2$).

Solution

Tension in the wire when the weight is at the lowest position

$$T = Mg + Mr \omega^2 \quad (\omega = 4\pi \text{ rad/s})$$

$$\begin{aligned}\therefore T &= 20 \times 10 + 20 \times 1 \times (4\pi)^2 \\ &= 200 + 3158 = 3358 \text{ N}\end{aligned}$$

$$\begin{aligned}l &= \frac{TL}{YA} = \frac{3358 \times 1}{2 \times 10^{11} \times 0.02 \times 10^{-4}} = 83950 \times 10^{-7} \text{ m} \\ &= 8.3950 \times 10^{-3} \text{ m} \\ &= 8.40 \text{ mm}\end{aligned}$$

9. A copper wire ($Y = 1 \times 10^{11} \text{ N/m}^2$) of length 8 m, and a steel wire ($Y = 2 \times 10^{11} \text{ N/m}^2$) of length 4 m, each of cross-section 0.5 cm^2 are fastened end to end and stretched with a tension of 500 N. Calculate the elastic potential energy of the system.

Solution

$$\text{Work done, } W = \frac{1}{2} \times \text{stretching force} \times \text{net stretch}$$

Here $l = l_1 + l_2$ (where l_1 and l_2 are the elongations in the copper wire and steel wire, respectively)

$$l_1 = \frac{MgL_1}{Y_{\text{cu}} A} = \frac{500 \times 8}{1 \times 10^{11} \times (0.5 \times 10^{-4})} = 0.8 \text{ mm}$$

$$l_2 = \frac{MgL_2}{Y_{\text{steel}} A} = \frac{500 \times 4}{2 \times 10^{11} \times (0.5 \times 10^{-4})} = 0.2 \text{ mm}$$

$$\text{Total stretch} = 1.0 \text{ mm}$$

$$\begin{aligned}\text{Elastic PE} &= \text{Work done in stretching} = \frac{1}{2} 500 \text{ N} \times (1 \times 10^{-3}) \text{ m} \\ &= 0.25 \text{ J}\end{aligned}$$

10. A litre of a perfect gas, at a pressure of 72 cm of Hg, is compressed isothermally to a volume of 900 cc. Calculate the stress, strain and the bulk modulus of the gas.

Solution

Let the pressure of the gas in the compressed state be p cm of Hg. As the process is isothermal, we have

$$p \times 900 = 72 \times 1000$$

$$p = 80 \text{ cm}$$

Stress = increase in pressure

$$\begin{aligned}&= (80 - 72) = 8 \text{ cm of Hg} = 0.08 \times 13.6 \times 10^3 \times 10 \\ &= 10.88 \times 10^3 \text{ N/m}^2\end{aligned}$$

$$\begin{aligned}\text{Strain} &= \frac{\text{change in volume}}{\text{original volume}} \\ &= \frac{1000 - 900}{1000} = 0.1\end{aligned}$$

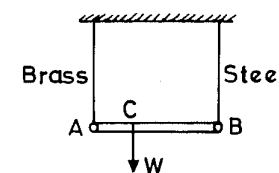
and bulk modulus of gas at constant temperature

$$= \frac{10.88 \times 10^3 \text{ N/m}^2}{0.1} = 10.88 \times 10^4 \text{ N/m}^2$$

EXERCISES

- The energy spent in stretching a wire of length L m and radius r m by l m is (Y is the Young's modulus of the material of the wire)
 - $\frac{Y\pi r^2 l^2}{2L}$
 - $\frac{Y\pi r^2 L}{2l}$
 - $\frac{Y\pi r^2 L}{l}$
 - $\frac{Y\pi r^2 l}{L}$
- A stone weighing 1 kg is tied to a string 1 m long and whirled in a horizontal circle with a speed of 2 m/s. If the breaking strength of the string is 9 N, the maximum speed with which the stone can be whirled is
 - 4 m/s
 - 18 m/s
 - 3 m/s
 - 6 m/s
- A spherical ball contracts in volume by 0.01% when subjected to a normal uniform pressure of 100 atmospheres. The bulk modulus of the material of the ball is
 - $1.034 \times 10^{12} \text{ N/m}^2$
 - $1.034 \times 10^{10} \text{ N/m}^2$
 - $1.034 \times 10^{11} \text{ N/m}^2$
 - $1.034 \times 10^9 \text{ N/m}^2$
- A metal bar of length L and area of cross-section A is heated through $T^\circ\text{C}$. If the coefficient of linear expansion of the metal is α , the value of force which must be applied to the ends to compress the bar length, keeping the temperature constant, is (Young's modulus of metal is Y)
 - $\frac{YA\alpha}{(1 + \alpha T)}$
 - $\frac{Y\alpha T}{A(1 + \alpha T)}$
 - $\frac{YA}{(1 + \alpha T)}$
 - $\frac{YA\alpha T}{(1 + \alpha T)}$
- Four wires of the same material are stretched by the same load. The dimensions of the wires are as given below. The one which has the maximum elongation is of
 - diameter 1 mm and length 1 m
 - diameter 2 mm and length 2 m
 - diameter 0.5 mm and length 0.5 m
 - diameter 3 mm and length 3 m
- If the radius of the suspension wire is doubled without changing the length in case of a torsion pendulum, its time period will be
 - double
 - half
 - the same
 - one fourth

- A spiral spring is stretched by a weight attached to it. The strain will be
 - bulk
 - shear
 - tensile
 - none of these
- Four wires of different material but the same area of cross-section are loaded by the same force. Their lengths and elongations are as follows. The one whose material's Young's modulus is the largest is
 - $L = 2 \text{ m}, l = 1 \text{ mm}$
 - $L = 1 \text{ m}, l = 0.25 \text{ mm}$
 - $L = 1.5 \text{ m}, l = 0.5 \text{ mm}$
 - $L = 2.5 \text{ m}, l = 1.5 \text{ mm}$
- The Young's modulus of a material is $7 \times 10^{10} \text{ N/m}^2$ and its bulk modulus is $11 \times 10^{10} \text{ N/m}^2$. Its Poisson's ratio is
 - 0.17
 - 0.43
 - 0.39
 - 0.26
- * A 2 m long light metal rod AB is suspended from the ceiling horizontally by means of two vertical wires of equal length tied to its ends. One wire is of brass and the other is of steel with $0.1 \times 10^{-4} \text{ m}^2$ cross-section. In order to have equal stresses in the two wires, a weight is hung from the rod. The position of the weight along the rod from end A should be
 - 66.6 cm
 - 133 cm
 - 44.4 cm
 - 155.6 cm
- * In the above problem, the Young's moduli for brass and steel are $1 \times 10^{11} \text{ N/m}^2$ and $2 \times 10^{11} \text{ N/m}^2$ respectively. For the strain in the two wires to be the same, the distance between the weight that should be hanged from the end A, has to be
 - 66.6 cm
 - 133 cm
 - 100 cm
 - 50 cm
- * A sphere of radius 11 cm and mass 20 kg is attached to the lower end of a steel wire which is suspended from the ceiling of a room. The point of support is 5.11 m above the floor. When the sphere is set swinging as a simple pendulum, its lowest point just grazes the floor. If the area of cross-section of the wire is $8 \times 10^{-7} \text{ m}^2$, $Y = 2 \times 10^{11} \text{ N/m}^2$ and the unstretched length of the wire is 5.10 m, the velocity of the sphere at the lowest point is
 - 3.5 m/s
 - 5.4 m/s
 - 0.55 m/s
 - 2.75 m/s
- * A steel wire of length 2.0 m and cross-sectional area $1 \times 10^{-8} \text{ m}^2$ is held between two rigid supports with a tension 200 N. If the wire is pulled 5 mm in the direction perpendicular to the wire, the change in the tension of the wire is ($Y = 2 \times 10^{11} \text{ N/m}^2$)
 - 5.2 N
 - 0.25 N
 - 5.0 N
 - 2.5 N
- A wire is stretched to double its original length and its area of cross-section is 1 cm^2 and $Y = 2 \times 10^{11} \text{ N/m}^2$. The force required to do so is
 - $2 \times 10^7 \text{ N}$
 - $2 \times 10^{11} \text{ N}$
 - $2 \times 10^{10} \text{ N}$
 - $2 \times 10^8 \text{ N}$
- Two wires of the same material and same length but of different radii are



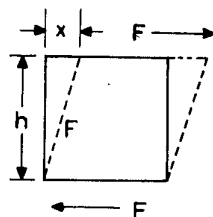
stretched by the same force. Wire A is stretched to double its length while wire B is stretched by a quarter of its original length. The ratio of radii of wires B and A respectively is

- (a) 8 : 1 (b) 2 : 1
(c) 4 : 1 (d) 16 : 1
16. One end of a wire of radius 5 mm and length 1 m is twisted through 45° . The angle of shear on its surface is
(a) 0.112 (b) 0.55
(c) 2.25 (d) 0.225
17. A square brass plate of side 1.0 m and thickness 0.005 m is subjected to a force F on each of its edges, causing a displacement of 0.02 cm. If the shear modulus of brass is 0.4×10^{11} N/m², the value of the force F is
(a) 4×10^3 N (b) 400 N
(c) 4×10^4 N (d) 1000 N
18. The volume of oil contained in a certain hydraulic press is 0.5 m³. It is subjected to a pressure of 3×10^7 N/m². If the compressibility of the oil is 25×10^{-11} m²/N, the decrease in the volume of the oil is
(a) 37.5×10^{-4} m³ (b) 37.5×10^{-3} m³
(c) 37.5×10^{-6} m³ (d) 37.5×10^{-2} m³
19. The Young's modulus of a material is 0.70×10^{11} N/m² and its Poisson's ratio is 0.16. Its bulk modulus is
(a) 2.0×10^{11} N/m² (b) 2.6×10^{11} N/m²
(c) 3×10^{11} N/m² (d) 0.3×10^{11} N/m²
20. The Young's modulus of a material is 2×10^{11} N/m² and its Poisson's ratio is 0.27. Its shear modulus is
(a) 0.79×10^{11} N/m² (b) 0.79×10^{10} N/m²
(c) 79×10^{10} N/m² (d) 79×10^{12} N/m²
21. The compressibility of water is 46.4×10^{-6} /atm. This means that
(a) the bulk modulus of water is 46.4×10^6 atm
(b) volume of water decreases by 46.4 one-millionths of the original volume for each atmosphere increase in pressure
(c) when water is subjected to an additional pressure of one atmosphere, its volume decreases by 46.4%
(d) when water is subjected to an additional pressure of one atmosphere, its volume is reduced to 10^{-6} of its original volume.
22. A brass rod of length 2 m and cross-sectional area 2.0 cm² is attached end to end to a steel rod of length L and cross-sectional area 1.0 cm². The compound rod is subjected to equal and opposite pulls of magnitude 5×10^4 N at its ends. If the elongations of the two rods are equal, the length of the steel rod (L) is ($Y_{\text{brass}} = 1.0 \times 10^{11}$ N/m² and $Y_{\text{steel}} = 2.0 \times 10^{11}$ N/m²)
(a) 1.5 m (b) 1.8 m
(c) 1 m (d) 2 m
23. In the above problem, the strains produced in brass and steel rods respectively are
(a) 1.25×10^{-3} and 2.5×10^{-3} (b) 2.5×10^{-3} and 2.5×10^{-3}
(c) 5×10^{-4} and 2.5×10^{-4} (d) 2.5×10^{-3} and 5×10^{-3}
24. A metal rod of length L , cross-sectional area A and Young's modulus Y , is subjected to a tension F . If stress and strain are represented by Q and P respectively, the elastic potential energy per unit volume is represented by
(a) $\frac{1}{2} Q^2 P^2$ or $\frac{Q^2 P^2}{Y}$ or $\frac{Q^2}{2Y}$ (b) $\frac{1}{2} QP$ or $\frac{P^2 Y}{2}$ or $\frac{Q^2}{2Y}$

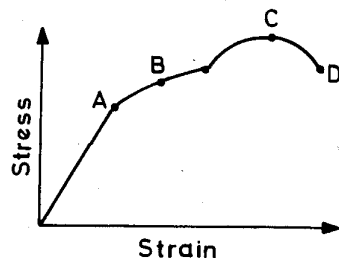
(c) QP or $P^2 Y$ or $\frac{Q^2}{Y}$

(d) $\frac{1}{2} P^2 Y$ or $\frac{1}{2} Q^2 Y$ or $\frac{1}{2} QP$

25. An elevator cable is to have a maximum stress of 7×10^7 N/m² to allow for appropriate safety factors. Its maximum upward acceleration is 1.5 m/s². If the cable has to support the total weight of 2000 kg of a loaded elevator, the area of cross-section of the cable should be
(a) 3.28 cm² (b) 2.38 cm²
(c) 0.328 cm² (d) 8.23 cm²
26. A 5 m long copper bar of square cross-section of area 0.2 cm² is stretched by a force of 500 N at each end. The total elongation of the bar is ($Y_{\text{Cu}} = 1 \times 10^{11}$ N/m²)
(a) 4.25 mm (b) 5.26 mm
(c) 6.25 mm (d) 2.65 mm
27. Atmospheric pressure is about 1.01×10^5 N/m². The force that the atmosphere exerts on a 2 cm² area on the top of your head is
(a) 50 N (b) 30 N
(c) 10 N (d) 20 N
28. A vertical wire 5 m long and of 0.0080 cm² cross-section has $Y = 2 \times 10^{11}$ N/m². An object weighing 2 kg is fastened to its end and stretches the wire elastically. If the object is now pulled down a little and released, it undergoes vertical SHM. Its period of vibration is
(a) 0.2 s (b) 0.05 s
(c) 0.1 s (d) 1.0 s
- 29.* A 50 kg motor rests on four cylindrical rubber blocks. Each block has a height of 4 cm and a cross-sectional area of 16 cm². The shear modulus of rubber is 2×10^6 N/m². A sideways force of 500 N is applied to the motor. The distance that the motor moves sideways is
(a) 0.156 cm (b) 1.56 cm
(c) 0.312 cm (d) 0.204 cm
30. After the motor referred to in the above problem is disturbed, it is released to vibrate back and forth. Its frequency of vibration is
(a) 26 Hz (b) 21 Hz
(c) 13 Hz (d) 18 Hz
31. A platform is suspended by four wires at its corners. Each wire is 3 m long and has a diameter of 2.0 mm. The Young's modulus of the material of wire is 1.8×10^{11} N/m². The distance through which the platform will descend a load of 50 kg symmetrically is placed symmetrically on it, is
(a) 0.32 mm (b) 0.65 mm
(c) 2.60 mm (d) 0.16 mm
32. A metal block is experiencing an atmospheric pressure of 1×10^5 N/m². When the same block is placed in a vacuum chamber, the fractional change in its volume is (the bulk modulus of metal is 1.25×10^{11} N/m²)
(a) 4×10^{-7} (b) 2×10^{-7}
(c) 8×10^{-7} (d) 1×10^{-7}
33. The compressibility of water is 5×10^{-10} m²/N. 500 cc of water is subjected to a pressure of 20×10^6 N/m². The decrease in its volume is
(a) 5 cc (b) 0.5 cc
(c) 1.5 cc (d) 0.05 cc
34. The body shown in the figure is a square aluminium plate of side 0.2 m. When a tangential force of magnitude 1.0×10^6 N is applied to the top face, the resulting shear is no greater than 0.01. The minimum thickness of the plate is ($\eta_{\text{Al}} = 0.3 \times 10^{11}$ N/m²)



- (a) 0.83 cm (b) 1.66 cm
(c) 0.66 cm (d) 0.066 cm
35. In the figure of problem 34, let the body be a square steel plate of side 10 cm and thickness 1 cm. To cause a shear strain of 0.01, the force required on each of the four sides is ($\eta_{\text{steel}} = 0.84 \times 10^{11} \text{ N/m}^2$)
(a) 840 N (b) $84 \times 10^5 \text{ N}$
(c) $84 \times 10^7 \text{ N}$ (d) $8.4 \times 10^5 \text{ N}$
36. A typical stress-strain curve for a ductile material under axial tension is shown in the figure. The points A, B, C and D are respectively
(a) elastic limit, proportional limit, ultimate strength and breaking point
(b) elastic limit, ultimate strength, breaking point, yield point
(c) proportional limit, yield point, ultimate strength and breaking point
(d) proportional limit, yield point, breaking point and ultimate strength.
37. A uniform steel bar of cross-sectional area A and length L is suspended so that it hangs vertically. The stress at the middle point of the bar is (ρ is the density of steel)
(a) $\frac{L}{2A} \rho g$ (b) $\frac{L\rho g}{2}$
(c) $\frac{LA}{\rho g}$ (d) $L\rho g$
38. A metal bar ($Y = 2 \times 10^{11} \text{ N/m}^2$ and $\rho = 8.0 \text{ g/cc}$) has a length of 1 m and area of cross-section of 0.01 m^2 . It is hanging vertically. The strain at the middle point of the bar is
(a) 2.0×10^{-7} (b) 1.0×10^{-7}
(c) 2.5×10^{-6} (d) 5.0×10^{-7}
39. A uniform bar of length L with an elastic modulus Y and thermal coefficient α is held between two rigid planes, one at each end of the bar. In this way the bar is prevented from expansion in these directions when it is heated. When the temperature of the bar is raised by ΔT °C, the stress developed in the bar is
(a) $\frac{Y\alpha}{\Delta T}$ (b) $\frac{YLa}{\Delta T}$
(c) $Y\alpha\Delta T$ (d) $\frac{Y\alpha\Delta T}{L}$



- 40.* A steel ball of mass m fits into the neck of a flask as shown. The flask contains a gas of volume V with compressibility K . If the cross-section of the neck is A and the ball is slightly displaced, its period of oscillation is



- (a) $2\pi \sqrt{\frac{mK}{VA}}$ (b) $2\pi \sqrt{\frac{Am}{VK}}$
(c) $\frac{2\pi}{A} \sqrt{\frac{m}{VK}}$ (d) $\frac{2\pi}{A} \sqrt{mVK}$
41. An iron rod of length 1 m and cross-sectional area 1 cm^2 and $Y = 1 \times 10^{11} \text{ N/m}^2$ is subjected to a force at the two ends so that it is elongated by 1 mm. The value of force is
(a) 10^4 N (b) 10^3 N
(c) 10^5 N (d) 10^6 N
42. The bulk modulus of water is $2.1 \times 10^9 \text{ N/m}^2$. The pressure required to increase the density of water by 0.1% is
(a) $2.1 \times 10^3 \text{ N/m}^2$ (b) $2.1 \times 10^6 \text{ N/m}^2$
(c) $2.1 \times 10^5 \text{ N/m}^2$ (d) $2.1 \times 10^7 \text{ N/m}^2$
43. A thick rubber rope has density $1.5 \times 10^3 \text{ kg/m}^3$, $Y = 5 \times 10^8 \text{ N/m}^2$, and length 8 m. When hung from the ceiling of a room the increase in its length due to its own weight is
(a) $96 \times 10^{-5} \text{ m}$ (b) $19.2 \times 10^{-5} \text{ m}$
(c) $9.6 \times 10^{-3} \text{ m}$ (d) $9.6 \times 10^{-2} \text{ m}$

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (a) | 2. (c) | 3. (c) | 4. (d) | 5. (c) |
| 6. (d) | 7. (b) | 8. (b) | 9. (c) | 10. (a) |
| 11. (c) | 12. (b) | 13. (d) | 14. (a) | 15. (b) |
| 16. (d) | 17. (c) | 18. (a) | 19. (d) | 20. (a) |
| 21. (b) | 22. (d) | 23. (b) | 24. (b) | 25. (a) |
| 26. (c) | 27. (d) | 28. (b) | 29. (a) | 30. (c) |
| 31. (b) | 32. (c) | 33. (a) | 34. (b) | 35. (d) |
| 36. (c) | 37. (b) | 38. (a) | 39. (c) | 40. (d) |
| 41. (a) | 42. (b) | 43. (a) | | |

Note: Problems marked with an asterisk should take about five to six minutes. All the rest should be solved in about three minutes each.

12

Heat

Heat is a form of energy. A useful but nonoperational definition is: heat is that which is transferred between a system and its surroundings as a result of temperature differences only. The unit of heat Q used to be defined quantitatively in terms of a specified change produced in a body during a specified process. For example, if the temperature of one kilogram of water is raised from 14.5° to 15.5°C by heating, we say that one kilocalorie (kcal) of heat has been added to the system. The calorie is also used as a heat unit.

$$1.000 \text{ kcal} = 1000 \text{ cal} = 3.968 \text{ Btu}$$

Heat Capacity

The ratio of the amount of heat energy ΔQ supplied to a body to its corresponding temperature rise ΔT is called the heat capacity of the body.

$$\text{Heat capacity} = \frac{\Delta Q}{\Delta T}$$

It simply means the energy that must be added as heat in order to raise the temperature of the body by one degree.

Specific Heat

The heat capacity per unit mass of a body is known as specific heat. It is a characteristic of the material of which the body is composed.

$$\text{Specific heat} = \frac{\text{heat capacity}}{\text{mass}} = \frac{\Delta Q}{m \Delta T}$$

The heat that must be given to a body of mass m , whose material has a specific heat capacity (c), to increase its temperature from T_i to T_f assuming $\Delta T \ll T_f - T_i$, is given by

$$Q = \sum \Delta Q = \sum_{T_i}^{T_f} mc \Delta T$$

$$\text{or } Q = m \int_{T_i}^{T_f} c \, dT$$

where c is a function of temperature.

At ordinary temperatures and over ordinary temperature intervals, specific heat can be considered to be constant.

Molar Heat Capacities of Solids

The amount of heat required per molecule to raise the temperature of a solid by a given amount is more or less the same for similar materials. Hence, if we express specific heat in cal/mol °C instead of cal/g°C, it is known as molar heat capacity. Dulong and Petit pointed out that the molar heat capacities of all substances (with a few exceptions) are close to 6 cal/mol °C. If M is the molecular weight, n the number of moles, and m the mass in grams, then

$$n = \frac{m}{M}$$

$$C = Mc = \frac{1}{n} \frac{dQ}{dT}$$

(the product Mc is called molar heat capacity)

$$dQ = nC \, dT$$

Latent Heat of Fusion

The latent heat of fusion (L_f) of a solid is the quantity of heat required to melt a unit mass of the solid at constant temperature. It is also equal to the quantity of heat given off by a unit mass of molten solid as it solidifies at the same temperature. If L_f is the latent heat of fusion of a substance and Q the heat required to melt a mass m at the same temperature, we have

$$Q = mL_f \quad (\text{units of } L_f \text{ are cal/g, kcal/kg or J/kg})$$

For example, the latent heat of ice is 80 cal/g, 80 kcal/kg or 336×10^3 J/kg.

The heat of vaporization (L_v) of a liquid is the quantity of heat required to vaporise a unit mass of the liquid at constant temperature. For water, it is 540 cal/g, 540 kcal/kg or 2268 kJ/kg.

The heat of sublimation of a solid substance is the quantity of heat required to convert a unit mass of the substance from the solid to the gaseous state at constant temperature.

Measurement of Temperature

There are three scales of measuring of temperature: (a) the Celsius scale, (b) the Fahrenheit scale, and (c) the Rankine scale.

$$\text{Kelvin scale temperature: } T = T_c + 273$$

$$T_F = \frac{9}{5} T_c + 32^\circ$$

$$T_R = \frac{9}{5} T = T_F + 459.68$$

Constant-Volume Gas Thermometer

$$T = (273.16 \text{ K}) \cdot \frac{p}{p_0}$$

where p_0 is the manometer pressure at the standard fixed point, the triple point of water, and p is the pressure at any other temperature, say T .

Thermocouples

A circuit that consists of two wires of different metals and connected with two junctions will develop a voltage within the circuit if the junctions are at different temperatures. This is the operating principle of the thermocouple. Common thermocouples are platinum-platinum rhodium, copper-constantan, chromel-alumel, etc.

Pyrometers

Very high temperatures—even above the melting points of most metals—can be determined by optical means that do not require contact with the sample. An electrically heated filament in a pyrometer is viewed through a suitable filter simultaneously with the sample, which might be, for example, the interior of a furnace. The “colour temperature” of the filament and the sample are matched.

Platinum Resistance Thermometer

The resistance of a material varies with temperature, $R_t = R_0 (1 + \alpha t)$ where α is the temperature coefficient of the material, t is the temperature upto which the material is heated and R_0 and R_t are the resistances at 0°C and $t^\circ\text{C}$ respectively. Any device which can measure resistance accurately can be used in the measurement of temperature.

Thermal Expansion**(i) Linear expansion**

When a solid is subjected to a rise in temperature ΔT , its increase in length ΔL is very nearly proportional to its original length L_0 multiplied by ΔT , i.e. $\Delta L = \alpha L_0 \Delta T$ where α is the proportionality constant called the coefficient of linear expansion. The value of α depends on the nature of the substance

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \quad (\text{units are per degree C or just } ^\circ\text{C}^{-1})$$

(ii) Area expansion

If an area A_0 expands to $A_0 + \Delta A$ when subjected to a temperature rise ΔT , then

$$\Delta A = \beta A_0 \Delta T$$

or $\beta = \Delta A/A_0 \Delta T$ where β is called the coefficient of area expansion. For isotropic solids (those that expand in the same way in all directions), $\beta = 2\alpha$ (approx.).

(iii) Volume expansion

If a volume V_0 expands to $V_0 + \Delta V$ when subjected to a temperature rise ΔT , then

$$\Delta V = \gamma V_0 \Delta T \Rightarrow \gamma = \frac{\Delta V}{V_0 \Delta T}$$

where γ is the coefficient of volume expansion. For isotropic solids, $\gamma = 3\alpha$ (approx.).

Thermal Stresses

Thermal stresses are set up in a rod that is not free to expand or contract. Suppose a rod of length L_0 and cross-section A has its ends rigidly fastened while the temperature is reduced by an amount ΔT . The fractional change in length if the rod were free to contract would be

$$\frac{\Delta L}{L_0} = \alpha \Delta T \text{ and } \frac{\Delta L}{L_0} = \frac{F}{AY}$$

where ΔL is positive. The tensile force F is determined by the requirement that the total fractional change in length, thermal expansion plus elastic strain, must be zero:

$$\alpha \Delta T + \frac{F}{AY} = 0 \Rightarrow F = -AY\alpha \Delta T$$

$$\text{or } \frac{F}{A} = -Y\alpha \Delta T$$

If ΔT represents an increase in temperature, then F and F/A become negative corresponding to compressive force and stress respectively.

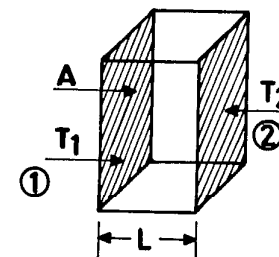
In case of a material enclosed in a very rigid container, a rise ΔT is accompanied by an increase in pressure Δp and is given by

$$\Delta p = B \cdot \gamma \cdot \Delta T$$

where B is bulk modulus and γ is the coefficient of volume expansion.

Heat Transfer

Consider a slab of material of thickness L and cross-sectional area A . The temperature of face 1 is T_1 and that of face 2 is T_2 . Thus the temperature difference across the slab is $\Delta T = T_1 - T_2$. The quantity $\Delta T/L$ is called the



temperature gradient. The quantity of heat transmitted from face 1 to face 2 in an interval of time Δt is given by

$$\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{L}$$

where K depends on the material of the slab and is called its thermal conductivity.

Units: $\text{W/m}\cdot\text{K}$ or $\text{cal cm}^{-1} (\text{C}^\circ)^{-1} \text{s}^{-1}$ or $\text{kcal m}^{-1} \text{K}^{-1} \text{s}^{-1}$

The thermal resistance of the slab is defined by the heat flow equation in the form

$$\frac{\Delta Q}{\Delta t} = \frac{A\Delta T}{R} \text{ where } R = \frac{L}{K} \quad (\text{units: } \text{m}^2 \cdot \text{K/W})$$

A black body is a body that absorbs all the radiant energy falling on it. At thermal equilibrium, a body emits as much energy as it absorbs.

Stefan-Boltzmann Law

Suppose a surface area A has absolute temperature T and radiates only a fraction ϵ of the energy a black body surface would radiate. Then ϵ is called the emissivity of the surface and the power radiated by the surface is given by

$$\frac{\Delta Q}{\Delta t} = \epsilon A \sigma T^4$$

where σ is called the Stefan-Boltzmann constant and T is the absolute temperature ($\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$).

If a body is at an absolute temperature T and the temperature of its surroundings is T_0 , then the net energy radiated per second by the body is

$$\frac{\Delta Q}{\Delta t} = \epsilon A \sigma (T^4 - T_0^4)$$

Ideal Gases

Equation of State

The state of a certain gas of mass m , pressure p , volume V and temperature T can be expressed as a functional relationship represented by $V = f(p, T, m)$.

An ideal or perfect gas is one that obeys the ideal gas law. At low to moderate pressures, and temperatures not too low, the following common gases can be treated as ideal: air, nitrogen, oxygen, helium, hydrogen and neon.

Ideal Gas Law

$$pV = nRT$$

where p is the pressure of n kilomoles of a gas whose volume is V and temperature T . Here R is the universal gas constant ($R = 8314 \text{ J/kmol} \cdot \text{K}$).

There are three special cases of the above law in which two quantities are kept constant and the rest are variables.

Boyle's Law (n, T constant), $p \cdot V = \text{constant}$

Charles's Law (n, p constant), $V/T = \text{constant}$

Gay-Lussac's Law (n, V constant), $p/T = \text{constant}$

Standard Temperature and Pressure

$$T = 273 \text{ K} = 0^\circ\text{C}, \quad p = 1.013 \times 10^5 \text{ N/m}^2 \text{ or Pa} = 1 \text{ Atm}$$

Under standard conditions, 1 kmol of an ideal gas occupies a volume of 22.4 m^3 .

If changes occur from (p_1, V_1, T_1) to (p_2, V_2, T_2) , then from the gas law,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \text{ (at constant } n)$$

ILLUSTRATIONS

1. Suppose the pressure of hydrogen in a constant-volume gas thermometer is 74.0 cm of Hg at the temperature of melting ice, 108.4 cm at the temperature of steam at normal pressure, and 87.2 cm at a temperature t of a liquid in which the bulb of the thermometer is dipped. Find the temperature of the liquid.

Solution

$$\frac{t}{100} = \frac{p_t - p_0}{p_{100} - p_0} \Rightarrow \frac{t}{100} = \frac{87.2 - 74.0}{108.4 - 74.0}$$

$$\therefore t = 38.4^\circ\text{C}$$

Similarly, in a platinum resistance thermometer,

$$\frac{t_p}{100} = \frac{R_t - R_0}{R_{100} - R_0}$$

Suppose $R_0 = 6.284 \Omega$.

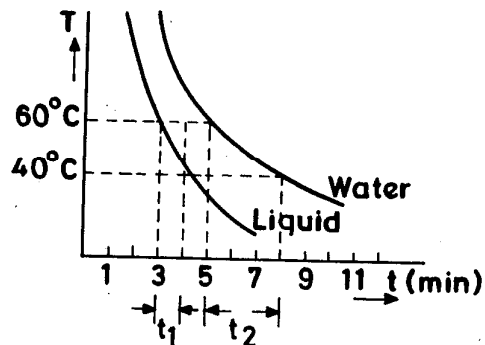
$$R_{100} = 7.321 \Omega$$

and the resistance at a temperature t is $R_t = 6.726 \Omega$ when the thermometer is placed in warm water. The temperature t_p is

$$t_p = \frac{6.726 - 6.284}{7.321 - 6.284} = 42.6^\circ\text{C}$$

2. The graphs of fall of temperature of a liquid (250 g) and water (200 g) (cooled under identical conditions having equal volumes) are shown in the

figure. Calculate the specific heat of the liquid (mass of the calorimeter is 100 g and specific heat = 0.1).



Solution

The general expression for the average heat lost per unit time is given by

$$\frac{(m_2 + W)(\theta_1 - \theta_2)}{t_2} = \frac{(m_1 s + W)(\theta_1 - \theta_2)}{t_1}$$

or
$$\frac{(m_2 + W)}{t_2} = \frac{(m_1 s + W)}{t_1}$$

From graph $\begin{cases} t_1 = 1 \text{ minute} \\ t_2 = 3 \text{ minutes} \\ \theta_1 - \theta_2 = (60 - 40)^\circ\text{C} \end{cases}$

$$\frac{(200 + 10)}{3} = \frac{(250 \times s + 10)}{1}$$

$$s = 0.24 \text{ cal/g}^\circ\text{C}$$

3. How much ice at -20°C must be dropped into 0.25 kg of water, initially at 20°C , in order for the final temperature to be 0°C when all the ice melts. (Neglect the heat capacity of the container.)

(Specific heat of ice = $2000 \text{ J} \cdot \text{kg}^{-1} (\text{C}^\circ)^{-1}$, ($L = 33.4 \times 10^4 \text{ J/kg}$).

Solution

Heat given out by water is spent in melting ice,

$$Q = (0.25) \times (4186 \text{ J} \cdot \text{kg}^{-1} \text{C}^\circ^{-1} (20^\circ - 0^\circ) \text{C}$$

$$= 2.09 \times 10^4 \text{ J}$$

Let m be the mass of ice added.

Therefore, the heat required to heat it from -20°C to 0°C is

$$Q = m \times 2 \times 10^3 \times 20 = m \times 4 \times 10^4 \text{ J kg}^{-1}$$

$$\text{Heat needed for melting ice} = m \times 33.4 \times 10^4$$

Equating total heat given by water to total heat taken by ice,

$$2.09 \times 10^4 = m \times 4 \times 10^4 + m \times 33.4 \times 10^4$$

$$m = \frac{2.09 \times 10^4}{37.4 \times 10^4} = 0.056 \text{ kg}$$

4. When a quantity of liquid bismuth at its melting point is transferred to a calorimeter containing oil, the temperature of the oil rises from 12.5°C to 27.6°C . The experiment is repeated under identical conditions except that bismuth is solid, the temperature of the oil rises to 18.1°C . The specific heat of bismuth is $0.032 \text{ cal/g}^\circ\text{C}$. Find the latent heat of fusion of bismuth (melting point of bismuth = 271°C).

Solution

Let W be the water equivalent of the calorimeter in grams, m_1 and m_2 the masses in grams of the bismuth and oil respectively and C the specific heat of the oil. When the liquid bismuth is transferred, it solidifies inside the oil, because the temperature is lower than the melting point. Since the heat given out by the bismuth on solidification plus that given out by the solid bismuth when the temperature falls from 271°C to 27.6°C is equal to the heat absorbed by the calorimeter and contents

$$m_1 L + m_1 \times 0.032 \times (271 - 27.6) = (m_2 C + W) (27.6 - 12.5) \quad (i)$$

When the solid bismuth at 271°C is transferred, no change of state occurs. Since the heat given out by the solid is equal to the heat absorbed by the calorimeter and its contents

$$m_1 \times 0.032 \times (271 - 18.1) = (m_2 C + W) (18.1 - 12.5)$$

$$\therefore m_1 \times 0.032 \times 252.9 = (m_2 C + W) (5.6) \quad (ii)$$

Dividing (i) by (ii) to eliminate all the unknown quantities except L , we get

$$\frac{L + 0.032 \times 243.4}{0.032 \times 252.9} = \frac{15.1}{5.6}$$

or $L = 14.0 \text{ cal/g}$

5. At very low temperatures, the molar heat capacity of rock salt varies with temperature according to "Debye's T^3 law". Thus,

$$C = k \cdot \frac{T^3}{\theta^3}$$

where k is $1940 \text{ J} \cdot \text{mol}^{-1} \text{K}^{-1}$ and $\theta = 281 \text{ K}$.

(a) How much heat is required to raise the temperature of 2 moles of rock salt from 10 K to 50 K?

(b) What is the mean molar heat capacity in this range?

(c) What is the true molar heat capacity at 50 K?

Solution

$$(a) \quad dQ = nCdT$$

$$dQ = nk \frac{T^3}{\Theta^3} dT$$

$$\begin{aligned} \text{or} \quad Q &= \frac{nk}{\Theta^3} \int_{T_1}^{T_2} T^3 dT = \frac{nk}{\Theta^3} \left(\frac{T_2^4 - T_1^4}{4} \right) \\ &= \frac{2 \times 1940}{(281)^3} \times \frac{50^4 - 10^4}{4} \\ &= 273 \text{ J} \end{aligned}$$

$$(b) \quad \text{Mean molar heat capacity} = \frac{1}{40} \int_{10}^{50} C(T) dT$$

$$= \frac{k}{\Theta^3} \frac{1}{40} \left[\frac{T^4}{4} \right]_{10}^{50} = \frac{k}{\Theta^3} \frac{1}{40} \times \frac{50^4 - 10^4}{4} = 3.42 \text{ J/mol.K}$$

$$\begin{aligned} (c) \quad \text{True molar heat capacity at } 50 \text{ K} &= \frac{k(50)^3}{\Theta^3} = \frac{1940 \times 50^3}{(281)^3} \\ &= 10.9 \text{ J/mol.K} \end{aligned}$$

6. The electric power input to a certain motor is 0.5 kW and the mechanical power output is 0.54 hp.

(a) What is the efficiency of the motor?

(b) How many calories of heat are developed in the motor in one hour of operation, assuming all electrical energy not converted to mechanical energy is converted to heat?

Solution

(a) 'Efficiency' means the fraction of electrical power that is converted to mechanical power.

$$0.5 \text{ kW} = 500 \text{ W and } 0.54 \text{ hp} = 402.8 \text{ W}$$

$$\therefore \text{Efficiency} = \frac{402.8}{500} = 0.80 \text{ or } 80\%$$

(b) Electrical energy that is not converted into mechanical energy produces heat.

20% of electrical power is not converted, i.e. 400 W are converted and 100 W are not converted into mechanical energy.

$$100 \text{ W} = 100 \text{ J/s and in one hour, heat produced is}$$

$$100 \times 60 \times 60 = 36 \times 10^4 \text{ J}$$

$$= \frac{36}{4.1856} \times 10^4 \text{ cal} = 8.6 \times 10^4 \text{ cal}$$

7. A flask of capacity 200 cc is completely filled with mercury when its temperature is 20°C. If it is heated to a temperature of 80°C, find the mass of mercury that will spill out of the flask. (The linear expansion of glass is $9 \times 10^{-6}/^\circ\text{C}$ and the volume expansion of mercury is $0.18 \times 10^{-3}/^\circ\text{C}$.)

Solution

$$\text{Change in volume of the flask only} = \gamma \times V \times \Delta t$$

(γ is the volume expansion of glass)

$$= 3 \times \alpha \times V \times \Delta t$$

$$= 3 \times 9 \times 10^{-6} \times 200 \times 60$$

$$= 324 \times 10^{-3} = 0.324 \text{ cc}$$

$$\begin{aligned} \text{Change in volume of mercury} &= 0.18 \times 10^{-3} \times 200 \times 60 = 2160 \times 10^{-3} \\ &= 2.16 \text{ cc} \end{aligned}$$

$$\text{Mass of mercury that will spill out} = (2.16 - 0.324) \times 13.6 = 25 \text{ g}$$

8. A clock is accurate at 8°C and has then a period of exactly 2 s. How much time will the clock lose in a month of 30 days if the average temperature of the month is 18°C. (α of pendulum wire material is 0.000011 per °C.)

Solution

If the length of the pendulum at 8°C is L_1 and the increased length at 18°C is L_2 , we have the relation

$$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}}$$

where T_1 and T_2 are the respective periods at 8°C and 18°C

$$\text{But } L_2 = L_1[1 + (18 - 8)\alpha]$$

$$\therefore \frac{T_2}{2} = \sqrt{\frac{L_1(1 + 10\alpha)}{L_1}} = \sqrt{1 + 10\alpha}$$

$$\frac{T_2}{T_1} = \sqrt{1 + 10 \times 0.000011} = \sqrt{1.00011}$$

$$T_2 = 2 \times \sqrt{1.00011} = 2.00011 \text{ s}$$

In 2.00011 s, the clock registers 2 s. Therefore, in 2.00011 s, the clock loses 0.00011 s. Therefore, in 1 s, the clock loses $(1/2) \times 0.00011 \text{ s}$ (to a good approximation).

$$\begin{aligned} \therefore \text{Time lost in a month} &= \frac{1}{2} \times 0.00011 \times 30 \times 24 \times 3600 \text{ s} \\ &= 143 \text{ s} = 2 \text{ min. } 23 \text{ s} \end{aligned}$$

9. An iron wire AB of length 3 m at 0°C is fixed rigidly at points A and B in a brass frame as shown. The diameter of the wire is 0.6 mm. If the temperature of the system is raised to 60°C, what extra stress will be produced in the wire?



$$(\alpha_{\text{brass}} = 0.000018/^{\circ}\text{C}, \alpha_{\text{iron}} = 0.000012/^{\circ}\text{C}, Y_{\text{iron}} = 2 \times 10^{11} \text{ N/m}^2)$$

Solution

$$Y = \frac{F/A}{e/L} \quad (e \text{ is elongation})$$

$$\text{or } F = \frac{Y A e}{L}$$

When the system is heated, the brass expands to a length

$$3(1 + 60\alpha) = 3(1 + 60 \times 0.000018) \\ = 3.00324 \text{ m}$$

The iron wire would normally have expanded to

$$3(1 + 60 \times 0.000012) = 3.00216 \text{ m}$$

Hence, net extension due to its attachment to the brass is

$$3.00324 - 3.00216 = 0.00108 \text{ m}$$

$$\text{Original length of iron wire} = 3.00216 \text{ m.}$$

$$\begin{aligned} \text{Extra tension in wire} = F &= \frac{Y A e}{L} \\ &= \frac{2 \times 10^{11} \times \pi (0.3 \times 10^{-3})^2 \times 0.00108}{3.00216} \\ &= 20.34 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Extra stress} &= \frac{\text{extra tension}}{\text{area of cross-section}} \\ &= \frac{20.3}{\pi (0.3 \times 10^{-3})^2} = 7.2 \times 10^7 \text{ N/m}^2 \end{aligned}$$

10. A glass weight thermometer is filled with mercury at 0°C and when heated to 100°C, 7.785 g of mercury overflows and 450.0 g remains in the thermometer. When glycerine is filled in the same weight thermometer, the corresponding figures are 2.173 g and 41.00 g.

(a) Given that the coefficient of expansion of mercury is 0.000183/°C, determine the coefficient of linear expansion of the glass.

(b) Find the coefficient of volume expansion of glycerine.

Solution

$$\begin{aligned} \text{(a) Coefficient of apparent expansion of mercury} &= \frac{m}{m_0 \times \Delta t} \\ &= \frac{7.785}{450.0 \times 100} = 0.000173/^{\circ}\text{C} \end{aligned}$$

We know that $\gamma_{\text{real}} = \gamma_{\text{app}} + \text{cubical expansion of glass, } \gamma_g$

$$\therefore \gamma_g = \gamma_{\text{real}} - \gamma_{\text{app}} = 0.000183 - 0.000173 = 0.000010/^{\circ}\text{C}$$

$$\therefore \alpha_g = \frac{1}{3} \gamma_g = 0.000033/^{\circ}\text{C}$$

$$\text{(b) } \gamma_{\text{app (glycerine)}} = \frac{2.173}{41.00 \times 100} = 0.000530/^{\circ}\text{C}$$

$$\begin{aligned} \gamma_{\text{glycerine}} &= \gamma_{\text{app}} + \gamma_g = 0.000530 + 0.000010 \\ &= 0.000540/^{\circ}\text{C} \end{aligned}$$

11. The height of a mercury barometer as read with a steel scale is 754 mm at 20°C. What will it read at 0°C?

$$(\alpha_{\text{steel}} = 0.000012/^{\circ}\text{C} \text{ and } \gamma_{\text{Hg}} = 0.000182/^{\circ}\text{C})$$

Solution

Let the true height at 20°C be h_t and h_0 be the required height at 0°C.

$$h_t = 754 (1 + 0.000012 \times 20) \quad \text{(i)}$$

Since the pressures at 0°C and 20°C are the same,

$$h_0 d_0 g = h_t d_t g$$

where d_0 and d_t are the densities of mercury at 0°C and 20°C respectively.

$$h_0 = h_t \frac{d_t}{d_0} = \frac{h_t}{1 + 20\gamma} \left(\frac{d_t}{d_0} = \frac{1}{1 + \gamma t} \right)$$

$$h_0 = \frac{754 (1 + 0.000012 \times 20)}{1 + 0.000182 \times 20} = 751.44 \text{ mm}$$

Thus, the height at 0°C = 751.44 mm.

12. A loaded glass bulb weighs 156.25 g in air, 57.50 g when immersed in a liquid at 15°C and 58.57 g when immersed at 52°C. Calculate the mean coefficient of real expansion of the liquid between 15°C and 52°C. The coefficient of linear expansion of glass is 0.000009/°C.)

Solution

$$\text{Upthrust on bulb at } 15^{\circ}\text{C} = 156.25 - 57.50 = 98.75 \text{ g wt}$$

$$\text{Upthrust on bulb at } 52^{\circ}\text{C} = 156.25 - 58.57 = 97.68 \text{ g wt}$$

But Upthrust = wt of liquid displaced

$$\therefore 98.75 = V_1 d_1 \text{ and } 97.68 = V_2 d_2$$

where V_1 and V_2 are the volumes of glass at 15°C and 52°C respectively, and d_1 and d_2 are the corresponding densities of the liquid.

$$\frac{V_1 d_1}{V_2 d_2} = \frac{98.75}{97.68} \quad (i)$$

$$\text{But } V_2 = V_1 [1 + 0.000027 (52 - 15)] \quad (\gamma = 3\alpha) \quad (ii)$$

If γ is the real coefficient expansion of the liquid

$$\frac{d_1}{d_2} = 1 + \gamma(52 - 15) \quad (iii)$$

Substituting the values of V_1/V_2 from [ii] and that of d_1/d_2 from [iii] in [i], we get

$$\frac{1 + 37\gamma}{1 + 37 \times 0.000027} = \frac{98.75}{97.68} \Rightarrow \gamma = 0.00032/^\circ\text{C}$$

13. A cylinder of capacity 5 litres contains oxygen at 27°C and the gauge pressure recorded is $20 \times 10^5 \text{ N/m}^2$ or Pa. The molecular mass of oxygen is 32 kg/k mol. and the atmospheric pressure is $1 \times 10^5 \text{ Pa}$. What is the mass of oxygen contained in the cylinder?

Solution

The absolute pressure of the gas

$$\begin{aligned} p &= (\text{gauge pressure}) + (\text{atmos. pressure}) \\ &= (20 \times 10^5 + 1 \times 10^5) \text{ Pa} \\ &= 21 \times 10^5 \text{ Pa} \end{aligned}$$

$$pV = \left(\frac{m}{M}\right) RT$$

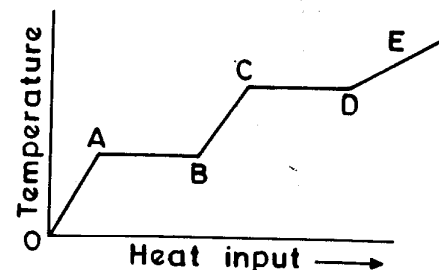
$$(21 \times 10^5 \text{ N/m}^2) \times (5 \times 10^{-3} \cdot \text{m}^3)$$

$$= \frac{m}{(32 \text{ kg/k mol})} \left(8314 \frac{\text{J}}{\text{k mol} \cdot \text{K}} \right) \cdot (300 \text{ K})$$

$$\begin{aligned} m &= \frac{21 \times 5 \times 10^2 \times 32}{8314 \times 300} \\ &= 0.135 \text{ kg} \end{aligned}$$

14. A solid material is supplied heat at a constant rate. The temperature of the material is changing with heat input as shown. Answer the following:

- What do regions AB and CD represent?
- If $CD = 2AB$, what do you infer?
- What does the slope of DE represent?
- Slope of OA > slope of BC. What does this indicate?



Solution

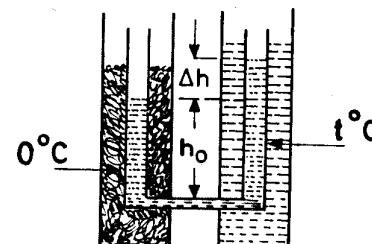
(a) The temperature in regions AB and CD remains constant. However, heat is supplied to the system, and is used to change the state. AB represents change of state from solid to liquid while CD represents the same from liquid to vapour.

(b) The latent heat of vapourization is twice the latent heat of fusion.

(c) The slope of DE shows inverse of heat capacity in the vapour state of the given solid.

(d) The fact that the slope of OA > slope of BC indicates that the specific heat of the solid is less than the specific heat in the liquid state.

15. Two vertical glass tubes filled with a liquid are connected at their lower ends by a horizontal capillary tube. One tube is surrounded by a bath containing ice and water at 0°C and the other by hot water at $t^\circ\text{C}$. The difference in height of the liquids in the two columns is Δh , and the height of the column at 0°C is h . Find the coefficient of volume expansion of the liquid.



Solution

Let the height of the liquid in the right tube be h .

$$\therefore h - h_0 = \Delta h \quad (i)$$

pressure at the interconnecting tube shall be equal and so

$$h_0 \rho_0 g = h \rho g \quad (ii)$$

$$\therefore h = \frac{h_0 \rho_0}{\rho}$$

$$\text{and } \Delta h = \frac{h_0 \rho_0}{\rho} - h_0 = h_0 \left[\frac{\rho_0}{\rho} - 1 \right]$$

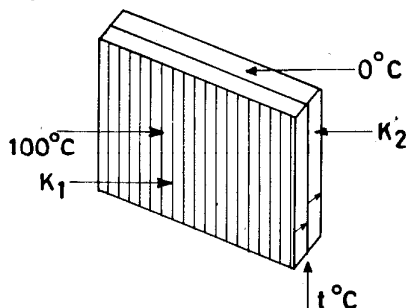
$$V = V_0 (1 + \gamma t) \quad (iii)$$

$$\frac{\rho_0}{\rho} = \frac{M/V_0}{M/V} = \frac{V}{V_0} = 1 + \gamma t \quad (iv)$$

$$\therefore \Delta h = h_0[1 + \gamma t - 1] = h_0 \gamma t$$

and hence $\gamma = \frac{\Delta h}{h_0 t}$

16. Two metal plates of an equal area of cross-section 100 cm^2 and equal thickness 5.0 mm are soldered together in such a manner that one face is maintained at 100°C while the other is kept at 0°C . $K_1 = 50 \text{ W/m.K}$ and $K_2 = 70 \text{ W/m.K}$. Find the temperature t of the soldered junction and calculate the rate of heat flow through the plates.



Solution

In equilibrium conditions, heat flowing through plate 1 equals that through plate 2. Then

$$K_1 A \frac{T_1 - t}{L_1} = K_2 A \frac{t - T_2}{L_2}$$

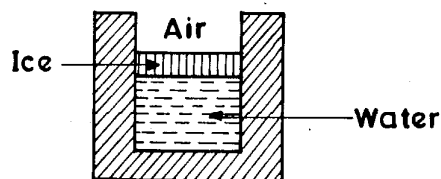
But $L_1 = L_2$

$$\therefore K_1(100 - t) = K_2(t - 0)$$

$$\begin{aligned} \text{or } t &= 100 \times \left(\frac{K_1}{K_1 + K_2} \right) = 100 \left(\frac{50}{50 + 70} \right) \\ &= \frac{100 \times 50}{120} = 41.7^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \frac{\Delta Q}{\Delta t} &= K_1 A \frac{T_1 - t}{L_1} = 50 \times (100 \times 10^{-4}) \times \frac{(100 - 41.7)}{5 \times 10^{-3}} \\ &= \frac{50 \times 100}{5 \times 10} \times 58.3 = 5830 \text{ J/s} \\ &= 5.8 \text{ kJ/s} \end{aligned}$$

17. A tank of water is placed in the open in cold weather until a 5.0 cm thick slab of ice forms on its surface. The air above the ice is at -10°C . Calculate the rate of formation of ice in centimetres per hour on the bottom surface of the slab. Take thermal conductivity, density and heat of fusion of ice to be $0.0040 \text{ cal/cm } ^\circ\text{C}$, 0.92 g/cc and 80 cal/g respectively. Assume that no heat enters or leaves the water through the walls of the tank.



Solution

(a) Let the thickness of the layer at $t = 0$ be x and the area of the surface be A . After an interval of time, dt , the thickness increases by dx . Let ρ be the density of ice, and L the latent heat of ice. Then

$$\text{Mass of ice formed in time } dt = A \cdot dx \cdot \rho$$

and Heat given by water $= m \cdot L = A dx \rho \cdot L \text{ cal}$
Heat is conducted through the ice layer already formed

$$\therefore \frac{\Delta Q}{\Delta t} = K \cdot A \left(\frac{\Delta T}{\Delta x} \right)$$

where $\Delta T/\Delta x$ is the temperature gradient through ice. Therefore,

$$A \rho dx L = K A \cdot \left(\frac{\Delta T}{\Delta x} \right) dt$$

$$\begin{aligned} \text{or } \frac{dx}{dt} &= \frac{KA}{L A \rho} \left(\frac{\Delta T}{\Delta x} \right) = \frac{K}{\rho L} \left(\frac{\Delta T}{\Delta x} \right) \\ &= \frac{0.0040}{0.92 \times 80} \left(\frac{10}{5} \right) = 0.39 \text{ cm/h} \end{aligned}$$

(b) Find an expression for the time required to increase the thickness of the ice layer from x_1 to x_2 .

Solution

$$\frac{dx}{dt} = \frac{K}{\rho L} \left(\frac{\Delta T}{\Delta x} \right) = \frac{K}{\rho L} \cdot \frac{\theta}{x}$$

(θ is the difference in temperature across a layer of thickness x .)

$$\text{or } dt = \frac{\rho L}{K \theta} x dx \quad \text{or } \int dt = \frac{\rho L}{K \theta} \int_{x_1}^{x_2} x dx$$

$$\text{or } t = \frac{1}{2} \frac{\rho L}{K \theta} (x_2^2 - x_1^2)$$

18. A metal cylinder of inner radius R_1 and outer radius R_2 carries a constant flow of hot water maintained at a temperature θ_1 . The outer temperature of the surroundings is θ_2 . If K is the thermal conductivity of the metal, find the rate of heat loss through the walls of the cylinder.

Solution

The rate of heat flow is given by

$$Q = K \cdot A \frac{d\theta}{dr} = K \cdot 2\pi r L \frac{d\theta}{dr}$$

where A is the surface area and L is the length of the cylindrical tube and $d\theta/dr$ is the temperature gradient.

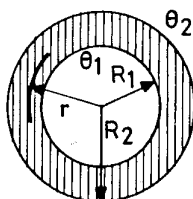
$$\frac{dr}{r} = \frac{2\pi KL}{Q} d\theta$$

If $\theta = \theta_1$ when $r = R_1$ and $\theta = \theta_2$ when $r = R_2$, then integrating and setting the limits, we get

$$\ln \frac{R_2}{R_1} = \frac{2\pi KL}{Q} (\theta_2 - \theta_1)$$

$$\text{or } Q = \frac{2\pi KL(\theta_2 - \theta_1)}{\ln(R_2/R_1)} \text{ cal/s}$$

19. If, in the above problem, we have a thick spherical shell where the inner surface of the shell is at a temperature θ_1 and the outer surface at θ_2 , find the rate of radial flow of heat.



Solution

Consider the heat flow through an elemental thin shell of radius r and thickness dr . The radial heat flow is given by

$$Q = KA \frac{d\theta}{dr} = K \cdot 4\pi r^2 \frac{d\theta}{dr}$$

$$\text{or } Q \frac{dr}{r^2} = K \cdot 4\pi d\theta \quad \text{or} \quad \int_{R_1}^{R_2} Q \frac{dr}{r^2} = 4\pi K \int_{\theta_1}^{\theta_2} d\theta$$

$$\text{or } Q \left[-\frac{1}{r} \right]_{R_1}^{R_2} = 4\pi K (\theta_2 - \theta_1)$$

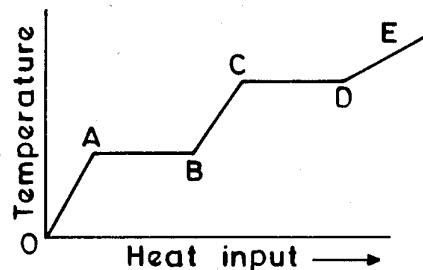
$$\text{or } Q = \frac{4\pi K R_1 R_2 (\theta_2 - \theta_1)}{(R_2 - R_1)}$$

EXERCISES

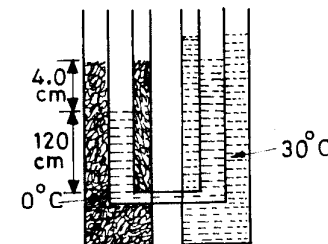
- A steel rod of diameter 1.0 cm, is clamped firmly at each end when its temperature is 25°C so that it cannot contract on cooling. The tension in the rod at 0°C is ($\alpha = 1 \times 10^{-5}/^\circ\text{C}$, $Y = 2 \times 10^{11} \text{ N/m}^2$)
 - 4000 N
 - 7000 N
 - 7400 N
 - 4700 N
- A glass bottle of capacity 50 cc at 0°C is filled with paraffin at 15°C. Given that the density of paraffin at 0°C is 0.82 g/cc, coefficient of expansion of paraffin for the range 0 to 15°C is 0.0009/°C and coefficient of linear expansion of glass is 0.000009/°C, the mass of paraffin in the bottle is
 - 420
 - 840
 - 16.80
 - 1680

- 40.5 g
 - 54.0 g
 - 50.4 g
 - 5.04 g
- A block of ice of mass $m = 10 \text{ kg}$ is moved back and forth over the flat horizontal surface of a large block of ice. Both blocks are at 0°C and the force that produces the back-and-forth motion acts only horizontally. The coefficient of friction between the two surfaces is 0.060. If 15.2 g of water is produced, the total distance travelled by the upper block relative to the lower is ($L_{\text{ice}} = 3.34 \times 10^5 \text{ J/kg}$)
 - 432 m
 - 863 m
 - 368 m
 - 216 m
 - Cooling water is supplied to a condenser at 20°C and leaves at 35°C. The supply of water is 500 kg per minute. The heat carried away in horse power is
 - 700
 - 150
 - 1000
 - 300
 - A liquid is found to cool from 50 to 45°C in 5 minutes and from 45 to 41.5°C in the next 5 minutes. The temperature of the surroundings is
 - 27.0°C
 - 40.3°C
 - 23.3°C
 - 33.3°C
 - A glass sinker has a mass M in air. When weighed in a liquid at temperature t_1 , the apparent mass is M_1 and when weighed in the same liquid at temperature t_2 , the apparent mass is M_2 . If the coefficient of cubical expansion of the glass is γ_g , then the real coefficient of expansion of the liquid is
 - $\gamma_s + \left(\frac{M_2 - M_1}{M - M_2} \right) \cdot \frac{1}{(t_2 - t_1)}$
 - $\gamma_s - \left(\frac{M_2 - M_1}{M - M_2} \right) \cdot \frac{1}{(t_2 - t_1)}$
 - $\gamma_s + \left(\frac{M - M_2}{M_2 - M_1} \right) \cdot \frac{1}{(t_2 - t_1)}$
 - $\gamma_s + \left(\frac{M_2 - M_1}{M_2 + M_1} \right) \cdot \frac{1}{(t_2 - t_1)}$
 - A mercury thermometer is to be made with glass tubing of internal bore 0.5 mm and the distance between the fixed points is to be 20 cm. The coefficient of expansion of mercury is 0.000180/°C and the coefficient of linear expansion of glass is 0.000009/°C. The internal volume of the bulb and stem below the lower fixed point is
 - 0.527 cc
 - 5.27 cc
 - 2.57 cc
 - 3.27 cc
 - A liquid takes 120 s to cool from 50°C to 40°C in a vessel in which the same volume of water takes 300 s to cool through the same range of temperature. If the mass of water is 100 g, the mass of liquid 85 g and the water equivalent of calorimeter 42 J/K, the specific heat of the liquid in J/kg. K is
 - 420
 - 840
 - 16.80
 - 1680
 - An apparatus to measure the density of a liquid consists of a glass bulb and a tube of uniform bore, and contains 150 g of mercury which extends into the tube at 0°C. The area of the cross-section of the bore is 0.8 mm² at 0°C. The density of mercury at 0°C is 13.6 g/cc, its coefficient of expansion is $1.82 \times 10^{-4}/^\circ\text{C}$, and the coefficient of linear expansion of glass is $1.1 \times 10^{-5}/^\circ\text{C}$. If the temperature is raised to 100°C, the rise of the mercury meniscus in the tube is
 - 10.54 cm
 - 30.50 cm
 - 20.54 cm
 - 25.54 cm

10. A 5.0 g bullet (specific heat of material of bullet = $128 \text{ J/kg } ^\circ\text{C}$) moving with a velocity of 200 m/s enters a sandbag and stops. If the entire kinetic energy of the bullet is changed into heat energy that is added to the bullet, then the rise in the temperature of the bullet is
 (a) 312.5°C (b) 156°C
 (c) 500°C (d) 624°C
11. A calorimeter (of water equivalent 50 g) contains 250 g of water and 50 g of ice at 0°C . 30 g of water at 80°C is added to it. The final condition of the system will be
 (a) the temperature of the system will be 4.2°C
 (b) the temperature of the system will still be 0°C and the entire ice will melt
 (c) the temperature will be 0°C and half of the ice will melt
 (d) the temperature will be 0°C and 20 g of ice will be left.
12. A 1000 W heater is used to generate steam (heat of vaporization at 100°C is $2.3 \times 10^6 \text{ J/kg}$). The quantity of water at 100°C that can be changed to steam in 5 minutes is (assume no heat losses)
 (a) 1.30 kg (b) 0.130 kg
 (c) 0.310 kg (d) 0.013 kg
13. 100 g of steam at 100°C is passed into 200 g of water and 20 g of ice at 0°C in a calorimeter whose water equivalent is 50 g. ($L_{\text{steam}} = 540 \text{ cal/g}$ and $L_{\text{ice}} = 80 \text{ cal/g}$). The observed result is
 (a) the temperature of the system becomes 169°C
 (b) half of the ice is melted and the temperature of the system remains 0°C
 (c) the temperature remains 100°C and 53 g of steam condenses
 (d) the temperature remains 100°C and the entire steam condenses.
14. 10 g of steam at 100°C is passed into 500 g of water and 100 g of ice at 0°C in a calorimeter whose water equivalent is 50 g. The final condition of the system is
 (a) the final temperature is 100°C and only 8 g of steam condenses
 (b) the final temperature is 100°C and all steam condenses
 (c) the final temperature is 0°C and all the ice melts
 (d) the final temperature is 0°C and 80 g of ice melts.
15. 5 kg of water is to be cooled from 373 to 273 K. The quantity of ice needed is
 (a) 62.5 kg (b) 6.25 kg
 (c) 5.62 kg (d) 0.625 kg
16. A solid substance is supplied heat at a constant rate and the variation of temperature with heat input is shown in the figure. Choose the correct statement:
 (a) AB and CD represent changes of phase from liquid to vapour and from solid to liquid respectively.
 (b) The latent heat of vaporization is half the latent heat of fusion.
 (c) The specific heat in the solid state is less than that in the liquid state.
 (d) The specific heat in the solid state is more than that in the liquid state.
17. A spherical body of diameter 4 cm is maintained at 500°C . Assuming that it behaves like a black body, the energy (in watts) radiated from the sphere is



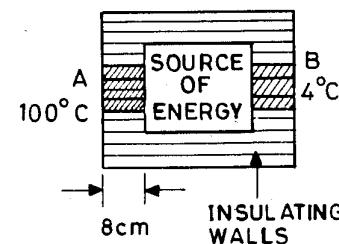
- (a) 205.12 (b) 10.26
 (c) 1026 (d) 102.6
18. The coefficients of linear expansion of steel and brass are $11 \times 10^{-6}/^\circ\text{C}$ and $19 \times 10^{-6}/^\circ\text{C}$ respectively. If their difference in lengths at all temperatures has to be kept constant at 30 cm, their lengths at 0°C should be
 (a) 71.25 cm and 41.25 cm (b) 82 cm and 52 cm
 (c) 92 cm and 62 cm (d) 62.25 cm and 32.25 cm
19. A circular hole in an aluminium plate ($\alpha = 23 \times 10^{-6}/^\circ\text{C}$) has a diameter of 4.0000 cm at 0°C . When the temperature of the plate is raised to 100°C , its diameter becomes
 (a) 4.0062 cm (b) 4.0034 cm
 (c) 4.0046 cm (d) 4.0092 cm
20. A steel rod is 4.000 cm in diameter at 30°C . A brass ring has an interior diameter of 3.992 cm at 30°C . In order that the ring just slides onto the steel rod, the common temperature of the two should be ($\alpha_{\text{steel}} = 11 \times 10^{-6}/^\circ\text{C}$ and $\alpha_{\text{brass}} = 19 \times 10^{-6}/^\circ\text{C}$)
 (a) 200°C (b) 350°C
 (c) 280°C (d) 300°C
21. When the temperature of a copper disc is raised by 100°C , its diameter increases by 0.18%. The coefficient of linear expansion of copper is
 (a) $36 \times 10^{-6}/^\circ\text{C}$ (b) $54 \times 10^{-6}/^\circ\text{C}$
 (c) $18 \times 10^{-6}/^\circ\text{C}$ (d) $36 \times 10^{-6}/^\circ\text{C}$
22. An aluminium sphere of 20.0 cm diameter is heated from 0 to 100°C . Its volume changes by ($\alpha_{\text{Al}} = 23 \times 10^{-6}/^\circ\text{C}$)
 (a) 2.89 cc (b) 28.9 cc
 (c) 29.8 cc (d) 9.28 cc
23. The temperature of a solid object whose rotational inertia is I and coefficient of linear expansion is α , is increased by ΔT . The change in its rotational inertia is
 (a) $2\alpha \Delta T$ (b) $\frac{1}{2} \alpha \Delta T$
 (c) $2\alpha I \Delta T$ (d) $\frac{1}{2} \alpha I \Delta T$
24. The temperature of a physical pendulum whose time period is t , is raised by ΔT . The change in its time period is
 (a) $\frac{1}{2} \alpha t \Delta T$ (b) $2\alpha t \Delta T$
 (c) $\frac{1}{2} \alpha \Delta T$ (d) $2\alpha \Delta T$
25. Two vertical glass tubes filled with a liquid are connected by a capillary tube as shown in the figure. The tube on the left is put in an ice bath at 0°C while the one on the right is kept at 30°C in a water bath. The difference in the levels of the liquid in the two tubes is 4.0 cm while the height of the liquid column at 0°C is 120 cm. The coefficient of volume expansion of the liquid is



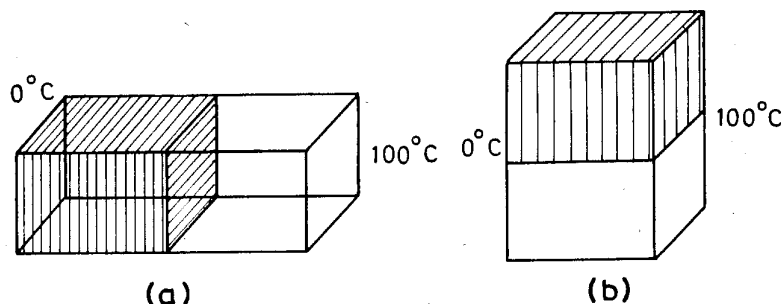
- (a) $22 \times 10^{-4}^\circ\text{C}$ (b) $1.1 \times 10^{-4}^\circ\text{C}$
(c) $11 \times 10^{-4}^\circ\text{C}$ (d) $2.2 \times 10^{-4}^\circ\text{C}$
26. 300 g of water at 25°C is added to 100 g of ice at 0°C . The final temperature of the mixture is
(a) 1.5°C (b) 2.5°C
(c) -2.5°C (d) 0°C
27. The earth receives on its surface radiation from the sun at the rate of 1400 W/m^2 . The distance of the sun from the surface of the earth is $1.5 \times 10^{11} \text{ m}$ and the radius of the sun is $7.0 \times 10^8 \text{ m}$. Treating the sun as a black body, it follows from the above data that the temperature of the sun's surface is
(a) 5000 K (b) 6000 K
(c) 5803 K (d) 8503 K
28. Two rods of different materials having coefficients of thermal expansion α_1 and α_2 and Young's Moduli Y_1 and Y_2 respectively are fixed between two rigid supports. The rods are heated so that they undergo the same increase of temperature. The rods do not bend. If $\alpha_1/\alpha_2 = 2/3$, the thermal stresses developed in the two rods are equal, provided $Y_1 : Y_2$ is equal to
(a) 2 : 3 (b) 1 : 1
(c) 3 : 2 (d) 4 : 9
29. One mole of a monatomic gas ($\gamma = 5/3$) is mixed with one mole of a diatomic gas ($\gamma = 7/5$). The value of γ of the mixture is
(a) 1.40 (b) 1.50
(c) 1.53 (d) 3.07
30. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2 . The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in a steady state. The effective thermal conductivity of the system is
(a) $K_1 + K_2$ (b) $\frac{K_1 \cdot K_2}{K_1 + K_2}$
(c) $\frac{K_1 + 3K_2}{4}$ (d) $\frac{3K_1 + K_2}{4}$
31. During an experiment, an ideal gas is found to obey an additional law— $VP^2 = \text{const}$. The gas is initially at temperature T and has volume V . When it expands to a volume $2V$, the temperature becomes
(a) $\sqrt{2} T$ (b) $\frac{1}{2} T$
(c) $\sqrt{3} T$ (d) $2T$
- 32.* The specific heat at constant pressure of an ideal gas, $C_p = 5R/2$. The gas is kept in a closed vessel of volume 0.0083 m^3 at 300 K and a pressure of $1.6 \times 10^6 \text{ N/m}^2$. $2.49 \times 10^4 \text{ J}$ of heat energy is supplied to the gas. The final temperature and the pressure respectively are
(a) 567.2 K and $6.3 \times 10^6 \text{ N/m}^2$
(b) 675.2 K and $3.6 \times 10^6 \text{ N/m}^2$
(c) 275.2 K and $2.3 \times 10^6 \text{ N/m}^2$
(d) 465.6 K and $4.2 \times 10^6 \text{ N/m}^2$
33. 70 calories of heat are required to raise the temperature of two moles of an ideal gas at constant pressure from 30 to 35°C . The amount of heat required (in calories) to raise the temperature of the same gas through the same range (30 to 35°C) at constant volume is
(a) 30 (b) 50
(c) 70 (d) 90
34. Steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C till the temperature of the calorimeter and its contents rises to 80°C . The mass (in kilograms) of the steam condensed is
(a) 0.130 (b) 0.650
(c) 0.065 (d) 0.135
35. Two bulbs of equal volume are connected by a narrow tube and are filled with a gas at 0°C and a pressure of 76 cm of Hg . One of the bulbs is then placed in melting ice and the other is placed in a water bath at 62°C . The new value of the pressure inside the bulb (volume of the connecting tube is negligible)
(a) 100.0 cm (b) 86.50 cm
(c) 83.75 cm (d) 90.4 cm
36. One mole of a monatomic gas is mixed with one mole of a diatomic gas. The molar specific heat of the mixture at constant volume is
(a) $0.16 \times 10^3 \text{ J/K mole}$ (b) $1.6 \times 10^3 \text{ J/K mole}$
(c) $6.16 \times 10^3 \text{ J/K mole}$ (d) $16.6 \times 10^3 \text{ J/K mole}$
37. A lead bullet just melts when stopped by an obstacle. Assuming that 25% of heat is absorbed by the obstacle and given that the initial temperature of the bullet is 27°C , m_p of lead = 327°C , specific heat of lead = $0.03 \text{ cal/g}^\circ\text{C}$ and latent heat of lead = 6 cal/g , and $J = 4.2 \text{ J/cal}$, the velocity of the bullet before it hits the obstacle is
(a) $5.2 \times 10^4 \text{ cm/s}$ (b) $4.1 \times 10^4 \text{ cm/s}$
(c) $8.2 \times 10^4 \text{ cm/s}$ (d) $6.2 \times 10^4 \text{ cm/s}$
38. A wall has two layers A and B, each made of different material. Both the layers have the same thickness. The thermal conductivity for A is twice that of B and, under thermal equilibrium, the temperature difference across the wall is 36°C . The temperature difference across the layer A is
(a) 6°C (b) 12°C
(c) 18°C (d) 24°C
- 39.* A composite rod is made by joining a copper rod end to end with a second rod of different material but of the same cross-section. At 25°C , the composite rod is 1 m in length, of which the copper rod constitutes 30 cm . At 125°C , the length of the composite rod increases by 1.91 mm . When the composite rod is held between rigid supports and is not allowed to expand, it is found that the lengths of the two constituents do not change with the rise of temperature. [Given: $\rho_{\text{cu}} = 9 \times 10^3 \text{ kg/m}^3$, $Y_{\text{cu}} = 1.3 \times 10^{11} \text{ N/m}^2$ and $\alpha_{\text{cu}} = 1.7 \times 10^{-5}^\circ\text{C}^{-1}$]
Young's modulus and the coefficient of expansion of the second rod respectively are
(a) $1.105 \times 10^{10} \text{ N/m}^2$ and $2 \times 10^{-5}^\circ\text{C}^{-1}$
(b) $1.4 \times 10^{12} \text{ N/m}^2$ and $2 \times 10^{-6}^\circ\text{C}^{-1}$
(c) $1.105 \times 10^{11} \text{ N/m}^2$ and $2 \times 10^{-5}^\circ\text{C}^{-1}$
(d) $2 \times 10^{11} \text{ N/m}^2$ and $1 \times 10^{-6}^\circ\text{C}^{-1}$
- 40.* A room is maintained at 20°C by a heater of resistance 20Ω connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm (K for glass is $0.2 \text{ cal/m}^\circ\text{C} \cdot \text{s}$). The temperature outside is
(a) 20°C (b) 10°C
(c) 12.2°C (d) 15.2°C

41. A brass scale of a barometer gives correct reading at 0°C . $\alpha_{\text{brass}} = 0.00002/^\circ\text{C}$. The barometer reads 75 cm at 27°C . The atmospheric pressure at 0°C is
 (a) 75.20 cm (b) 75.62 cm
 (c) 75.04 cm (d) 75.16 cm
42. 5 g of water at 30°C and 5 g of ice at -20°C are mixed together in a calorimeter. The water equivalent of calorimeter is negligible and specific heat and latent heat of ice are $0.5 \text{ cal/g } ^\circ\text{C}$ and 80 cal/g respectively. The final temperature of the mixture is
 (a) 0°C (b) -8°C
 (c) -4°C (d) 2°C
43. A piece of metal weighs 46 g in air. When it is immersed in a liquid of specific gravity 1.24 at 27°C , it weighs 30 g. When the temperature of the liquid is raised to 42°C , the metal piece weighs 30.5 g. If the specific gravity of the liquid at 42°C is 1.20, the coefficient of linear expansion of the metal is
 (a) $1.4 \times 10^{-5}/^\circ\text{C}$ (b) $2.3 \times 10^{-5}/^\circ\text{C}$
 (c) $4.3 \times 10^{-5}/^\circ\text{C}$ (d) $3.4 \times 10^{-5}/^\circ\text{C}$
44. A 1-litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. If $\alpha_g = 9 \times 10^{-6}/^\circ\text{C}$ and $\gamma_{\text{Hg}} = 1.8 \times 10^{-4}/^\circ\text{C}$, the volume of mercury in the flask is
 (a) 300 cc (b) 100 cc
 (c) 150 cc (d) 200 cc
45. A 'cold box' in the shape of a cube of edge 50 cm is made of 'thermocool' material 4.0 cm thick. If the outside temperature is 30°C , the quantity of ice that will melt each hour inside the 'cold box' is (the thermal conductivity of the thermocool material is 0.050 W/m.K)
 (a) 1.52 kg (b) 0.250 kg
 (c) 2.520 kg (d) 0.512 kg
46. The body of an unclothed person has a surface area of 1.50 m^2 , and an emissivity of 0.80. His skin temperature is 37°C . He stands in an air-conditioned room where the temperature is maintained at 17°C . The amount of heat he loses per minute is ($\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$)
 (a) 8.8 kJ (b) 4.4 kJ
 (c) 2.2 kJ (d) 3.3 kJ
47. A sphere of radius 4 cm acts like a black body and is in equilibrium with its surroundings. It absorbs 40 kW of power radiated to it from the surroundings. The temperature of the sphere in kelvin is
 (a) 2600 (b) 2400
 (c) 3000 (d) 2000
48. A 3 cm thick metal plate ($K = 100 \text{ W/k.m}$) is sealed with a glass plate ($K = 0.80 \text{ W/K.m}$) and both have the same area. The exposed face of the metal plate is at 100°C while the exposed face of glass plate is at 10°C . If the temperature of the interface is 70°C , the thickness of the glass plate is
 (a) 0.80 mm (b) 1.5 cm
 (c) 0.48 mm (d) 0.64 mm
49. Water contained in a closed thin-walled cylindrical copper tank, of radius 30 cm and height 1 m, is maintained at 60°C by means of an electric heater immersed in water, the outside temperature being 20°C . The tank's outer curved surface is covered with 1 cm thick felt ($K_{\text{felt}} = 9 \times 10^{-5} \text{ cal/s. cm. } ^\circ\text{C}$). Neglect all other losses. The wattage of the heater is
 (a) 100 (b) 1000
 (c) 220 (d) 284

- 50.* A brick wall 12 cm thick is covered on one side with a layer of cork 3 cm thick and the difference in temperature between the exposed surfaces is 20°C . Given that the thermal conductivities of cork and brick are 1.0×10^{-4} and 12×10^{-4} respectively (cgs units), the heat conducted per square metre per hour is
 (a) 24000 cal. (b) 12000 cal.
 (c) 18000 cal. (d) 72000 cal.
- 51.* If each square centimetre of the sun's surface radiates energy at the rate of $1.5 \times 10^3 \text{ cal s}^{-1} \text{ cm}^{-2}$ and Stefan's constant is $5.7 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$, the temperature of the sun's surface in degrees Celsius is (assume Stefan's Law applied to the radiation)
 (a) 5924 (b) 2594
 (c) 9425 (d) 5492
- 52.* The sun's rays are focussed by a concave mirror of diameter 12 cm fixed with its axis towards the sun onto a copper calorimeter, where they are absorbed. If the thermal capacity of the calorimeter and its contents is $59 \text{ cal/}^\circ\text{C}$ and the temperature rises by 8°C in 2 minutes, the heat received in 1 minute by a square metre of the earth's surface when the rays are incident normally is
 (a) 20860 cal (b) 25540 cal
 (c) 10430 cal (d) 51180 cal
53. An aluminium container of mass 100 g contains 200 g of ice at -20°C . Heat is added to the system at the rate of 100 cal/s. The temperature of the system after 4 minutes will be (specific heat of ice = 0.5 and $L_{\text{ice}} = 80 \text{ cal/g}$, specific heat of Al = $0.2 \text{ cal/g/}^\circ\text{C}$)
 (a) 40.5°C (b) 25.5°C
 (c) 30.3°C (d) 35.0°C
54. A closed cubical box is made of a perfectly insulating material walls of thickness 8 cm and the only way for heat to enter or leave the box is through two solid metallic cylindrical plugs, each of cross-sectional area 12 cm^2 and length 8 cm, fixed in the opposite walls of the box. The outer surface A of one plug is maintained at 100°C while the outer surface B of the plug is maintained at 4°C . The thermal conductivity of the material of the plug is $0.5 \text{ cal/}^\circ\text{C/cm}$. A source of energy generating 36 cal/s is enclosed inside the box. Assuming the temperature to be the same at all points on the inner surface, the equilibrium temperature of the inner surface of the box is
 (a) 62°C (b) 46°C
 (c) 76°C (d) 52°C
55. Two 50-g ice cubes are dropped into 200 g of water in a glass. If the water was initially at a temperature of 25°C and the temperature of ice is -15°C , the final temperature of water is (specific heat of ice = $0.50 \text{ cal/g/}^\circ\text{C}$ and $L = 80 \text{ cal/g}$).
 (a) 0°C (b) -12.5°C
 (c) -125°C (d) 5°C
56. A chamber containing a gas was evacuated till the vacuum attained was 10^{-14} m of Hg. If the temperature of the chamber was 30°C , the number of molecules that remain in it per cubic metre is



- (a) 3.2×10^{11} (b) 3.2×10^{12}
 (c) 2.3×10^{12} (d) 2.3×10^{11}
57. Specific heat of helium at constant volume is 3129 J/kg/K. Given that one gram molecule of the gas occupies 22.42 litres at NTP, the specific heat at constant pressure is
 (a) 2797 J/kg/K (b) 2452 J/kg/K
 (c) 5243 J/kg/K (d) 7279 J/kg/K
58. A gas is suddenly compressed to half its original volume. If the original temperature of the gas is 27°C and the ratio of two specific heats for the gas is 1.5, the temperature of the gas rises by
 (a) 54°C (b) 78°C
 (c) 124°C (d) 65°C
- 59.* A copper ring has a diameter of exactly 25.0000 mm at its temperature of 0°C . An aluminium sphere has a diameter of exactly 25.0500 mm at its temperature of 100°C . The sphere is placed on top of the ring and the two are allowed to come to thermal equilibrium, no heat being lost to the surroundings. The sphere just passes through the ring at the equilibrium temperature. The ratio of the mass of the sphere to the mass of the ring is
 (Given: $\alpha_{\text{Cu}} = 17 \times 10^{-6}/^\circ\text{C}$, $\alpha_{\text{Al}} = 23 \times 10^{-6}/^\circ\text{C}$, specific heat of Cu = 0.0923 cal/g $^\circ\text{C}$ and specific heat of Al = 0.215 cal/g $^\circ\text{C}$)
 (a) 1/5 (b) 23/108
 (c) 23/54 (d) 216/23
60. Two identical square rods of metal are welded end to end as shown in Fig. (a). Assume that 10 cal of heat flows through the rods in 2 min. Now the rods are welded as shown in Fig. (b). The time it would take for 10 cal to flow through the rods now, is



- (a) -0.75 min (b) 0.5 min
 (c) 1.5 min (d) 1 min
61. Three metal rods made of copper, aluminium and brass, each 20 cm long and 4 cm in diameter, are placed end to end with aluminium between the other two. The free ends of copper and brass are maintained at 100 and 0°C respectively. Assume that the thermal conductivity of copper is twice that of aluminium and four times that of brass. The equilibrium temperatures of the copper-aluminium and aluminium-brass junctions are respectively
 (a) 68°C and 75°C (b) 75°C and 68°C
 (c) 57°C and 86°C (d) 86°C and 57°C
62. In the above problem, if the lengths of the rods are doubled and the area of cross-section is reduced to half, the junction temperatures of the Cu-Al and Al-brass junctions will be

- (a) one-fourth of what they were (b) half of what they were
 (c) the same as they were (d) twice what they were
63. 1 g of air at NTP is compressed adiabatically to half its volume. Given that the density of air is 0.00129 g/cc and γ for air is 1.4, the work done is
 (a) -6.4 J (b) 12.8 J
 (c) -9.6 J (d) -12.8 J
64. 1 kg of a gas at 250°C expands adiabatically and its temperature falls by 125°C . The work done during expansion is 9.2 J. The specific heat of the gas at constant volume is
 (a) 0.002715 (b) 0.01752
 (c) 0.02125 (d) 0.7152
65. 1 litre of hydrogen at 0°C is suddenly compressed to half its volume. The ratio of the two specific heats for hydrogen is 1.4. The change in temperature of the gas is
 (a) 54°C (b) 68.6°C
 (c) 78.2°C (d) 87.2°C
66. A pendulum clock has an iron pendulum 1 m long ($\alpha_{\text{iron}} = 0.000010/^\circ\text{C}$). If the temperature rises by 10°C , the number of seconds the clock loses per day due to this change of temperature will be
 (a) 8 (b) 10
 (c) 4 (d) 5
67. A quartz beaker of volume 1 litre contains an 800 cc block of copper and 200 cc of alcohol. The temperature of the beaker and its contents is raised by 60°C . The amount of alcohol that overflows is (coeff. of cubical exp. of alcohol and copper respectively are $75 \times 10^{-5}/^\circ\text{C}$ and $5.1 \times 10^{-5}/^\circ\text{C}$)
 (a) 11.4 cc (b) 16.5 cc
 (c) 21.5 cc (d) 24.5 cc
68. The approximate volume occupied by 1 g of carbon dioxide at 30°C and 3 atmospheres pressure is
 (a) 300 cc (b) 200 cc
 (c) 100 cc (d) 500 cc
69. A wooden box is made of a 2 cm thick plank and has inner dimensions of 100 cm $\times$ 60 cm $\times$ 60 cm. The external temperature is 25°C . If the box is used as an ice box, the rate at which ice will melt is ($K_{\text{wood}} = 0.0004$ cal/cm/s $^\circ\text{C}$ and $L_{\text{ice}} = 80$ cal/g)
 (a) 2.95 g/s (b) 3.95 g/s
 (c) 2.02 g/s (d) 1.95 g/s
70. A boiler is made of a copper plate 2.4 mm thick with an inside coating of a 0.2 mm thick layer of tin. The surface area exposed to gases at 700°C is 400 cm 2 . The maximum amount of steam that could be generated per hour at atmospheric pressure is ($K_{\text{Cu}} = 0.9$ and $K_{\text{tin}} = 0.15$ cal/cm/s $^\circ\text{C}$ and $L_{\text{steam}} = 540$ cal/g)
 (a) 5000 kg (b) 1000 kg
 (c) 4000 kg (d) 200 kg
71. The diameter of a steel ball is 9 cm. This is 0.012 cm too large to permit it to fit into a hole in a brass plate when the temperature is 20°C . In order to just fit the ball in the hole, the temperature of both should be [$\alpha_{\text{steel}} = 11 \times 10^{-6}/^\circ\text{C}$ and $\alpha_{\text{brass}} = 25 \times 10^{-6}/^\circ\text{C}$]
 (a) 95°C (b) 180°C
 (c) 300°C (d) 250°C
72. A 2000 cc aluminium tank is filled with water at 80°C . When the tank and its contents are cooled down to 10°C , the amount of water that can be added is ($\gamma_{\text{Al}} = 7.2 \times 10^{-5}/^\circ\text{C}$, $\gamma_{\text{water}} = 34 \times 10^{-5}/^\circ\text{C}$)

- (a) 28.0 cc (b) 37.5 cc
(c) 82.0 cc (d) 42.2 cc

73. An insulated tub is divided into two sections by a watertight partition of dimensions 20 cm × 40 cm made of copper 3 mm thick. On one side is water which is boiling and on the other side is a mixture of 2 kg of crushed ice being constantly stirred in 5 kg of water. All the ice will melt in ($K_{Cu} = 0.92 \text{ cal s}^{-1} \text{ cm}^{-1} \text{ }^\circ\text{C}^{-1}$)

- (a) 1.32 s (b) 1.6 s
(c) 0.33 s (d) 0.65 s

74. A jar of diameter 10 cm and height 20 cm is made of glass 2 mm thick. The lid of the jar is of copper 1 mm thick and the water in the jar is accidentally frozen. To melt the ice, the jar is put into a tank of water at 20°C. The time it takes is (Given: $\rho_{ice} = 0.92 \text{ g/cc}$, $K_g = 0.0025 \text{ cgs units}$, $K_{Cu} = 0.92 \text{ cgs units}$)

- (a) 30.0 s (b) 40.0 s
(c) 8.1 s (d) 16.2 s

75. A lead bullet of unknown mass is fired at a speed of 200 m/s into a tree in which it stops. Assuming that two-thirds of the heat produced goes into the bullet and one-third into the wood, the temperature of the bullet rises by [Given: specific heat of lead = 0.130 J/g°C]

- (a) 106°C (b) 140°C
(c) 200°C (d) 180°C

76. A 100 W lamp is immersed in an insulated container holding 500 g of alcohol at 20°C. The time it takes to warm the alcohol to 50°C is (specific heat of alcohol = 0.572 cal/g °C)

- (a) 150 s (b) 354 s
(c) 100 s (d) 200 s

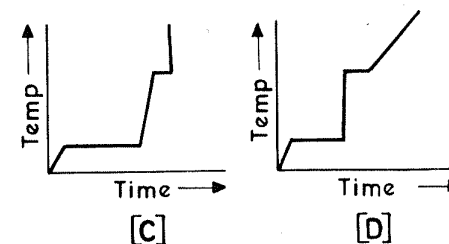
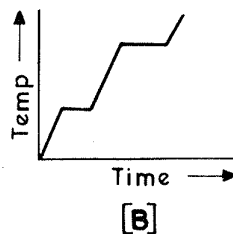
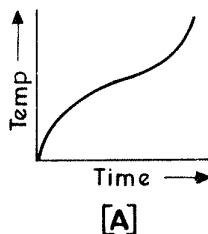
77. The melting point of lead is 327°C. A lead bullet is fired into a target and, on penetration, it melts completely. If half of the heat produced is absorbed by the bullet, the speed of the bullet is (initial temperature of bullet = 27°C, specific heat of lead = 130 J/kg°C, and $L = 2.5 \times 10^4 \text{ J/kg}$)

- (a) 500 m/s (b) 300 m/s
(c) 506 m/s (d) 188 m/s

78. Sunrays are allowed to fall on a lens of diameter 200 cm. They are then brought to focus on a calorimeter containing 20 g of ice. If the absorption by the lens is negligible, the time required to melt all the ice is ($L = 80 \text{ cal/g}$, amount of heat from the sun received on 1 cm² at the earth's surface = 1.9 cal/min)

- (a) 6.4 min (b) 3.2 min
(c) 7.2 min (d) 2.7 min

79. Ice at -25°C is crushed and placed in a container surrounded by a heating coil which supplies heat at uniform rate. The container is insulated so that heat does not reach the ice from any other source. The correct plot of temperature versus time is



80. A certain incandescent light bulb operates at 3000 K. The total surface area of the filament is 0.05 cm² and the emissivity is 0.3. The electric power that must be supplied to the filament is

- (a) 6.89 W (b) 8.69 W
(c) 9.68 W (d) 8.96 W

81. A small blackened solid copper sphere of radius 2 cm is placed in an evacuated enclosure whose walls are kept at 100°C. In order to keep the temperature of the enclosure constant at 127°C, the rate at which energy must be supplied to the sphere is

- (a) 3.56 W (b) 2.87 W
(c) 1.78 W (d) 0.89 W

82. A wire undergoes an infinitesimal change from an initial equilibrium state to a final equilibrium state. (Change in length = dL and change in temperature = $d\theta$). The change in the tension of the wire due to this change is dT , given by (where α is coefficient of linear expansion, A the area of cross-section, Y the Young's Modulus and L the original length of the wire)

(a) $\frac{AY}{L} dL + \alpha AY d\theta$ (b) $AYdL + L\alpha AY d\theta$

(c) $\frac{AYdL}{L} - \alpha AY d\theta$ (d) $\alpha AY d\theta - \frac{AYdL}{L}$

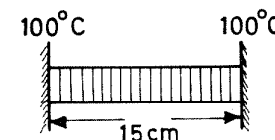
83. A metal wire of cross-sectional area 0.0085 cm² under a tension of 20 N and at a temperature of 20°C is stretched between two rigid supports 1.2 m apart. If the temperature is reduced to 8°C, the final tension in the wire is ($\alpha = 1.5 \times 10^{-5}/^\circ\text{C}$ and $Y = 2.0 \times 10^{11} \text{ N/m}^2$)

- (a) 6.30 N (b) 63.0 N
(c) 36.0 N (d) 50.6 N

84. The fundamental frequency of vibration of a wire of length L , mass M and tension T is given by $f_1 = (1/2L) \sqrt{TL/m}$. Refer to problem 83, the ratio of frequencies at temperatures 20°C and 8°C shall be

- (a) 3/4 (b) $\frac{39}{100}$
(c) $\frac{63}{100}$ (d) $\frac{21}{50}$

85. A brass rod with diameter 2 cm and length 15 cm at 20°C just fits between two parallel rigid surfaces which are at 100°C ($\alpha_{brass} = 1.90 \times 10^{-5}/^\circ\text{C}$, $Y_{brass} = 9.0 \times 10^{10} \text{ N/m}^2$)



- The compressive force acting on the rod is
 (a) 4.30×10^4 N (b) 3.40×10^4 N
 (c) 500 N (d) 3000 N
86. A metallic rod of length L and diameter D is fixed between two rigid supports as in problem 85 and a temperature difference of $\Delta\theta$ is responsible for causing a compressive force F on the rod. The length of the rod is doubled. Keeping all other factors the same, the compressive force becomes
 (a) $2F$ (b) $F/2$
 (c) F (d) $F/4$
87. A metal block of mass 2 kg ($\rho = 5 \times 10^3$ kg/m³) is suspended from an ideal spring of force constant 25 N/m. The spring-block system is immersed in 500 g of water contained in a vessel such that the block is at a height h from the bottom of the vessel. The system is released so as to sink to the bottom, resulting in a rise of temperature of 0.004°C of water. The height h through which the block went down is (given: specific heat of the block = 250 J/kg $^\circ\text{C}$ and that of water = 4200 J/kg $^\circ\text{C}$, heat capacity of vessel and spring are negligible. Take $g = 10$ m/s²).
 (a) 0.35 m (b) 0.16 m
 (c) 1.60 m (d) 0.65 m
88. A hole is drilled through a steel plate at room temperature (20°C) and its diameter is 2.0 cm. The temperature is then raised to 160°C , and the hole becomes larger by a fraction $\Delta r/r$ whose value is ($\alpha_{\text{steel}} = 1 \times 10^{-5}/^\circ\text{C}$ and r is the radius of the hole)
 (a) 2.8×10^{-4} (b) 14×10^{-4}
 (c) 28×10^{-4} (d) 1.4×10^{-4}
89. A cylindrical steel plug is inserted into a circular hole of diameter 2.60 cm in a brass plate. When the plug and the plate are at a temperature of 20°C , the diameter of the plug is 0.010 mm smaller than that of the hole. The temperature at which the plug will just fit in is (given: $\alpha_{\text{steel}} = 11 \times 10^{-6}/^\circ\text{C}$ and $\alpha_{\text{brass}} = 19 \times 10^{-6}/^\circ\text{C}$)
 (a) -48°C (b) -20°C
 (c) 10°C (d) -4°C
90. A tungsten wire with a diameter of 0.20 mm is stretched until the tension is 50 N. The wire is then clamped to a stout aluminium rod when both are at a temperature of 20°C . The tension in the wire when both are brought to a temperature of 150°C is ($Y_{\text{Tungsten}} = 34 \times 10^{10}$ N/m², $\alpha_T = 4.5 \times 10^{-6}/^\circ\text{C}$)
 (a) 85.0 N (b) 57.5 N
 (c) 100 N (d) 56.2 N
91. A metal block is made from a mixture of 2.4 kg of aluminium 1.6 kg of brass and 0.8 kg of copper. The amount of heat required to raise the temperature of this block from 20° to 80°C is the (specific heats of aluminium, brass and copper are 0.216, 0.0917 and 0.0931 cal/kg $^\circ\text{C}$ respectively)
 (a) 88.8 cal (b) 44.4 cal
 (c) 22.2 cal (d) 33.3 cal
92. An iron sheet with an area of 1 m² and a thickness of 5 mm is cooled from 600 to 250°C by sprayed water delivered at 25°C . Assume that water is converted to steam at 100°C upon contact with the sheet. If the specific heat of iron is 0.145 cal/g $^\circ\text{C}$, the amount of water required to cool the sheet is ($\rho_{\text{iron}} = 7.8 \times 10^3$ kg/m³ and $L_{\text{steam}} = 540$ cal/g)
 (a) 3.2 kg (b) 6.2 kg
 (c) 5.4 kg (d) 9.4 kg
93. Water is heated from 10 to 90°C in a residential hot water heater at a rate of 70 litres per minute. Natural gas with a density of 1.2 kg/m³ is used in the heater, which has a transfer efficiency of 32%. The gas consumption rate in cubic metres per hour is (heat of combustion for natural gas is 8400 cal/kg)
 (a) 223.2 (b) 111.6
 (c) 61.10 (d) 21.6
94. A closed cubical box with sides of length 0.50 m is constructed of wood of thickness of 2.0 cm. When the outside temperature is 20.0°C , an electric heater inside the box must supply 110 W of power to maintain the temperature of the air inside the box at a uniform 85°C . The average thermal conductivity of wood is
 (a) 45×10^{-3} cal/m.s. $^\circ\text{C}$ (b) 54×10^{-3} cal/m.s. $^\circ\text{C}$
 (c) 45×10^{-4} cal/m.s. $^\circ\text{C}$ (d) 54×10^{-4} cal/m.s. $^\circ\text{C}$
95. A very thin spherical shell of silver has a radius of 5.00 cm at a temperature of 5°C . If the shell is heated until it has a uniform temperature of 35°C , the change in radius is ($\alpha_{\text{Ag}} = 1.9 \times 10^{-5}/^\circ\text{C}$)
 (a) 8.2×10^{-4} cm (b) 2.85×10^{-3} cm
 (c) 0.28×10^{-3} cm (d) 5.2×10^{-4} cm
96. A peg of whiskey of volume 120 ml (half ethyl alcohol and half water by mass) originally at a temperature of 25°C is cooled by adding 20 g of ice at 0°C . If all the ice melts, the final temperature of the drink is (assume there is no heat loss, density of whiskey = 0.919 g/cc, specific heat of alcohol = 0.572 cal/g $^\circ\text{C}$)
 (a) 5.3°C (b) 3.5°C
 (c) 0°C (d) 6.2°C
97. A completely enclosed calorimeter made of copper with a wall thickness of 1.0 mm has a surface area of 335 cm². Mechanical work is delivered to the water at a rate of $1/4$ hp by means of stirring system. When equilibrium is achieved, the water in the calorimeter has a uniform temperature. Given that the thermal conductivity of copper is 0.092 cal/s.m. $^\circ\text{C}$. The difference between the water temperature and the outside temperature is
 (a) 7.2°C (b) 4.14°C
 (c) 14.4°C (d) 8.6°C
98. A spherical shell of copper is completely filled with a liquid at a temperature $t^\circ\text{C}$. The bulk modulus of the liquid is K and coefficient of volume expansion is γ . If the temperature of the liquid and the shell is increased by ΔT , then the outward pressure dp on the shell that results from the temperature increase is given by (α is the coefficient of linear expansion of the material of the shell)
 (a) $K(\gamma - 3\alpha)\Delta T$ (b) $K(3\alpha - \gamma)\Delta T$
 (c) $3\alpha(K - \gamma)\Delta T$ (d) $\gamma(3\alpha - K)\Delta T$
99. A lead bullet of weight 45 g is fired with a velocity of 250 m/s at a steel plate. The bullet is stopped completely by the plate, with nine-tenths of its initial kinetic energy appearing as heat in the bullet. The temperature increase of the bullet (specific heat of lead = 0.03 cal/g $^\circ\text{C}$)
 (a) 330°C (b) 220°C
 (c) 400°C (d) 180°C
100. Two models of a windowpane are made. In one model, two identical glass panes of thickness 3 mm are separated with an air gap of 3 mm. This composite system is fixed in the window of a room. The other model consists of a single glass pane of thickness 3 mm, the temperature difference being the same as for the first model. The ratio of the heat flow for the double pane to that for the single pane is ($K_{\text{glass}} = 2.5 \times 10^{-4}$ cal/s.m. $^\circ\text{C}$ and $K_{\text{air}} = 6.2 \times 10^{-6}$ cal/s.m. $^\circ\text{C}$)

(a) 1/20
(c) 1/100

(b) 1/70
(d) 1/50

Answers

- | | | | | |
|---------|---------|---------|---------|----------|
| 1. (a) | 2. (a) | 3. (b) | 4. (a) | 5. (d) |
| 6. (a) | 7. (c) | 8. (d) | 9. (c) | 10. (b) |
| 11. (d) | 12. (b) | 13. (c) | 14. (d) | 15. (b) |
| 16. (c) | 17. (d) | 18. (a) | 19. (d) | 20. (c) |
| 21. (c) | 22. (b) | 23. (c) | 24. (a) | 25. (c) |
| 26. (d) | 27. (c) | 28. (c) | 29. (c) | 30. (c) |
| 31. (a) | 32. (b) | 33. (b) | 34. (a) | 35. (c) |
| 36. (d) | 37. (b) | 38. (b) | 39. (c) | 40. (d) |
| 41. (c) | 42. (a) | 43. (d) | 44. (c) | 45. (d) |
| 46. (a) | 47. (b) | 48. (c) | 49. (d) | 50. (c) |
| 51. (d) | 52. (a) | 53. (b) | 54. (c) | 55. (a) |
| 56. (a) | 57. (d) | 58. (c) | 59. (c) | 60. (b) |
| 61. (d) | 62. (c) | 63. (a) | 64. (b) | 65. (d) |
| 66. (c) | 67. (a) | 68. (b) | 69. (d) | 70. (c) |
| 71. (a) | 72. (b) | 73. (d) | 74. (c) | 75. (a) |
| 76. (b) | 77. (c) | 78. (d) | 79. (b) | 80. (a) |
| 81. (c) | 82. (c) | 83. (d) | 84. (c) | 85. (a) |
| 86. (c) | 87. (d) | 88. (b) | 89. (a) | 90. (d) |
| 91. (b) | 92. (a) | 93. (b) | 94. (d) | 95. (b) |
| 96. (a) | 97. (c) | 98. (a) | 99. (b) | 100. (d) |

Note: Problems marked with an asterisk should take about five minutes. Rest should take about three minutes each.

13**Kinetic Theory of Gases****Ideal Gases**

An ideal or perfect gas is one that obeys the ideal gas law given below. At low to moderate pressures and at temperatures not too low, some common gases such as air, nitrogen, oxygen, helium, hydrogen and neon are considered to be ideal.

Ideal Gas Law

The absolute pressure p of n kilomoles of gas contained in a volume V is related to the absolute temperature T by the equation

$$pV = nRT$$

where R is the same for all gases and has a value of $8314 \text{ J/kmol} \cdot \text{K} = 1.986 \text{ cal/mol} \cdot \text{K}$. This is called the universal gas constant. If the volume of a gas contains a mass m of gas that has a molecular or atomic mass M , then $n = m/M$. The ideal gas law can be expressed in terms of three variables, P , V and T , of which two are held constant.

Boyle's Law (n , T constant) : $pV = \text{constant}$

Charles's Law (n , p constant) : $\frac{V}{T} = \text{constant}$

Gay-Lussac's Law (n , V constant) : $\frac{P}{T} = \text{constant}$

Absolute Zero

The unique temperature at which p and V would reach zero is called absolute zero.

NTP or STP (Normal or Standard Temperature and Pressure)

$$T = 273.15 \text{ K} = 0^\circ\text{C}, p = 1.013 \times 10^5 \text{ Pa or N/m}^2 = 1 \text{ Atm.}$$

Dalton's Law of Partial Pressure

The total pressure of a mixture of gases (ideal and nonreactive) is equal to the sum of the individual partial pressures, each obtained as if it were the only gas in the volume V .

Gas-law problems involving a change of condition from (p_1, V_1, T_1) to (p_2, V_2, T_2) are solved by expressing the gas law as

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \text{ (at constant } n \text{)}$$

Microscopic Definition of an Ideal Gas

From the microscopic point of view, an ideal gas is defined by making the following assumptions:

1. A gas consists of particles, called molecules.
2. Molecules are in random motion and obey Newton's laws of motion.
3. The total number of molecules is large.
4. The volume of the molecules themselves is a negligibly small fraction of the volume occupied by the gas.
5. No appreciable forces act on the molecules except during a collision.
6. Collisions are elastic and are of negligible duration.

Avogadro's Number (N_0)

This is the number of particles (or molecules or atoms) in 1 kmol. of a substance, and is same for all substances.

$$N_0 = 6.023 \times 10^{26} \text{ particles/kmol.}$$

$$\text{Mass of an atom or molecule, } m_0 = \frac{M}{N_0}$$

where M is the molecular mass of the substance.

The average translational kinetic energy of a gas molecule is $3kT/2$ where $k = R/N_0 = 1.381 \times 10^{-23} \text{ J/K}$, called Boltzmann's constant and T is the absolute temperature.

$$\text{Average of } \frac{1}{2} m_0 V^2 = \frac{3}{2} kT$$

Kinetic Calculation of Pressure

If ρ is the density of gas, and V_{rms} is the square root of the average of square of the velocity for a molecule, then the pressure exerted by the gas is given by

$$p = \frac{1}{3} \rho V_{\text{rms}}^2 \quad \text{or} \quad V_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

Root mean square speed

$$V_{\text{rms}} = \sqrt{\frac{3kT}{m_0}} \quad \text{or} \quad \sqrt{\frac{3RT}{M}}$$

Relation between Kinetic Energy of Molecules and Temperature

$$\frac{1}{2} M V_{\text{rms}}^2 = \frac{3}{2} RT$$

This means that the total kinetic energy per mole of the molecules of an ideal gas is proportional to the temperature. It may also be expressed as

$$\frac{1}{2} m V_{\text{rms}}^2 = \frac{3}{2} kT$$

$$\begin{aligned} \text{where } k &= \frac{R}{N_0} = \frac{8.314 \text{ J/mol.K}}{6.023 \times 10^{23} \text{ molecules/mol}} \\ &= 1.380 \times 10^{-23} \text{ J/molecule} \cdot \text{K} \end{aligned}$$

Internal Energy of an Ideal Gas

If an ideal gas contains N molecules, its internal energy,

$$U = \frac{3}{2} NkT = \frac{3}{2} nRT$$

Some other Useful Relations

1. $\Delta U = nC_v \Delta T$, C_v is the specific heat at constant volume
2. $p \Delta V = nR \Delta T$, at constant pressure
3. $C_p - C_v = R$
4. $\Delta T = \frac{p \Delta V + V \Delta p}{nR}$
5. $pV^\gamma = \text{constant}$
6. $V = \sqrt{\frac{\gamma p}{\rho}}$
7. $\gamma = \frac{C_p}{C_v}$

Monatomic and Polyatomic Gases, Their Energies and Degrees of Freedom

Monatomic	Three degrees of freedom, ($n=3$)	$E = \frac{3}{2} RT$, $\gamma = \frac{5}{3}$	$\gamma = \frac{n+2}{n}$ $\gamma = \frac{5}{3}$
Diatomic	Five degrees of freedom, ($n=5$)	$E = \frac{5}{2} RT$, $\gamma = \frac{7}{5}$	$\gamma = \frac{7}{5}$
Triatomic	Six degrees of freedom, ($n=6$)	$E = \frac{6}{2} RT$, $\gamma = \frac{4}{3}$	$\gamma = \frac{8}{6}$

For more complex molecules, γ approaches unity but is always greater than one.

Mean Free Path

The average distance between successive collisions is called mean free path

(λ). If d is the diameter of the molecule (assumed to be spheres) and n_v is the number of molecules per unit volume, then

$$\lambda = \frac{1}{\sqrt{2} \pi n_v d^2}$$

ILLUSTRATIONS

1. Calculate the work done per mole by an ideal gas which expands isothermally from an initial volume V_i to a final volume V_f .

Solution

$$W = \int_{V_i}^{V_f} p \cdot dV = \int_{V_i}^{V_f} \frac{nRT}{V} dV$$

As the process is isothermal, T is constant.

$$\frac{W}{n} = RT \int_{V_i}^{V_f} \frac{dV}{V} = RT \ln \frac{V_f}{V_i}$$

Refer to the figure. The work done is the area of the shaded portion under the curve.

2. Calculate the root mean square velocity of the molecules of nitrogen at 0°C . The density of nitrogen at NTP is 1.25 g/litre .

Solution

$$p = 76 \times 13.6 \times 981 \text{ dynes/cm}^2 = 1.013 \times 10^5 \text{ N/m}^2$$

$$\rho = 1.25/1000 \text{ g/cc} = 1.25 \text{ kg/m}^3$$

$$V_{\text{rms}} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 \times 1.013 \times 10^5}{1.25}} \text{ m/s} = \sqrt{2.4 \times 10^5}$$

$$= \sqrt{24} \times 10^2 = 4.9 \times 10^2 = 490 \text{ m/s}$$

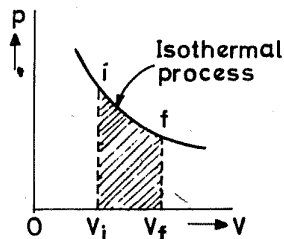
3. Assuming that the speed of sound in a gas is the same as the root mean square speed of molecules, find how the speed of sound for an ideal gas would depend on the temperature.

Solution

The density of a gas is given by

$$\rho = \frac{nM}{V}$$

where M is the molecular weight (grams/mole) and n is the number of moles.



The ideal gas law is given by

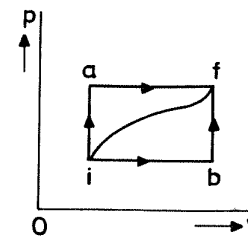
$$pV = nRT \text{ and } V_{\text{rms}} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3RT}{M}}$$

Therefore, the speed of sound V_1 at a temperature T_1 is related to the speed of sound V_2 in the same gas at temperature T_2 according to the equation

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

4. A system is taken from an initial state i to a final state f as shown in the figure. When it goes from i to f via iaf it is found that $Q = 70 \text{ cal}$ and $W = 40 \text{ cal}$, and along the path ibf , $Q = 40 \text{ cal}$.

- What is W along the path ibf ?
- If $W = -15 \text{ cal}$ for the curved path fi , what is Q for this path?
- If internal energy at $i(U_i) = 20 \text{ cal}$, what is U_f ?
- If $U_b = 25 \text{ cal}$, what is Q for the process ib ?



Solution

$$U_f - U_i = \Delta U = Q - W$$

(a) For path iaf ,

$$\Delta U = 70 - 40 = 30 \text{ cal}$$

Now ΔU is independent of path taken and is, therefore, the same for ibf .

$$30 = Q - W$$

$$30 = 40 - W \Rightarrow W = 10 \text{ cal}$$

(b) For the return path fi ,

$$\Delta U = -30 \text{ cal} \quad \text{and} \quad W = -15 \text{ cal}$$

$$\therefore Q = \Delta U + W = -30 - 15 = -45 \text{ cal}$$

(c) $U_i = 20 \text{ cal}$ and $\Delta U = 30 \text{ cal}$

$$\begin{aligned} \therefore U_f &= \Delta U + U_i \\ &= 30 + 20 = 50 \text{ cal} \end{aligned}$$

(d) $U_i = 20 \text{ cal}$ and $U_b = 25 \text{ cal}$

$$\text{Now } W_{ib} = W_{ibf} = 10 \text{ cal}$$

$$\therefore \Delta U = Q - W \quad \text{or} \quad 5 = Q - 10 \quad \text{or} \quad Q = 15 \text{ cal}$$

5. At what temperature is the average translational kinetic energy of a molecule equal to the kinetic energy of an electron accelerated from rest through a potential difference of 1 V ?

Solution

To accelerate an electron from rest through a potential difference of 1V, the energy required is 1 eV and $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.

$$\text{Average translational kinetic energy of a molecule} = \frac{3}{2} kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times T$$

$$\therefore 1.6 \times 10^{-19} = \frac{3}{2} \times 1.38 \times 10^{-23} T \Rightarrow T = 7729 \text{ K}$$

6. The root mean square speed of oxygen molecules is 460 m/s at 0.00°C . The molecular weight of oxygen is 32 g/mole. If the molecular weights of argon and helium are 40 and 4 respectively, find their root mean square speeds at 50°C .

Solution

For oxygen,

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

$$\therefore 460 = \sqrt{\frac{3 \times R \times 273}{0.032}} \text{ (at } 0^\circ\text{C)}$$

For helium,

$$V_{\text{rms}} = \sqrt{\frac{3 \times R \times 323}{0.004}}$$

$$= \sqrt{\left(\frac{323}{273}\right) \left(\frac{0.032}{0.004}\right)} \times 460 = 1415 \text{ m/s}$$

Similarly, for argon at 50°C ,

$$V_{\text{rms}} = \sqrt{\left(\frac{323}{273}\right) \left(\frac{0.032}{0.04}\right)} \times 460 = 447.5 \text{ m/s}$$

7. 1.000 mole of a gas occupies a volume of $2.54 \times 10^{-2} \text{ m}^3$ at a pressure of $9.480 \times 10^4 \text{ N/m}^2$ when its temperature is 290.0 K . The same mass of gas requires 125.0 cal to raise its temperature from 290.0 K to 315.0 K while the volume is held constant. If $C_p/C_v = 1.430$, find the value of J .

Solution

$$PV = nRT \Rightarrow R = \frac{9.48 \times 10^4 \times 2.54 \times 10^{-2}}{1.0 \times 290} = 8.3 \text{ J/mole K}$$

C_v = heat required to raise the temperature of a mole of a gas through 1 K at constant volume

$$125 = (315 \text{ K} - 290 \text{ K}) \times C_v \times n$$

$$C_v = \frac{125 \text{ cal}}{1 \text{ mole} \times 25 \text{ K}} = 5 \text{ cal/mol.K}$$

$$\frac{C_p}{C_v} = 1.430$$

$$\therefore C_p = 1.430 C_v = 7.150$$

$$R = C_p - C_v = 7.150 - 5.0 = 2.150 \text{ cal/mole.K}$$

$$J = \frac{8.3 \text{ J/mole K}}{2.15 \text{ cal/mole K}} = 3.86 \text{ J/cal}$$

Note: The value of J is lower than the standard value of 4.18. The discrepancy can be attributed to the values of R that have been obtained, one from the gas equation and the other from the values of C_p and C_v . The value obtained from the gas equation is close to the accepted value. Hence the error must be in the determination of either C_p or C_v .

8. One mole of neon gas fills a 1-litre flask at 15°C . The neon found in nature consists of three isotopes with the following proportion by mass:

^{20}Ne : 90.92 per cent (19.9924 u) ($1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$)

^{21}Ne : 0.26 per cent (20.9938 u)

^{22}Ne : 8.82 per cent (21.9914 u)

(a) What is the total internal energy of the gas?

(b) Determine $(V_{\text{rms}})^2$ for each isotope.

(c) What is the molecular mass of naturally occurring neon?

(d) Calculate the partial pressure of each isotopic component.

Solution

(a) Total internal energy of 1 mol. of neon (a monatomic gas) at 15°C is

$$U = \frac{3}{2} nRT = \frac{3}{2} (1 \text{ mol.}) (8.314 \text{ J/mol} \cdot \text{K}) (288 \text{ K}) \\ = 3594 \text{ J}$$

(b) $[V_{\text{rms}}^2]_{20} = \frac{3kT}{m_0}$, where m_0 is the mass of a single molecule (for neon, a single atom)

$$= \frac{3(1.380 \times 10^{-23} \text{ J/K})(288 \text{ K})}{19.992 \times 1.6605 \times 10^{-27} \text{ kg}} = 3.594 \times 10^5 \text{ m}^2/\text{s}^2$$

Similarly,

$$[V_{\text{rms}}^2]_{21} = 3.423 \times 10^5 \text{ m}^2/\text{s}^2$$

$$[V_{\text{rms}}^2]_{22} = 3.267 \times 10^5 \text{ m}^2/\text{s}^2$$

(c) The molecular mass of naturally occurring neon is

$$M(\text{Ne}) = (0.9092 \times 19.9924 \text{ u}) + (0.0026 \times 20.9938 \text{ u}) \\ + (0.0882 \times 21.9914 \text{ u}) \\ = 20.171 \text{ u}$$

$$(d) P_i = \frac{1}{3} \rho_i \langle V_i^2 \rangle \text{ and } \rho_i = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{(0.9092) (19.9924 \text{ g/mole}) (1 \text{ mol})}{10^{-3} \text{ m}^3}$$

$$\therefore p(^{20}\text{Ne}) = \frac{1}{3} \left(\frac{18.177 \times 10^{-3} \text{ kg}}{10^{-3} \text{ m}^3} \right) \times 3.594 \times 10^5 \text{ m}^2/\text{s}^2$$

$$= 2.1776 \times 10^6 \text{ N/m}^2$$

Similarly

$$p(^{21}\text{Ne}) = 0.0062 \times 10^6 \text{ N/m}^2$$

$$\text{and } p(^{22}\text{Ne}) = 0.2112 \times 10^6 \text{ N/m}^2$$

9. Argon gas at atmospheric pressure and at 20°C is confined in a vessel with a volume of 1 m³. The effective hard-sphere diameter of argon atom is $3.10 \times 10^{-10} \text{ m}$.

(a) Determine the mean free path.

(b) Find the pressure (at 20°C) when $\lambda = 1 \text{ m}$

Solution

$$(a) \text{ Mean free path, } \lambda = \frac{kT}{\sqrt{2} \pi p d^2}$$

where p is the pressure of the gas.

$$\therefore \lambda = \frac{(1.381 \times 10^{-23} \text{ J/K}) (293 \text{ K})}{\sqrt{2} \pi (1.013 \times 10^5 \text{ N/m}^2) (3.10 \times 10^{-10} \text{ m})^2}$$

$$= 9.36 \times 10^{-8} \text{ m} = 93.6 \text{ nm}$$

(b) From the expression of λ , we observe that

$$\lambda_1 \propto \frac{1}{p_1} \text{ and } \lambda_2 \propto \frac{1}{p_2} \Rightarrow p_1 \lambda_1 = p_2 \lambda_2$$

$$p_2 = \frac{p_1 \lambda_1}{\lambda_2} = \frac{(1 \text{ atm}) (9.36 \times 10^{-8} \text{ m})}{(1 \text{ m})} = 9.36 \times 10^{-8} \text{ atm}$$

10. A cylinder contains 3 litres of air at 2 atm pressure and at a temperature of 300 K. The following operations are performed on the air:

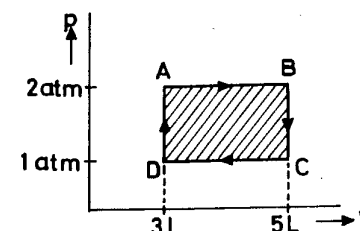
- heating at constant pressure to 500 K
- cooling to 250 K at constant volume
- cooling at constant pressure to 150 K
- heating at constant volume to 300 K

Draw a p - V diagram and find the values of p and V at the end of each process. Calculate the net work done.

Solution

The initial conditions are represented by point A of the p - V diagram shown here. $P_A = 2 \text{ atm}$, $V_A = 3 \text{ litres}$ and $T_A = 300 \text{ K}$.

In process (a), heating is done at constant pressure and the temperature rises to 500 K. Line AB represents the process.



$$\frac{V_B}{T_B} = \frac{V_A}{T_A} \text{ (Charles's law)} \Rightarrow V_B = 5 \text{ litres}$$

In process (b), cooling takes place at constant volume and the temperature comes down to 250 K. This is represented by BC.

$$\frac{p_C}{T_C} = \frac{p_B}{T_B} \text{ or } p_C = \frac{T_C}{T_B} p_B \Rightarrow p_C = \frac{250}{500} \times 2 = 1 \text{ atm. pressure}$$

In process (c), the gas is cooled at constant pressure to 150 K, shown by CD. Again Charles's law gives

$$\frac{V_D}{T_D} = \frac{V_C}{T_C} \Rightarrow V_D = \frac{T_D}{T_C} V_C = \frac{150}{250} \times 5 = 3 \text{ litres}$$

Process (d) is represented by DA, heating at constant volume. It is observed that the air returns to its original condition represented by A.

Net work done is shown by the area of the shaded region of the p - V diagram

$$W = AB \times AD = (5 - 3) \text{ litres} \times (2 - 1) \text{ atm.}$$

$$= 2000 \times 10^{-6} \text{ m}^3 \times 1.013 \times 10^5 \text{ N/m}^2$$

$$= 2026 \times 10^{-1} \text{ N} \cdot \text{m} = 202.6 \text{ J}$$

11. The volume of an oxygen cylinder is 50 L. As oxygen is withdrawn from the cylinder, the reading of a pressure gauge drops from 20 to 8 atm. and the temperature of the gas remaining in the cylinder drops from 30 to 10°C.

- How many kilograms of oxygen were there in the cylinder originally?
- How many kilograms of oxygen were withdrawn?
- What will be the volume of oxygen withdrawn from the cylinder at a pressure of 4 atm. and a temperature of 25°C?

Solution

(a) As the pressures given are recorded in the gauge, the actual pressures are 21 and 9 atm.

Let n_1 be the number of moles originally present. Then,

$$n_1 = \frac{p_1 V}{RT_1} = \frac{21 \text{ atm.} \times 50 \text{ L}}{(0.0821 \text{ L atm. mol}^{-1} \cdot \text{K}^{-1}) (303 \text{ K})} = 42.2 \text{ mol.}$$

The original mass was therefore

$$m_1 = Mn_1 = (32 \text{ g mol}^{-1}) (42.2 \text{ mol.}) = 1350.4 \text{ g} = 1.350 \text{ kg}$$

(b) The number of moles remaining in the cylinder is

$$n_2 = \frac{p_2 V}{RT_2} = \frac{9 \text{ atm.} \times 50 \text{ L}}{(0.0821 \text{ L atm. mol}^{-1} \cdot \text{K}^{-1}) (283 \text{ K})} = 19.4 \text{ mol.}$$

The mass of oxygen remaining in the cylinder is

$$m_2 = Mn_2 = 32 \times 19.4 = 620.8 \text{ g} = 0.621 \text{ kg}$$

$$\therefore \text{Mass of oxygen withdrawn} = m_1 - m_2 = 1.350 - 0.621 \\ = 0.729 \text{ kg}$$

(c) The number of moles withdrawn is

$$n_1 - n_2 = 42.2 - 19.4 = 22.8 \text{ mol.}$$

The volume occupied would be

$$V = \frac{nRT}{p} = \frac{22.8 \times 0.821 \times 298}{4} = 1394.5 \text{ L}$$

12. A balloon whose volume is 500 m^3 is to be filled with hydrogen at atmospheric pressure.

(a) If hydrogen is stored in cylinders of volume 0.05 m^3 at an absolute pressure of $15 \times 10^5 \text{ Pa}$, how many cylinders are required?

(b) What is the total weight that can be supported by the balloon, in air at standard conditions? ($p_{\text{air}} = 1.3 \text{ kg/m}^3$ and $\rho_{\text{H}} = 9 \times 10^{-2} \text{ kg/m}^3$)

Solution

(a) The volume of the balloon is 500 m^3 and hydrogen is at a pressure of $1.013 \times 10^5 \text{ N/m}^2$.

The hydrogen cylinders are at a pressure of $15 \times 10^5 \text{ Pa}$.

$$\therefore 15 \times 10^5 \times V = 500 \times 1.013 \times 10^5 \Rightarrow V = 33.77 \text{ m}^3$$

$$\therefore \text{Number of cylinders} = \frac{33.77}{0.05} = 675$$

(b) The weight that can be supported by the balloon is given by

$$mg = V(\rho - \sigma)g$$

where ρ is the density of air and σ is the density of hydrogen.

$$\therefore mg = 500(1.3 - 0.09) \times 9.8 = 5929 \text{ N}$$

EXERCISES

1. Choose the correct statement from among the following.

- (a) A monoatomic gas has three degrees of freedom because it undergoes translational as well as rotational motion.

- (b) A diatomic molecule undergoes both translational and rotational motion, and has six degrees of freedom.
(c) A diatomic molecule is capable of rotating energetically about each of three mutually perpendicular axes.
(d) The molecule of a polyatomic gas is capable of rotating energetically about each of three mutually perpendicular axes, and undergoes both translational and rotational motion.

2. Dry air at 15°C and 1 atm. consists of N_2 with a partial pressure of $0.791 \times 10^5 \text{ N/m}^2$ and a mass fraction of air of 75.5%, oxygen (O_2) with a partial pressure of $0.212 \times 10^5 \text{ N/m}^2$ and a mass fraction of 23.2%, plus a number of other small molecular fractions. The density of air is 1.225 kg/m^3 . The ratio of root mean square speeds of nitrogen and oxygen molecules is
(a) 1.07 (b) 1.70
(c) 0.70 (d) 0.17
3. 1 kg of a gas is at a pressure of 2 atm. and a temperature of 530 K. Heat is supplied at constant volume till pressure reaches a value of 4 atm. The heat absorbed by the gas is (specific heat of gas is $0.17 \text{ cal/g}^\circ\text{C}$)
(a) $9 \times 10^6 \text{ cal}$ (b) 90 kcal
(c) 900 cal (d) 9000 cal
4. A bubble has a volume of 3 mm^3 at a depth of 20 m in a lake of pure water. If the bubble slowly rises up to the surface of the lake, its volume will be (atmospheric pressure is 100 kPa)
(a) 7.24 mm^3 (b) 4.44 mm^3
(c) 8.88 mm^3 (d) 6.66 mm^3
5. A 25-cm-long tube closed at one end is lowered, with the open end down, into a freshwater lake. If one-third of the tube is filled with water, the distance between the surface of the lake and level of water in the tube is
(a) 6.41 m (b) 5.82 m
(c) 5.10 m (d) 4.20 m
6. A cylinder contains 20 kg of N_2 gas ($M = 28 \text{ kg/kmol.}$) at a pressure of 5 atm. The mass of hydrogen ($M = 2 \text{ kg/kmol.}$) at a pressure of 3 atm. contained in the same cylinder is.
(a) 1.08 kg (b) 0.86 kg
(c) 0.68 kg (d) 1.68 kg
7. A tank of capacity 50000 cm^3 contains an ideal gas ($M = 40 \text{ kg/kmol.}$) at a gauge pressure of 550 kPa and a temperature of 27°C . Assuming the atmospheric pressure to be 100 kPa, the mass of the gas in the tank is
(a) 2.50 kg (b) 0.25 kg
(c) 0.052 kg (d) 0.52 kg
8. The following data is given for carbon dioxide gas:
(i) mass of one mole of CO_2 is $44 \times 10^{-3} \text{ kg}$.
(ii) density of CO_2 at NTP is 1.98 kg/m^3 .
(iii) Avogadro's number is 6.03×10^{23} per mole.
A correct calculation made using the above data gives:
(a) The number of molecules in 1 kg of the gas is 1.37×10^{40} .
(b) The mass of one molecule of the gas is $7.31 \times 10^{-28} \text{ kg}$.
(c) The number of molecules in 1 m^3 of the gas at NTP is 2.7×10^{25} .
(d) The root mean square speed of the molecule is 0.0168 m/s .
9. 32 g of O_2 at NTP occupies 22.3 litres. The root mean square speed of the molecules at 27°C is (atm. pressure is $1 \times 10^5 \text{ N/m}^2$)
(a) 480.14 m/s (b) 408.41 m/s
(c) 840.14 m/s (d) 204.41 m/s

10. The temperature at which the root mean square speed of a gas will be half its value at 0°C is (assume the pressure remains constant)
 - (a) 86.4°C
 - (b) -204.75°C
 - (c) -104.75°C
 - (d) 68.25°C
11. If the root mean square speed of the molecules of hydrogen at NTP is 1.84 km/s , the rms speed of oxygen molecules at NTP is (molecular weights of hydrogen and oxygen are 2 and 32 respectively)
 - (a) 320 m/s
 - (b) 640 m/s
 - (c) 460 m/s
 - (d) 230 m/s
12. A 5-mg droplet of liquid nitrogen is enclosed in a 50-mL tube which is sealed at very low temperature. When the tube is warmed to 30°C , the nitrogen pressure in the tube is (molecular weight of nitrogen is 28 kg/kmol .)
 - (a) 9987 N/m^2
 - (b) 7998 N/m^2
 - (c) 9897 N/m^2
 - (d) 8997 N/m^2
13. A 500-mL sealed cylinder contains nitrogen at a pressure of 1 atm. A tiny glass tube lies at the bottom of the cylinder. Its volume is 0.50 mL and it contains hydrogen at a pressure of 4.5 atm. The glass tube is broken so that hydrogen also fills the cylinder. The new pressure in the cylinder is (1 atm. = $1 \times 10^5\text{ N/m}^2$)
 - (a) 76.34 cm Hg
 - (b) 82.40 cm Hg
 - (c) 94.24 cm Hg
 - (d) 104.34 cm Hg
14. A tank attached to an air compressor contains 25 litres of air at a temperature 25°C and gauge pressure of $5.0 \times 10^5\text{ N/m}^2$. The mass of air in the tank is (given that the average molecular mass of air is 28.8 g/mol .)
 - (a) 0.414 kg
 - (b) 0.441 kg
 - (c) 0.175 kg
 - (d) 1.404 kg
15. Two flasks are connected by an initially closed stopcock. One flask contains 250 cc of krypton gas at 500 mm Hg while the other contains 450 cc of helium at 950 mm Hg. On opening the stopcock, the two gases mix and the final pressure of the system becomes (assume constant temperature)
 - (a) 897 mm Hg
 - (b) 778 mm Hg
 - (c) 789 mm Hg
 - (d) 987 mm Hg
16. An air bubble of volume V_0 is released from the bottom of a pure water lake at a depth of 15.0 m. The temperature at the bottom is 5°C . The bubble comes to the surface of the lake where the temperature is 15°C . The volume of the air bubble at the surface is
 - (a) $5.62 V_0$
 - (b) $2.56 V_0$
 - (c) $1.2 V_0$
 - (d) $6.25 V_0$
17. The molecules of an ideal gas behave like spheres of radius $4 \times 10^{-10}\text{ m}$. If the pressure is $1 \times 10^5\text{ N/m}^2$, $K = 1.38 \times 10^{-23}\text{ J/K}$ and the temperature is 0°C , the mean free path is
 - (a) $0.33 \times 10^{-8}\text{ m}$
 - (b) $2.4 \times 10^{-8}\text{ m}$
 - (c) $1.64 \times 10^{-8}\text{ m}$
 - (d) $1.33 \times 10^{-8}\text{ m}$
18. The mean free path of some gas molecules (assumed to be spherical) of radius $4 \times 10^{-10}\text{ m}$ is 30 cm at 10°C . The pressure of the gas is
 - (a) 0.46 mPa
 - (b) 2.6 mPa
 - (c) 4.6 mPa
 - (d) 6.4 mPa
19. The rms speed of an oxygen molecule ($M = 32\text{ kg/kmol}$) in air at 27°C is
 - (a) 483 m/s
 - (b) 580 m/s
 - (c) 390 m/s
 - (d) 650 m/s
20. A nitrogen gas molecule has some rms speed at 0°C on the surface of the

earth. With this speed, it goes straight up. If there are no collisions with other molecules, the molecule will rise up to a height of

- (a) 8.2 km
- (b) 12.4 km
- (c) 10.6 km
- (d) 15.2 km

21.* Mark the incorrect statement:

- (a) The average kinetic energy of a molecule of a gas at a temperature of 300 K is $6.21 \times 10^{-21}\text{ J}$.
- (b) The total random kinetic energy of the molecules in 1 mole of a gas at a temperature of 300 K is 3741 J.
- (c) The root mean square speed of a hydrogen molecule at 300 K is 1927 m/s.
- (d) The number of molecules in 1 m^3 of air at atmospheric pressure and 0°C is 2.69×10^{28} .
[Given: $K = 1.38 \times 10^{-23}\text{ J/K}$, $R = 8.314\text{ J. mol}^{-1}\text{ K}^{-1}$, $N_0 = 6.023 \times 10^{23}\text{ molecules.mol}^{-1}$, atm. pressure = $1 \times 10^5\text{ Pa or N/m}^2$]

22. Five gas molecules chosen at random are found to have speeds of 500, 600, 700, 800 and 900 m/s.

- (a) The root mean square speed and the average speed are the same.
- (b) The root mean square speed is 14 m/s higher than the average speed.
- (c) The root mean square speed is 14 m/s lower than the average speed.
- (d) The root mean square speed is $\sqrt{14}$ m/s higher than the average speed.

23. A helium storage tank has a capacity of 0.5 m^3 and the gas pressure is 50 atm. at 27°C . The mass of helium gas in the tank is [1 atm = $1 \times 10^5\text{ N/m}^2$ and $R = 8.314\text{ (N.m}^{-2}\text{)m}^3\text{ mol}^{-1}\text{ K}^{-1}$]

- (a) 4.008 kg
- (b) 2.004 kg
- (c) 1.001 kg
- (d) 0.4008 kg

24. A container of capacity 5 L provided with a stopcock contains nitrogen at a temperature of 300 K and pressure of 1 atm. The system is heated to 400 K with the stopcock open to the atmosphere. The stopcock is then closed and the system is cooled to its original temperature. The amount of nitrogen (in grams) left in the container is

- (a) 4.84
- (b) 3.48
- (c) 4.31
- (d) 8.34

25. 5 L of argon under a pressure of 4 atm. and at a temperature of 27°C is heated until both pressure and volume are doubled. The final temperature of the gas and the quantity of argon (in grams) are respectively (atomic weight of argon is 40 and 1 atm. = $1 \times 10^5\text{ N/m}^2$)

- (a) 800 K and 32.49
- (b) 1200 K and 32.49
- (c) 1200 K and 23.94
- (d) 800 K and 32.49

26. 2 g of helium gas is contained in a flask at an absolute pressure of 20 atm and at a temperature of 47°C . It was detected that due to leakage in the flask, the pressure was reduced to half and the temperature decreased to 27°C . The quantity of helium (in grams) that leaked out is

- (a) 1.390
- (b) 0.393
- (c) 0.290
- (d) 0.933

27. In a very good vacuum system in the laboratory, the vacuum attained was 10^{-13} atm . If the temperature of the system was 300 K, the number of molecules present in a volume of 1 cc is

- (a) 2.4×10^6
- (b) 4.2×10^4
- (c) 1.2×10^4
- (d) 0.4×10^4

28. The temperature at which the rms speed of oxygen molecules is equal to the rms speed of hydrogen molecules at 0°C is

- (a) 841 K (b) 4368 K
(c) 1842 K (d) 2418 K
29. The surface of the sun has a temperature of about 6000 K and consists largely of hydrogen. The rms speed and the average kinetic energy of a hydrogen molecule are respectively
(a) 6580 m/s and 0.125×10^{-19} J
(b) 8650 m/s and 1.25×10^{-19} J
(c) 8065 m/s and 2×10^{-19} J
(d) 8056 m/s and 5×10^{-19} J
30. An oxygen molecule is raised by 1 m near the surface of earth, thereby increasing its potential energy. The temperature at which this increase equals the average kinetic energy of oxygen molecules is
(a) 5.025 K (b) 2.500 K
(c) 0.250 K (d) 0.025 K
31. The rms speed of a molecule is 1 m/s at a temperature of 300 K. The mass of the molecule is
(a) 1.24×10^{-24} kg (b) 4.2×10^{-22} kg
(c) 2.4×10^{-20} kg (d) 1.24×10^{-20} kg
32. The mean free path of nitrogen at 0°C and 1 atm. pressure is 0.80×10^{-5} cm. There are 3×10^{19} molecules/cm³ at this temperature and pressure. The diameter of the molecule is
(a) 3.06×10^{-8} cm (b) 6.03×10^{-8} cm
(c) 0.63×10^{-8} cm (d) 3.06×10^{-10} cm
33. If the wavelength of sound be of the order of the mean-free path in oxygen at 1 atm. pressure and at 0°C and the diameter of the oxygen molecule be 3.06×10^{-8} cm, the frequency of the sound wave is
(a) 36×10^{12} cps (b) 3.6×10^{10} cps
(c) 3.6×10^9 cps (d) 6.3×10^{10} cps
34. The rms speed of the molecules of a gas at 27°C is 2×10^3 m/s. The rms speed at 87°C is
(a) 220 m/s (b) 1100 m/s
(c) 1800 m/s (d) 2200 m/s
35. One gram mole of nitrogen at 27°C and 1 atm. pressure is contained in a vessel and the molecules are moving with their rms speeds. The number of collisions per second which the molecules make with an area of 1 m^2 on the vessel's wall is
(a) 2×10^{27} (b) 2×10^{20}
(c) 2×10^{10} (d) 2×10^{24}
36. A vessel containing one gram mole of oxygen at 27°C and 1 atm. pressure is thermally insulated and moves with a constant speed V_0 . It is then suddenly stopped and this results in a rise of temperature of the gas by 1°C . The speed of the vessel V_0 has the value
(a) 63.03 m/s (b) 30.63 m/s
(c) 36.03 m/s (d) 33.06 m/s
37. A vessel contains 14 g of hydrogen and 96 g of oxygen at STP. The volume of the vessel is
(a) 2.3 m^3 (b) 4.6 m^3
(c) 0.46 m^3 (d) 0.23 m^3
38. The temperature at which the rms speed of a gas molecule is the same as that of a molecule of another gas at 27°C is (the molecular weights of the first and the second gas are 64 and 32 respectively)

- (a) 172°C (b) 327°C
(c) 142°C (d) 372°C
39. For an ideal gas having molecular diameter d , at pressure p and temperature T , the expression for the mean free path is
(a) $\frac{1}{\sqrt{2} \pi d^2} \cdot \frac{p}{kT}$ (b) $\frac{1}{4 \pi d^2} \cdot \frac{kT}{p}$
(c) $\frac{1}{\sqrt{2} \pi d^2} \cdot \frac{kT}{p}$ (d) $\frac{3p}{2 \pi d^2 \cdot kT}$
40. For a polyatomic gas the specific heat at constant volume is (given that density of gas at NTP is $7.95 \times 10^{-4} \text{ g/cm}^3$, 1 atm. = $1 \times 10^5 \text{ N/m}^2$)
(a) 0.334 cal/g/ $^\circ\text{C}$ (b) 0.433 cal/g/ $^\circ\text{C}$
(c) 0.043 cal/g/ $^\circ\text{C}$ (d) 0.034 cal/g/ $^\circ\text{C}$
41. The temperature of 2 g of nitrogen occupying a volume of 820 cm^3 at a pressure of 2 atm. is
(a) 3°C (b) 27°C
(c) 37°C (d) 14°C
42. The volume occupied by 10 g of oxygen at a pressure of 750 mm Hg and at a temperature of 20°C is
(a) $1.5 \times 10^{-3} \text{ m}^3$ (b) $7.6 \times 10^{-3} \text{ m}^3$
(c) $6.7 \times 10^{-3} \text{ m}^3$ (d) $15.12 \times 10^{-3} \text{ m}^3$
43. The pressure of air inside a tightly sealed bottle was 1 atm. at a temperature of 7°C . The cork flew out of the bottle when it was heated. If the pressure in the bottle was 1.3 atm. when the cork flew out, the temperature at that instant was
(a) 73°C (b) 65°C
(c) 81°C (d) 91°C
44. A cylinder contains 6.4 kg of oxygen and its walls can withstand a pressure of $16 \times 10^5 \text{ N/m}^2$ at a temperature 20°C . The smallest volume that the cylinder can have is
(a) $1.3 \times 10^{-2} \text{ m}^3$ (b) $3.1 \times 10^{-2} \text{ m}^3$
(c) $13 \times 10^{-2} \text{ m}^3$ (d) $31 \times 10^{-2} \text{ m}^3$
45. A cylinder contained 10 kg of gas at a pressure of 10^7 N/m^2 . On taking out a certain quantity of gas from the cylinder, the final pressure comes down to $2.5 \times 10^6 \text{ N/m}^2$. If the temperature remains constant in the process, the weight of the gas taken out is
(a) 5.7 kg (b) 15.0 kg
(c) 22.5 kg (d) 7.5 kg
46. Air fills a room in winter at 7°C and in summer at 37°C . If the pressure is the same in winter and summer, the ratio of the weight of the air filled in winter and that in summer is
(a) 2.2 (b) 1.75
(c) 1.1 (d) 3.3
47. A vertical cylinder closed from both ends is equipped with an easily moving piston dividing the volumes into two parts, each containing one mole of air. In equilibrium at $T_0 = 300 \text{ K}$, the ratio of the volumes of the upper and lower parts is 4. The temperature at which this ratio will be 3 is
(a) 320 K (b) 420 K
(c) 480 K (d) 500 K
48. A vessel of volume V is evacuated by means of a piston air pump. One piston stroke captures the volume ΔV . The pressure in the vessel is to be reduced to

($1/n$) of its original value p_0 . If the process is assumed to be isothermal and air is considered an ideal gas, the number of strokes needed in the process is

- (a) $\frac{\ln n}{\ln(1 - \Delta V/V)}$ (b) $\frac{\ln n}{\ln(1 + \Delta V/V)}$
 (c) $\frac{\ln(1 - \Delta V/V)}{\ln n}$ (d) $\frac{\ln(1 + \Delta V/V)}{\ln n}$

49. A vessel contains a mixture of nitrogen ($m_1 = 7.0$ g) and carbon dioxide ($m_2 = 11$ g) at a temperature of 290 K and pressure $p_0 = 1.0$ atm. If the gases are assumed to be ideal, the density of the mixture is (molecular masses of N_2 and CO_2 are 28 and 44 respectively)
 (a) 1.5×10^{-3} g/cc (b) 3×10^{-3} g/cc
 (c) 2.5×10^{-3} g/cc (d) 1.5×10^{-4} g/cc
50. An ideal monoatomic gas is compressed (no heat being added or removed in the process) so that its volume is halved. The ratio of the new pressure to the original pressure is
 (a) 2.5 (b) 7.13
 (c) 6.34 (d) 3.17

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (b) | 4. (c) | 5. (c) |
| 6. (b) | 7. (d) | 8. (c) | 9. (a) | 10. (b) |
| 11. (c) | 12. (d) | 13. (a) | 14. (c) | 15. (c) |
| 16. (b) | 17. (d) | 18. (c) | 19. (a) | 20. (b) |
| 21. (d) | 22. (b) | 23. (a) | 24. (c) | 25. (b) |
| 26. (d) | 27. (a) | 28. (b) | 29. (b) | 30. (d) |
| 31. (d) | 32. (a) | 33. (c) | 34. (d) | 35. (a) |
| 36. (c) | 37. (d) | 38. (b) | 39. (c) | 40. (a) |
| 41. (a) | 42. (b) | 43. (d) | 44. (d) | 45. (d) |
| 46. (c) | 47. (b) | 48. (b) | 49. (a) | 50. (d) |

Note: Problems marked with an asterisk should take about five minutes. All the rest should be solved in about three minutes each.

14

Thermodynamics

Heat Energy (ΔQ)

A useful but nonoperational definition of heat is: 'that which is transferred between a system and its surroundings as a result of temperature difference only'. Heat always flows from a hot to a cold body and if two bodies are in thermal equilibrium, no net heat transfer takes place from one to the other. This means that their temperatures must be the same.

Zerth Law of Thermodynamics

It states that if each of two objects is in thermal equilibrium with a third body, then the two are in thermal equilibrium with each other.

The Mechanical Equivalent of Heat

Heat has a mechanical aspect also. Both heat and work are forms of energy and there is, therefore, a definite relationship between them, called the mechanical equivalent of heat. As a consequence of this relationship, we have $1 \text{ cal} = 4.186 \text{ J}$.

The Work Done by a System

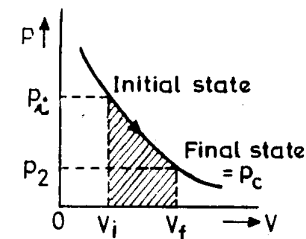
A gas expands from an initial state defined by pressure p_i and volume V_i . In this expansion, suppose a piston is pushed by a small distance ds , then

$$dW = \vec{F} \cdot \vec{ds} = pA \cdot ds = p \cdot dV$$

where dV is the differential change in volume of the gas. To obtain the total work done, we must know how p varies with the displacement. We, therefore, compute the integral

$$W = \int dW = \int_{V_i}^{V_f} p \cdot dV$$

The shaded region in this p - V diagram shows the work done.



In a cyclic process, the work output per cycle done by a fluid is equal to the area enclosed by the p - V diagram representing the cycle.

Work done by a system depends not only on the initial and final states but also on the intermediate states, that is, on the path of the process.

Heat lost or gained by a system depends not only on the initial and final states but also on the intermediate states, that is, on the path of the process.

Internal Energy (U)

The internal energy (U) of a system is the total energy content of the system. It is the sum of the kinetic, potential, chemical, electrical, nuclear and all other forms of energy possessed by the atoms and molecules of the system.

The First Law of Thermodynamics

Let heat absorbed by the system be Q and the work done by the system be W . As mentioned earlier, both Q and W are path-dependent. But the quantity $Q - W$ remains the same, whatever be the path or paths followed, provided the initial and final states are the same. These initial and final states are called equilibrium states. Thus, there exists a function of the thermodynamic coordinates whose final value minus its initial value equals the change $Q - W$ in the process. We call this function the internal energy function (U).

$$\Delta U = U_f - U_i \quad \text{and} \quad \Delta U = Q - W$$

or we can write this in the differential form as

$$dU = dQ - dW$$

Thus the law states that every thermodynamic system in an equilibrium state possesses a state variable called the internal energy whose change dU in a differential process is given by the above equation.

Isobaric Process

A process taking place at constant pressure is said to be isobaric. The first law becomes:

$$dU = dQ - p dV$$

Isovolumic Process

This process takes place at constant volume and the first law becomes

$$dQ = dU$$

$$\text{as } dW = 0$$

Isothermal Process

This is a constant-temperature process. In the case of an ideal gas, $dU = 0$ in an isothermal process and so the first law reduces to $dQ = dW$. In this process,

if the pressure and volume of an ideal gas change from (p_1, V_1) to (p_2, V_2) , where we hold the Boyle's Law and $p_1 V_1 = p_2 V_2$, then the first law of thermodynamics takes the form

$$dQ = dW = p_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$$

where $\ln$ is the logarithm to the base e .

Adiabatic Process

This process is one in which no heat is transferred to or from the system. For such a process, $dQ = 0$, and so we have

$$dU = -dW$$

The negative sign here shows that any work done by the system is done at the expense of the internal energy and any work done on the system results in the increase of internal energy.

For an ideal gas, changing conditions (p_1, V_1, T_1) to (p_2, V_2, T_2) in an adiabatic process, the following relations are obtained:

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad \text{and} \quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

where γ is the ratio of specific heats of gas at constant pressure and at constant volume ($C_p/C_v = \gamma$).

Enthalpy

Special thermodynamic functions are defined as a matter of convenience. The simplest such function is enthalpy (H), explicitly defined for any system by the expression

$$H = U + pV$$

Since both U and pV have units of energy, H also has energy units. Moreover, since U , p and V are all properties of a system, H must be a property too. In differential form, we have

$$dH = dU + d(pV)$$

Heat Capacity

Only for a reversible process, where the path is fully specified, is it possible to relate heat to a property of the system. On this basis, heat capacity is defined as

$$C = \left(\frac{\delta Q}{\delta T} \right)$$

$C_V = \left(\frac{\delta Q}{\delta T} \right)_V$ and represents the amount of heat required to increase the temperature by δT when the system is held at constant volume.

$C_V = \left(\frac{\partial U}{\partial T} \right)_V$ is an alternative definition of C_V .

$C_p = \left(\frac{\delta Q}{\delta T} \right)_p$ and also $C_p = \left(\frac{\partial H}{\partial T} \right)_p$

We may also write:

$$dU = C_V dT \text{ (const. } V) \text{ and } dH = C_p dT \text{ (const. } p)$$

Internal energy of a closed pVT system may be considered a function of T and V .

$$U = U(T, V)$$

$$\therefore dU = \left(\frac{\partial U}{\partial T} \right)_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

$$\text{or } dU = C_V dT + \left(\frac{\partial U}{\partial V} \right)_T dV$$

Functional Relationship

Let us consider a polytropic process in which the relation $pV^n = \text{constant}$ throughout the process.

$$\int_1^2 p dV = \frac{p_2 V_2 - p_1 V_1}{1 - n}$$

This result is valid for any exponent n except $n = 1$. When $n = 1$,

$$pV = \text{const} = p_1 V_1 = p_2 V_2 \quad \text{and} \quad \int_1^2 p dV = p_1 V_1 \ln \frac{V_2}{V_1}$$

First Law in Terms of C_p and C_V

$$dQ = nC_V dT \quad \rightarrow \text{(constant volume)}$$

$$\left. \begin{aligned} dQ &= nC_p dT \\ dW &= nR dT \\ dU &= nC_V dT \end{aligned} \right\} \quad \begin{aligned} &\text{(constant pressure)} \\ &\Rightarrow nC_V dT = nC_p dT - nR dT \end{aligned}$$

Second Law of Thermodynamics

"It is impossible for any system to undergo a process in which it absorbs heat

from a reservoir at a single temperature and convert it completely into mechanical work, while ending in the same state in which it begins."

Efficiency of the Carnot Engine

If T_H is the temperature at which the gas expands isothermally and T_c the temperature attained after adiabatic expansion, then the efficiency of the engine

$$e = 1 - \frac{T_c}{T_H} = \frac{T_H - T_c}{T_H}$$

Also if Q_1 is the heat supplied to the system during isothermal expansion and Q_2 is the heat rejected during the isothermal compression, then the efficiency

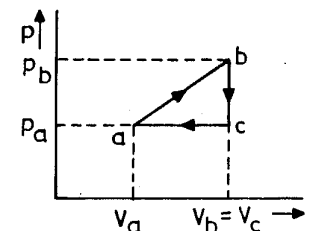
$$e = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Also, if the engine operates between the temperatures T_1 and T_2 K, then

$$e = \frac{T_1 - T_2}{T_1}$$

ILLUSTRATIONS

1. A gas system consists of n moles of an ideal gas whose initial state is defined by (p_a, V_a, T_a) . The system is carried along the path ab where, at b , the pressure, volume and temperature are p_b, V_b and T_b respectively, and $p_b = 2p_a$ and $V_b = 2V_a$. The pressure is then reduced at constant volume and comes to a state (p_c, V_c, T_c) . Finally, the system undergoes isobaric compression and comes back to the state $(p_a, V_a$ and $T_a)$.



- Calculate the work done in the process comprising ab , bc and ca .
- What is the net work done during the complete cycle?
- Calculate the T_b and T_c in terms of T_a .

Solution

$$(a) \quad W_{ab} = \int_{V_a}^{V_b} p \cdot dV = \frac{p_a}{V_a} \int_{V_a}^{2V_a} V \cdot dV = \frac{1}{2} \frac{p_a}{V_a} V^2 \Big|_{V_a}^{2V_a} = \frac{3}{2} p_a V_a$$

$W_{bc} = 0$, as there is no change in volume

$$W_{ca} = \int_{V_c}^{V_a} p dV = p_a \int_{2V_a}^{V_a} dV = -p_a V_a$$

The negative sign means that $W_{ca} < 0$ and the work was done on the gas during the compression.

$$(b) \quad W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = \left(\frac{3}{2} + 0 - 1 \right) p_a V_a = \frac{1}{2} p_a V_a$$

(This is just the area enclosed within the triangle.)

$$(c) \quad nR = \frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} = \frac{(2p_a)(2V_a)}{T_b} \Rightarrow T_b = 4T_a$$

Also,

$$\frac{p_a V_a}{T_a} = \frac{p_c V_c}{T_c} = \frac{2p_a (2V_a)}{T_b} \Rightarrow T_c = 2T_a$$

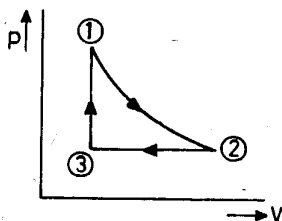
2. 1 g of nitrogen undergoes the following sequence of processes in a piston-cylinder system:

- An adiabatic expansion in which the volume doubles
- A constant-pressure process in which the volume is reduced to its initial value.
- A constant-volume compression back to initial state.

The initial temperature of nitrogen is 150°C and pressure is 5 atm. Calculate the net work done on the gas in this sequence of processes.

Solution

$p_1 = 5 \text{ atm} = 5.066 \times 10^5 \text{ N/m}^2$, $T_1 = 150^\circ\text{C} = 423 \text{ K}$
and $m = 1 \text{ g} = 10^{-3} \text{ kg}$. For nitrogen, gas constant will be



$$R = \frac{8315}{28} = 297 \text{ J/kg} \cdot \text{K}$$

$$V_1 = \frac{mRT_1}{p_1} = \frac{(0.001)(297)(423)}{5.066 \times 10^5} = 2.48 \times 10^{-4} \text{ m}^3$$

As $V_2 = 2V_1$,

$$\therefore V_2 = 2 \times 2.48 \times 10^{-4} = 4.96 \times 10^{-4} \text{ m}^3$$

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^\gamma = (5.066 \times 10^5) \left(\frac{1}{2} \right)^{1.4} = 1.92 \times 10^5 \text{ N/m}^2$$

Work done in the process 1-2

$$W_{1-2} = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} = \frac{[(1.92) \times (4.96) - (5.066) \times (2.48)](10)}{1.4 - 1} = -76.05 \text{ J}$$

Work done in the process 2-3,

$$W_{2-3} = - \int_{V_2}^{V_3} p dV = -p_2 (V_3 - V_2) = -(1.92 \times 10^5)(2.48 - 4.96)(10^{-4}) = 47.62 \text{ J}$$

$W_{3-1} = 0$, as the volume remains constant.

$$W_{\text{total}} = W_{1-2} + W_{2-3} + W_{3-1} = -76.05 + 47.62 + 0 = -28.43 \text{ J}$$

The negative sign indicates that there is a net work output from the three-process cycle.

3. In the above problem, the specific heats for nitrogen are $C_p = 1038.3 \text{ J/kg } ^\circ\text{C}$ and $C_v = 741.1 \text{ J/kg } ^\circ\text{C}$. Calculate the net heat added in the cycle.

In order to calculate heat added, we must know the temperature at each stage of cycle, i.e. T_2 and T_3 (T_1 is already given).

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = (423) (1/2)^{0.4} = 320.6 \text{ K}$$

Because $p_2 = p_3$,

$$T_3 = T_2 \frac{V_3}{V_2} = (320.6) \times \left(\frac{1}{2} \right) = 160.3 \text{ K}$$

As process 1-2 is adiabatic,

$$Q_{1-2} = 0$$

$$Q_{2-3} = mC_p(T_3 - T_2) = (0.001)(1038.3)(160.3 - 320.6) = -166.4 \text{ J}$$

$$Q_{3-1} = mC_v(T_1 - T_3) = (0.001)(741.1)(423 - 160.3) = 194.8 \text{ J}$$

$$\text{Net heat added} = Q = Q_{1-2} + Q_{2-3} + Q_{3-1} = 0 - 166.4 + 194.8 = 28.4 \text{ J}$$

Note: See that it is the negative of the work done calculated in Illustration 2.

4. A gas expands in a cylinder according to the relation $pV^{1.3} = \text{constant}$. The initial volume of the gas is 30 L and initial pressure is 30 atm. The final pressure is 15 atm. Calculate the work done on the face of piston by the pressure force of the gas.

Solution

$$W = \int_{V_1}^{V_2} p dV \text{ and } pV^{1.3} = \text{constant (say C) or } p = CV^{-1.3}$$

$$\therefore W = \int_{V_1}^{V_2} CV^{-1.3} dV = \frac{-C}{0.3} \left[V^{-0.3} \right]_{V_1}^{V_2}$$

$$\text{But } C = p_1 V_1^{1.3} = p_2 V_2^{1.3}$$

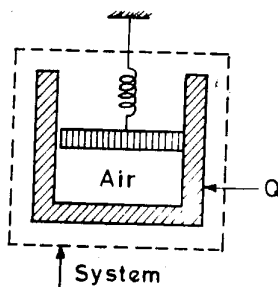
$$\text{so that } W = \frac{p_2 V_2 - p_1 V_1}{-0.3}$$

The final volume is

$$\begin{aligned} V_2 &= V_1 \left(\frac{p_1}{p_2} \right)^{1/1.3} = (30 \times 10^3 \times 10^{-6} \text{ m}^3) \left(\frac{30}{15} \right)^{1/1.3} \\ &= 30 \times 10^{-3} (2)^{0.77} \text{ m}^3 = 30 \times 10^{-3} \times 1.71 \\ &= 51.3 \times 10^{-3} \text{ m}^3 \end{aligned}$$

$$\therefore W = - \frac{15 \times 10^5 \times V_2 - 30 \times 10^5 \times 30 \times 10^{-3}}{0.3} = 4.35 \times 10^4 \text{ J}$$

5. Air is contained in a piston-cylinder arrangement as shown with a cross-sectional area of 4 cm^2 and an initial volume of 20 cc . The air is initially at a pressure of 1 atm . and temperature of 20°C . The piston is connected to a spring whose spring constant is $K = 10000 \text{ N/m}$, and the spring is initially undeformed. How much heat must be added to the air to increase the pressure to 3 atm . (For air, $C_V = 718 \text{ J/kg } ^\circ\text{C}$, molecular weight of air = 28.97)



Solution

$$F = kx$$

(i)

$$\text{and } W = \int F \cdot dx = \Delta PE \quad (\Delta PE \text{ is change in potential energy})$$

The thermodynamic system consists of both the spring and the piston cylinder. For this system, external work is zero.

$$\therefore Q = \Delta U = (\Delta PE)_{\text{spring}} + (\Delta E)_{\text{air}}$$

$$\begin{aligned} (\Delta U)_{\text{air}} &= m_a C_V \Delta T_a = (\Delta E)_{\text{air}} \Rightarrow m_a = \frac{p_1 V_1 M_a}{RT_1} \\ &= \frac{(1 \times 10^5)(20 \times 10^{-6})(28.97)}{(8315)(293)} = 2.41 \times 10^{-5} \text{ kg} \end{aligned}$$

Pressure changes from 1 atm to 3 atm . Thus, the change in pressure is

$(2 \times 1 \times 10^5)$ and therefore the force exerted on the piston is $F = (2 \times 1 \times 10^5 \times 4 \times 10^{-4}) = 80 \text{ N}$

The displacement of the spring is, therefore,

$$\begin{aligned} \frac{F}{K} &= \frac{80 \text{ N}}{10000 \text{ N/m}} \\ &= 0.0080 \text{ m} \end{aligned}$$

Additional volume swept out by piston = $A \Delta x$

$$= (4 \times 10^{-4} \text{ m}^2) (0.0080 \text{ m}) = 3.2 \times 10^{-6} \text{ m}^3$$

$$\therefore \text{Final volume} = V_2 = (V_1 + 3.2 \times 10^{-6}) \text{ m}^3$$

$$\begin{aligned} \text{Final temperature, } T_2 &= 293 \times \left(\frac{3}{1} \right) \left(\frac{23.2 \times 10^{-6}}{20.0 \times 10^{-6}} \right) \\ &= 1020 \text{ } ^\circ\text{K} = 747 \text{ } ^\circ\text{C} \end{aligned}$$

Change in the potential energy of spring,

$$(\Delta PE)_{\text{spring}} = \frac{1}{2} Kx^2 = \frac{1}{2} \times (10000 \text{ N/m}) (0.0080 \text{ m})^2 = 0.32 \text{ J}$$

Change in the internal energy of air,

$$(\Delta U)_{\text{air}} = (2.38 \times 10^{-5} \text{ kg}) (718 \text{ J/kg } ^\circ\text{C}) (747^\circ - 20^\circ\text{C}) = 12.43 \text{ J}$$

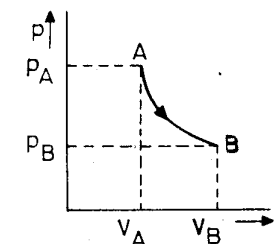
$$\therefore Q = 12.43 \text{ J} + 0.32 \text{ J} = 12.75 \text{ J}$$

6. 5 moles of an ideal gas is carried by a quasi-state isothermal process at 500 K to twice its volume.

(a) How much work was done by the gas (W_{AB})?

(b) How much heat was supplied to the gas (Q_{AB})?

(c) Calculate the pressure ratio p_f/p_i in the process.



Solution

(a) Work done in isothermal expansion

$$\begin{aligned} &= nRT \ln \frac{V_B}{V_A} \\ &= (5 \text{ moles}) (8.314 \text{ J/mol} \cdot \text{K}) (500 \text{ K}) (\ln 2) \\ &= 14407 \text{ J} \end{aligned}$$

(b) For an isothermal process, $U_{AB} = 0$. Therefore,

$$Q_{AB} = W_{AB} = 14407 \text{ J}$$

(c) $T_A = T_B$. We have

$$\frac{p_B}{p_A} = \frac{V_A}{V_B} = \frac{1}{2}$$

7. In the above problem, a constant-volume process is used to reduce the original pressure p_A to the same final pressure p_B . Determine the new values of W'_{AB} , U'_{AB} and Q'_{AB} .

Solution

For a constant-volume process, $W'_{AB} = 0$.

$$V'_B = V_A \quad \text{and} \quad p'_B = p_B = \frac{1}{2} p_A$$

$$T'_B = T_A \frac{p'_B}{p_A} = (500 \text{ K}) \times \frac{1}{2} = 250 \text{ K}$$

$$U'_{AB} = \frac{1}{2} \nu n R (T'_B - T_A)$$

where ν is the number of degrees of freedom. It is 3 for a monatomic gas.

$$\therefore U'_{AB} = \frac{1}{2} \times 3 \times (5 \text{ mole}) (8.314 \text{ J/mol} \cdot \text{K}) (250 \text{ K} - 500 \text{ K})$$

$$U'_{AB} = -15589 \text{ J}$$

Using the first Law, we get

$$Q'_{AB} = W'_{AB} + U'_{AB} = -15589 \text{ J}$$

8. 1 g of water at atmospheric pressure has a volume of 1 cc and on boiling becomes 1671 cc of steam. The heat of vaporization of water is 540 cal/g. Find the change in its internal energy in this process.

Solution

Heat required for vaporization is $Q = mL = 1 \times 540 \text{ cal}$

$$\begin{aligned} W &= p \Delta V = (1 \times 10^5 \text{ N/m}^2) (1671 - 1) \times 10^{-6} \text{ m}^3 = 167 \text{ J} \\ &= \frac{167}{4.186} \text{ cal} = 39.9 = 40 \text{ cal} \end{aligned}$$

$$\begin{aligned} \Delta U &= Q - W = mL - p \Delta V \\ &= 540 - 40 = 500 \text{ cal} \end{aligned}$$

Since this quantity is positive, it means that the internal energy of the system increases during this process. Thus, of the 540 cal supplied, 500 cal are added to the internal energy and 40 cal are used in doing external work to expand the system.

9. Gas is contained in a cylinder-piston system as shown in the figure. Over the piston, small weights are placed. The initial pressure of the gas is $2 \times 10^5 \text{ N/m}^2$ and initial volume is 0.04 m^3 . When the cylinder is heated

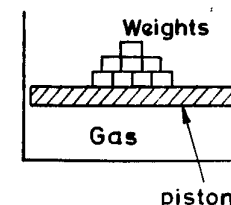
externally, the volume of the gas increases to 0.1 m^3 while the pressure remains constant.

(a) Calculate the work done by the system during the process.

(b) Everything remaining same, now when the piston is rising, the weights are removed from the piston at such a rate that the temperature remains constant during the process. What is the work done in this process?

(c) In the same system, during the heat transfer, the weights are removed at such a rate that the expression $pV^{1.3} = \text{constant}$ describes the relation between pressure and volume during the process and the final volume is 0.1 m^3 . Calculate the work done.

(d) Consider the system and initial state to be the same as in (a), (b) and (c) but let the piston be held by a pin so that the volume remains constant. In addition, let heat be transferred from the system until the pressure drops to $1 \times 10^5 \text{ N/m}^2$. Calculate the work done.



Solution

$$(a) \quad W_{1-2} = \int_1^2 p dV = 2 \times 10^5 (0.1 - 0.04) = 12.0 \text{ kJ}$$

(b) $pV = mRT$, and since it is an isothermal process, $n = 1$

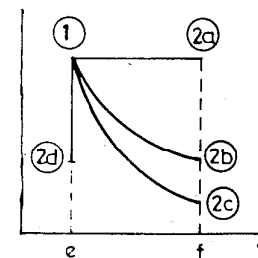
$$W_{1-2} = \int_1^2 p dV = p_1 V_1 \ln \frac{V_2}{V_1} = 2 \times 10^5 \times 0.04 \times \ln \frac{0.1}{0.04} = 7.3 \text{ kJ}$$

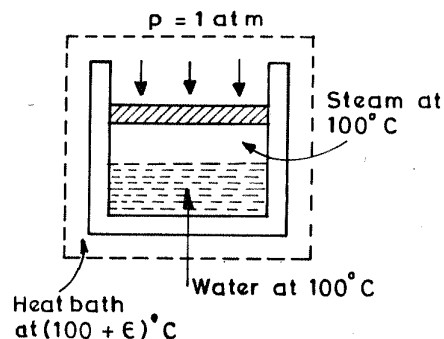
(c) Here, $n = 1.3$, and

$$p_2 = 2 \times 10^5 \left(\frac{0.04}{0.1} \right)^{1.3} = 60.77 \times 10^3 \text{ N/m}^2$$

$$\begin{aligned} W_{1-2} &= \int_1^2 p dV = \frac{p_2 V_2 - p_1 V_1}{1 - 1.3} \\ &= \frac{60.77 \times 10^3 \times 0.1 - 200 \times 10^3 \times 0.04}{-0.3} \\ &= 6.41 \text{ kJ} \end{aligned}$$

(d) Since the volume remains constant, the work done is zero. These processes are shown in the p - V diagram. 1-2a is a constant-pressure process and work done $W_{1-2} = \text{area (1-2a-f-e-1)}$. 1-2b represents the process in which $pV = \text{constant}$, 1-2c represents the process in which $pV^{1.3} = \text{constant}$, and 1-2d is a constant volume process.





10. A certain quantity (m) of water is placed in a cylinder that is closed with a frictionless tight-fitting piston. The cylinder has perfectly conducting walls and the entire assembly is immersed in a heat bath that has an adjustable temperature T . The external pressure on the piston is maintained at 1 atm. The system under consideration consists entirely of the original mass of water. The temperature of the heat bath is increased slowly so that water also attains the same temperature. Finally, the heat bath temperature is raised to just above 100°C ($100 + \epsilon$) and maintained constant so that heat flows into water and causes vaporization. Calculate the change in internal energy per unit mass.

Solution

The amount of heat required to vapourize $= Q = mL_V$. Due to the production of steam, the volume of the system increases and the piston is raised. At 1 atm and 100°C , the density of steam is ρ_s and density of water is ρ_w . Then the volume expansion of the system when a mass m is converted into steam at 100°C is

$$\Delta V = m \left(\frac{1}{\rho_s} - \frac{1}{\rho_w} \right) \quad \text{and} \quad W = p_0 \Delta V = mp_0 \left(\frac{1}{\rho_s} - \frac{1}{\rho_w} \right)$$

$$\Delta U = Q - W = mL_V - mp_0 \left(\frac{1}{\rho_s} - \frac{1}{\rho_w} \right)$$

$$\text{or} \quad \frac{\Delta U}{m} = L_V - p_0 \left(\frac{1}{\rho_s} - \frac{1}{\rho_w} \right)$$

Putting the values of L_V , p_0 and ρ_s and ρ_w in this equation, we can calculate the change in internal energy per unit mass of water that evaporates

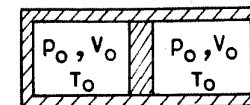
$$L_V = 2.257 \times 10^6 \text{ J/kg}, \quad p_0 = 1 \times 10^5 \text{ Pa}, \quad \rho_s = 0.598 \text{ kg/m}^3$$

and $\rho_w = 1040 \text{ kg/m}^3$. Therefore, we get

$$\frac{\Delta U}{m} = (2.257 \times 10^6) - (1 \times 10^5) \left(\frac{1}{0.598} - \frac{1}{1040} \right) = 2.088 \times 10^6 \text{ J/kg}$$

Notice that heat required to convert 1 kg of water to steam at 1 atm. and 100°C is $2.257 \times 10^6 \text{ J}$ while only $2.088 \times 10^6 \text{ J}$ appears as change in the internal energy. The remainder of the energy, $0.169 \times 10^6 \text{ J}$ is the work done by steam in expanding at constant pressure.

11. Two moles of monatomic gas occupy two chambers of a cylinder-piston system in which the piston is free to move and the walls of the cylinder and the piston are made of insulating material. The initial volumes, pressures and temperatures of the two chambers are the same— p_0 , V_0 and T_0 . The chamber in the left is heated internally by some device resulting in expansion of the gas pushing the piston to the right. The gas in the right chamber is compressed until the pressure becomes 32 times the initial pressure. Calculate: (a) the work done in compression, (b) the final temperatures of the two chambers, (c) the change in internal energy of the left chamber, and (d) the heat absorbed by the left chamber.



Solution

(a) No heat is absorbed or lost by the right chamber and hence the process is adiabatic compression. Let V_R be the volume and p_R the pressure of the right chamber after compression. Therefore,

$$p_0 V_0^\gamma = p_R V_R^\gamma \Rightarrow V_R = V_0 \left(\frac{p_0}{p_R} \right)^{1/\gamma} = V_0 \left(\frac{1}{32} \right)^{3/5} = \frac{V_0}{8}$$

$$\begin{aligned} \text{Work done on the gas} &= - \int_{V_0}^{V_R} p \, dV = p_0 V_0^\gamma \int_{V_0}^{V_R} \frac{dV}{V^\gamma} \\ &= - p_0 V_0^\gamma \left[\frac{V^{1-\gamma}}{1-\gamma} \right]_{V_0}^{V_0/8} \\ &= \left(\frac{p_0 V_0^{5/3}}{2/3} \right) \left[\frac{1}{(V_0/8)^{2/3}} - \frac{1}{(V_0)^{2/3}} \right] \\ &= \frac{9}{2} p_0 V_0 = \frac{9}{2} RT_0 \end{aligned}$$

(b) $p_R V_R = 32 p_0 \times \frac{V_0}{8}$ and since $n = 1$ in each chamber, $T_R = 4T_0$. For the mechanical equilibrium of the piston,

$$p_L = p_R \quad \text{and} \quad V_L = V_0 + \frac{7}{8} V_0 = \frac{15}{8} V_0$$

$$p_L V_L = 32 p_0 \times \frac{15}{8} V_0 = 60 n R T_0 \Rightarrow T_L = 60 T_0$$

$$(c) \quad \Delta U = n C_V dT = 1 \times \frac{3}{2} R \times 59 T_0 = 88.5 RT_0$$

$$(d) \quad \Delta Q = \Delta U + \Delta W = 88.5 RT_0 + 4.5 RT_0 = 93.0 RT_0$$

12. A 'floating' piston is fixed in a rigid cylinder which can move without friction. Initially, the piston is positioned such that it divides the cylinder in two halves on each side of the piston. The cylinder holds on each side 1 kmol of the same ideal gas at 300 K and 200 kPa. The temperature of A is raised slowly by means of a heating system till it reaches 500 K. The cylinder and piston are perfectly insulated and are of negligible heat capacity. Calculate the amount of heat added to the system. ($C_V = 12.56 \text{ J/mol} \cdot \text{K}$ and $C_p = 20.88 \text{ J/mol} \cdot \text{K}$)



Solution

$$\Delta U = Q - W \quad (W = 0 \text{ as no force external to the system moves}).$$

$$Q = n_A \Delta U_A + n_B \Delta U_B \quad (\text{here } n_A = n_B = 1000 \text{ mol})$$

$$\text{and } \Delta U = C_V \Delta T \quad (\text{for an ideal gas with constant heat capacity}).$$

$$\therefore Q = 1000 \times (C_V \Delta T_A + C_V \Delta T_B)$$

As heat is added to A, the temperature and pressure rise in A and the gas expands. This results in the compression of gas in B, and since there is no transfer of heat, the process in B is adiabatic. For a reversible adiabatic compression of an ideal gas with constant heat capacities, we have the relation

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma}$$

The total volume of the cylinder is V and is constant and is given by

$$V = \frac{(n_A + n_B) RT_1}{P_1} \quad (\text{initial condition})$$

Also,

$$V = \frac{(n_A RT_{A2} + n_B RT_{B2})}{P_2} \quad (\text{final condition})$$

Equating these two expressions and putting $n_A = n_B = 1$, we get

$$\begin{aligned} 2 \frac{p_2}{p_1} &= \frac{T_{A2}}{T_1} + \frac{T_{B2}}{T_1} \\ &= \frac{T_{A2}}{T_1} + \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} \end{aligned}$$

Putting the given values,

$$T_{A2} = 500 \text{ K}, T_1 = 300 \text{ K}, \frac{\gamma-1}{\gamma} = 1 - \frac{12.56}{20.88} = 0.40$$

Thus,

$$\frac{p_2}{p_1} = \frac{1}{2} \left[\frac{500}{300} + \left(\frac{p_2}{p_1} \right)^{0.4} \right] \Rightarrow \frac{p_2}{p_1} = 1.406$$

$$T_{B2} = T_1 \left(\frac{p_2}{p_1} \right)^{\gamma-1/\gamma} = 300 \times (1.406)^{0.4} = 300 \times 1.146 = 343.8$$

$$\therefore Q = 1000 \times 12.56 \times [(500 - 300) + (343.8 - 300)] = 3062 \text{ kJ}$$

Note: Such problems involve time-consuming calculations and students should try a quick trial solution. The purpose of including this illustration is precisely to show the complicated calculations.

13. A cylinder fitted with a piston contains 1 kg of methane gas at 700 kPa, 40°C. The piston cross-sectional area is 0.5 m² and the total external force restraining the piston is directly proportional to the cylinder volume squared. Heat is now transferred to the methane until its temperature reaches 1000°C. Calculate the heat transferred during the process, the work done by the methane and the final pressure inside the cylinder (C_V for methane is 50 J/mol · K)

Solution

$$n = \frac{m}{M} = \frac{1}{16} \text{ kmol} \quad \text{and} \quad p_1 = 700 \text{ kPa} = \frac{KV_1^2}{0.5}$$

$$\therefore KV_1^2 = 350 \text{ kPa} \text{ and } \Delta U = \left(\frac{1}{16} \right) \times 1000 (50 \text{ J/mol} \cdot \text{K}) \times (1273 - 313)$$

$$= \frac{1000}{16} \times 50 \times 960 = 3000 \text{ kJ}$$

$$\Delta U = Q - W \text{ and } W = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} 2KV^2 \, dV$$

$$\therefore W = \frac{2K}{3} [V_2^3 - V_1^3]$$

Also,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \text{or} \quad \frac{2KV_1^2 \times V_1}{313} = \frac{2KV_2^2 \times V_2}{1273}$$

$$\therefore V_2^3 = \left(\frac{1273}{313} \right) V_1^3 \Rightarrow V_2 = (1.595) V_1$$

$$\text{and } \left(\frac{V_2}{V_1} \right)^2 = (1.595)^2 \Rightarrow p_2 = p_1 \frac{V_2^2}{V_1^2} = 700 \text{ kPa} \times (1.595)^2$$

$$p_2 = 1780.8 \text{ kPa}$$

16 g of methane at STP will have a volume of 22.4 litres and therefore 1 kg of methane will have a volume of 1.4 m^3 (at STP). Then volume at 700 kPa and 313 K is

$$\frac{p_1 V_1}{T_1} = \frac{p_0 V_0}{T_0} \Rightarrow \frac{700 \times 10^3 \times V_1}{313} = \frac{100 \times 10^3 \times 1.4}{273}$$

$$\therefore V_1 = \frac{313 \times 1.4}{7 \times 273} \text{ m}^3 = 0.23 \text{ m}^3$$

$$\therefore K = \frac{(350 \times 1000) \text{ N/m}^2}{(0.23)^2 \text{ m}^6} = 6.6 \times 10^6 \text{ N/m}^8$$

$$W = \frac{2}{3} K (V_2^3 - V_1^3) = \frac{2}{3} K \left(\frac{1273}{313} - 1 \right) V_1^3$$

$$= \frac{2 \times 6.6 \times 10^6 \times 960 \times (0.23)^3}{313} \text{ J} = 4.9 \times 10^5 \text{ J}$$

$$Q = \Delta U + W = 3 \times 10^6 \text{ J} + 4.9 \times 10^5 \text{ J} = 3.5 \text{ MJ}$$

14. 0.5 kg of air is compressed reversibly and adiabatically from 80 kPa, 60°C to 0.4 MPa and is then expanded at constant pressure to the original volume. Sketch the process on the p - V diagram and compute the heat transfer and work transfer for the whole path (C_p for air = $1.005 \text{ kJ/kg} \cdot \text{K}$, $\gamma_{\text{air}} = 1.4$ and $R_{\text{air}} = 0.287 \text{ kJ/kg} \cdot ^\circ\text{C}$)

Solution

The p - V diagram for the whole process is shown

$$p_1 V_1 = mRT_1 \Rightarrow V_1 = \frac{mRT_1}{p_1}$$

$$= \frac{1 \times 0.287 \times 333}{2 \times 80} = 0.597 \text{ m}^3$$

Since the process 1-2 is reversible and adiabatic,

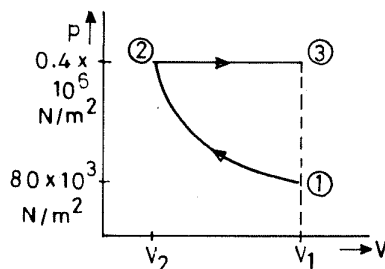
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} = \left(\frac{400}{80} \right)^{(1.4-1)/1.4} = (5)^{2/7} \Rightarrow T_2 = 527 \text{ K}$$

For process 1-2, the work done is

$$W_{1-2} = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1} = \frac{1/2 \times 0.287 \times (333 - 527)}{0.4}$$

$$= -69.6 \text{ kJ}$$

Again,



$$p_1 V_1^\gamma = p_2 V_2^\gamma \text{ or } \frac{p_1}{p_2} = \left(\frac{V_2}{V_1} \right)^\gamma = \frac{80}{400} = \frac{1}{5}$$

$$\frac{V_2}{V_1} = \left(\frac{1}{5} \right)^{1/\gamma} = \left(\frac{1}{5} \right)^{1/1.4} = \frac{1}{3.162} \Rightarrow V_2 = \frac{0.597}{3.162}$$

$$= 0.189 \text{ m}^3$$

For process 2-3,

$$W_{2-3} = p_2 (V_1 - V_2) = 400 (0.597 - 0.189) = 163.2 \text{ kJ}$$

$$\text{Total work transfer} = W_{1-2} + W_{2-3} = -69.6 + 163.2 = 93.6 \text{ kJ}$$

For states (2) and (3), $\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$ $\begin{bmatrix} p_2 = p_3 \\ V_3 = V_1 \end{bmatrix}$

$$\therefore T_3 = T_2 \frac{V_3}{V_2} = 527 \times 3.162 = 1667 \text{ K}$$

$$\left[\frac{V_3}{V_2} = \frac{V_1}{V_2} = \frac{0.597}{0.189} \right]$$

Total heat transfer to the gas,

$$Q = Q_{1-2} + Q_{2-3} = Q_{2-3} = mC_p (T_3 - T_2)$$

$$= \frac{1}{2} \times 1.005 (1667 - 527)$$

$$= 572.85 \text{ kJ}$$

15. A mass of air is initially at 260°C and 700 kPa and occupies 0.028 m^3 . The air is expanded at constant pressure to 0.084 m^3 . A polytropic process with $n = 1.50$ is then carried out followed by a constant-temperature process which completes a cycle. All the processes are reversible. Sketch the cycle in the p - V diagram. Find (a) the heat received, (b) the heat rejected in the cycle, and (c) the efficiency of the cycle ($C_v = 0.718 \text{ kJ/kg} \cdot \text{K}$)

Solution

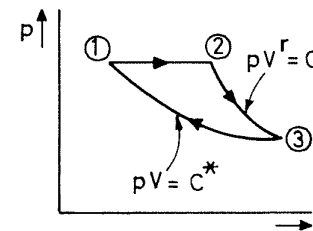
$$p_1 = 700 \text{ kPa},$$

$$T_1 = 260 + 273 = 533 \text{ K}$$

$$T_3 = 533 \text{ K}$$

$$V_1 = 0.028 \text{ m}^3,$$

$$V_2 = 0.084 \text{ m}^3$$



$$p_1 V_1 = mRT_1 \Rightarrow m = \frac{700 \times 0.028}{0.287 \times 533} = 0.128 \text{ kg}$$

Also,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \text{ or } \frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1} = \frac{V_2}{V_1} \text{ (as } p_2 = p_1)$$

$$= \frac{0.084}{0.028} = 3$$

$$\therefore T_2 = 3 \times 533 = 1599 \text{ K}$$

Again,

$$\frac{p_2}{p_3} = \left(\frac{T_2}{T_3}\right)^{n/(n-1)} = \left(\frac{1599}{533}\right)^{1.5/0.5} = (3)^3 = 27$$

$$Q_{1-2} = mC_p (T_2 - T_1) = 0.128 \times 1.005 \times (1599 - 533) = 137.13 \text{ kJ}$$

$$Q_{2-3} = \Delta U + \int p dV = mC_v \Delta T + \frac{mR(T_3 - T_2)}{n-1}$$

$$= mC_v \left(\frac{n-\gamma}{n-1}\right) (T_3 - T_2)$$

$$= 0.128 \times 0.718 \times \frac{1.5-1.4}{1.5-1} (533 - 1599)$$

$$= 0.128 \times 0.718 \times \frac{0.1}{0.5} (-1066) = -19.59 \text{ kJ}$$

For process 3-1,

$$dQ = dU + dW = dW$$

$$Q_{3-1} = W_{3-1} = \int_3^1 p dV = mRT_1 \ln \frac{V_1}{V_3} = mRT_1 \ln \frac{p_3}{p_1}$$

$$\therefore Q_{3-1} = 0.128 \times 0.287 \times 533 \times \ln \left(\frac{1}{27}\right)$$

$$= -0.128 \times 0.287 \times 533 \times 3.2959$$

$$= -64.53 \text{ kJ}$$

(a) Heat received in the cycle, $Q_1 = 137.13 \text{ kJ}$

(b) Heat rejected in the cycle, $Q_2 = 19.59 + 64.53 = 84.12 \text{ kJ}$

(c) The efficiency of the cycle

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{84.12}{137.13} = 1 - 0.61 = 0.39 \text{ or } 39\%$$

EXERCISES

- 1/10 kg of oxygen gas at 150 kPa at 20°C is contained in a cylinder fitted with a piston. Weights are slowly added to the piston and the gas is compressed isothermally to a final pressure of 600 kPa. The work done during the process is

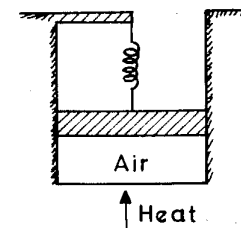
(a) $1.055 \times 10^4 \text{ J}$

(c) $2.1 \times 10^6 \text{ J}$

(b) $21 \times 10^4 \text{ J}$

(d) $2.1 \times 10^3 \text{ J}$

2. A system consists of a cylinder, piston and a spring as shown. The initial volume of the cylinder is 100 L and its pressure is 100 kPa so that it just balances the atmosphere pressure plus the piston weight. In this position, the spring connected to the piston exerts no force on it. Heat is now transferred to the system so as to expand air to double its volume, at which the pressure in the cylinder is 300 kPa. The work done by the system is (in case of a spring $F = -kx$)



- (a) 200 J (b) $2 \times 10^6 \text{ J}$
 (c) 20 kJ (d) 2 kJ
3. In the above problem, the percentage of work that is done against the spring is
 (a) 50 (b) 20
 (c) 75 (d) 100
 4. The work required to compress 5 mol. of air at 20°C and 1 atm. to 1/10 of the original volume in an isothermal process is
 (a) 280 J (b) $2.8 \times 10^3 \text{ J}$
 (c) 28 J (d) $2.80 \times 10^4 \text{ J}$
 5. The work required to compress 5 mol. of air at 20°C and 1 atm. to 1/10 of the original volume in an adiabatic process is (C_v for air = 20.68 J/mol. K, $\gamma_{\text{air}} = 1.403$)
 (a) 40.63 J (b) $4.63 \times 10^4 \text{ J}$
 (c) $4.63 \times 10^5 \text{ J}$ (d) 463 J
 6. A room of volume 1000 m³ contains air at a pressure of 10^5 N/m^2 and its temperature is 300 K. In the room, there is a balloon which also contains air at 300 K. Suddenly, the balloon bursts. Before any air can escape through any openings, the pressure in the room becomes $1.3 \times 10^5 \text{ N/m}^2$ while the temperature still remains at 300 K. If the volume of the balloon was 50 m³, the initial pressure of the air in the balloon is
 (a) $2 \times 10^5 \text{ N/m}^2$ (b) $7 \times 10^5 \text{ N/m}^2$
 (c) $4 \times 10^5 \text{ N/m}^2$ (d) $5 \times 10^5 \text{ N/m}^2$
 7. 10 kg of CO at 40°C occupies 3.0 m³. An additional mass of CO is then very slowly added to raise the pressure of the tank to $10 \times 10^5 \text{ N/m}^2$. If the temperature of the gas remains constant, the mass of extra gas added is
 (a) 22.3 kg (b) 33.2 kg
 (c) 23.2 kg (d) 3.22 kg
 8. Water initially at $5 \times 10^4 \text{ N/m}^2$ and 100°C is contained in a piston and cylinder arrangement with an initial volume of 3 m³. The water is then slowly compressed according to the relation $pV = \text{constant}$ until a final pressure of 10^6 N/m^2 is reached. The work done in this process is
 (a) -4.49 kJ (b) 4.49 kJ
 (c) 44.94 kJ (d) -449.4 kJ
 9. A balloon that is initially flat is inflated by filling it from a tank of compressed air. The final volume of the balloon is 5 m³. The barometer reads 95 kPa. The work done in this process is

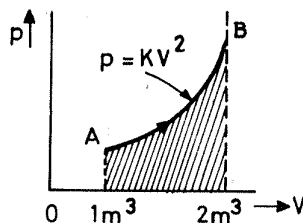
- (a) $475 \times 10^5 \text{ J}$ (b) $4.75 \times 10^7 \text{ J}$
 (c) $4.75 \times 10^8 \text{ J}$ (d) $4.75 \times 10^5 \text{ J}$

10. A cylinder fitted with a piston contains propane gas at a pressure of $1 \times 10^5 \text{ N/m}^2$, and temperature of 300 K. At this stage, the volume is 200 L. The gas is now slowly compressed according to the relation $pV^{1.1} = \text{constant}$ until the final temperature reaches 340 K. The work done during the process is

- (a) 267 J (b) -267 J
 (c) -26.7 kJ (d) 2.67 kJ

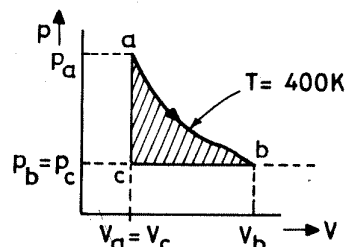
11. A sample of an ideal gas is expanded to twice its original volume of 1 m^3 in a quasi-static process for which $p = KV^2$ where K is a constant whose value is 5.0 atm/m^6 as shown in the figure. The work done by the expanding gas is

- (a) 11.78 J (b) 1178 J
 (c) $11.78 \times 10^5 \text{ J}$ (d) $11.78 \times 10^3 \text{ J}$



12. A 1-mol. sample of an ideal gas is carried around the thermodynamic cycle shown in the figure. The cycle consists of three steps:

- (i) an isothermal expansion ($a \rightarrow b$),
 (ii) an isobaric compression ($b \rightarrow c$), and
 (iii) a constant volume increase in pressure ($c \rightarrow a$). If $T_a = 400 \text{ K}$, $p_a = 4 \text{ atm}$, and $p_b = p_c = 1 \text{ atm}$, the work done by the gas per cycle is



- (a) 211.6 J (b) 1158 J
 (c) 705.3 J (d) 2116 J

13. Nitrogen gas ($m = 1.00 \text{ kg}$) is confined in a cylinder with a movable piston exposed to a normal atmospheric pressure. A quantity of heat $Q = 25.0 \text{ cal}$ is added to the gas in an isobaric process and the internal energy of the gas increases by 8.0 cal. The work done by the gas is

- (a) 7.1 J (b) $7.1 \times 10^4 \text{ J}$
 (c) $7.1 \times 10^3 \text{ J}$ (d) 71 J

14. The work done by an ideal gas with constant heat capacities during a quasi-static adiabatic expansion is given by the following expressions out of which one is incorrect. The one which is incorrect is

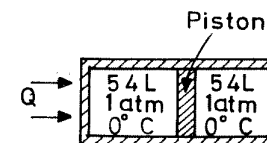
- (a) $W = C_V(\theta_i - \theta_f)$ (b) $W = \frac{p_i V_i - p_f V_f}{\gamma - 1}$
 (c) $W = \frac{p_i V_i}{\gamma - 1} \left[1 - \left(\frac{p_f}{p_i} \right)^{(\gamma-1)/\gamma} \right]$ (d) $W = \frac{V_i p_i}{\gamma - 1} \left[1 + \left(\frac{p_i}{p_f} \right)^{\gamma-1} \right]$

15. Helium ($\gamma = 5/3$) at 300 K and 1 atm. pressure is compressed quasi-statically and adiabatically to a pressure of 5 atm. If helium behaves like an ideal gas, the final temperature is

- (a) 591 K (b) 461 K
 (c) 489 K (d) 571 K

16. A horizontal insulated cylinder contains a frictionless non-conducting piston. On each side of the piston are 54 L of an inert monoatomic ideal gas at 1 atm and 0°C . Heat is slowly added from the left side until the piston compresses the gas to the right and the final pressure becomes 7.59 atm. The quantity of heat added on the left side is

- (a) $-8 \times 10^5 \text{ J}$ (b) $1.08 \times 10^5 \text{ J}$
 (c) $2.4 \times 10^5 \text{ J}$ (d) $1.2 \times 10^4 \text{ J}$



17. An exhausted chamber with nonconducting walls is connected through a valve to the atmosphere where the pressure is p_0 and the temperature is T_0 . The valve is opened slightly, and air flows into the chamber until the pressure within the chamber is p_0 . Assuming the air to behave like an ideal gas with constant heat capacities, the final temperature of the air in the chamber is

- (a) γT_0 (b) $\frac{T_0}{\gamma}$

- (c) $\left(\frac{1}{\gamma - 1} \right) T_0$ (d) $\left(\frac{\gamma}{1 - \gamma} \right) T_0$

18. A 1 kW electrical heater is placed in a chamber of capacity 10,000 L filled with air at a pressure of 1 atm. temperature of 30°C . The heater is allowed to operate for 15 minutes and the chamber is insulated. The temperature of the air at the end of this period is

- (a) 458 K (b) 314 K
 (c) 415 K (d) 395 K

19. On p - V coordinates, the slope of an isothermal curve of a gas at a pressure $P = 1 \text{ MPa}$ (megapascal) and volume $V = 0.0025 \text{ m}^3$ is equal to -400 MPa/m^3 . If $C_p/C_v = 1.4$, the slope of the adiabatic curve passing through this point is

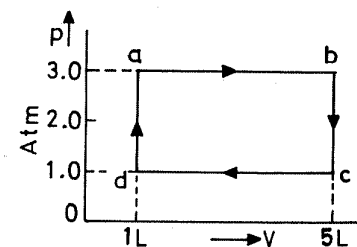
- (a) -560 MPa/m^3 (b) -56 MPa/m^3
 (c) 56 MPa/m^3 (d) 560 MPa/m^3

20. A reversible engine converts one-sixth of the heat input to work. When the temperature of the sink is reduced by 62°C , its efficiency is doubled. The temperature of the source and sink respectively are

- (a) 77 and 44°C (b) 88 and 54°C
 (c) 66 and 24°C (d) 99 and 37°C

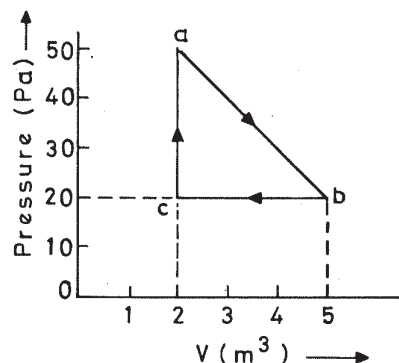
21. An ideal gas is carried around the cycle shown in the figure which consists of alternating constant-volume and constant-pressure paths. The amount of net heat supplied per cycle is

- (a) 8 J
 (b) 400 J
 (c) 80 J
 (d) 800 J



22. 1 g of water becomes 1671 cc of steam when boiled at a pressure of 1 atm. The heat of vaporization at this pressure is 2256 J/g. The increase in the internal energy is

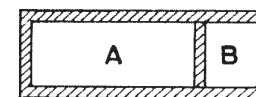
- (a) 208 J (b) 2087 J
(c) 870 J (d) 1042 J
23. 100 g of water is heated from 25 to 45°C. If the specific heat of water is 4184 J/kg. K, the change in the internal energy of water is [ignore the expansion of water]
(a) 6.2 kJ (b) 2.4 kJ
(c) 8.4 kJ (d) 4.8 kJ
24. A spring of force constant 400 N/m supports a mass of 0.5 kg which is immersed in 1 kg of water. The specific heat of the material of the supported mass is 450 J/kg. K. The spring is now stretched 20 cm, and after thermal equilibrium is reached, the mass is released so that it oscillates up and down. When the oscillations stop, the change in temperature of water recorded is
(a) 18.0 K (b) 0.18 K
(c) 0.018 K (d) 0.0018 K
25. A cube of side 5 cm, made of iron, and having a mass of 1500 g, is heated from 25°C to 400°C. The specific heat for iron is 0.12 cal/g°C and the coefficient of volume expansion is $3.5 \times 10^{-5}/^\circ\text{C}$, the change in the internal energy of the cube is (atm. pressure = $1 \times 10^5 \text{ N/m}^2$)
(a) 320 kJ (b) 282 kJ
(c) 141 kJ (d) 423 kJ
26. A reversible process for a closed PVT system is shown in the figure. The work done per cycle is



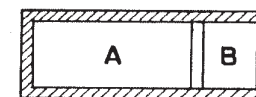
- (a) 45 J (b) 54 J
(c) 22.5 J (d) 32.5 J
27. Only one of the four statements given here is wrong. Tick the wrong one.
(a) A block weighing 50 kg is dragged through a distance of 100 m. If the coefficient of friction between the weight and the surface is 0.4, 4700 calories of heat are generated.
(b) An automobile weighing 500 kg is travelling at the speed of 100 km/h when the brakes are applied and it stops. The heat generated is 46 kcal.
(c) A lead bullet of mass 20 g travelling with a speed of 700 m/s strikes a metal plate and comes to a stop. The heat generated is 1170 cal.
(d) The height of Niagara Falls is 51.8 m. When water falls through this height, its temperature rises by 1.2°C.
28. Only one of the four statements given here is wrong. Tick the wrong one.
(a) A metallic wire of initial length L_0 is stretched within elastic limits. The

area of cross-section is A , modulus of elasticity is Y and strain is e . The amount of heat produced in stretching is $\frac{AYL_0}{2J}(e)^2$, where J is the mechanical equivalent of heat.

- (b) A 40 W incandescent lamp is placed in a calorimeter containing 200 cc of water. The temperature rise per minute is 2.9°C.
(c) A spring with a spring constant of 1 kg/cm is held compressed by 5 cm, and placed in acid and dissolved. The extra heat evolved by the chemical reaction by virtue of compression is 0.3 cal.
(d) An electric mixer rotates 1000 times a minute. The torque acting on it is 1 N.m. The heat generated per minute is 150 cal.
29. A 0.5 hp motor is stirring 4 kg of water. Assuming there is no heat loss except in heating the water, the time it will take to raise the temperature of the water by 5°C is
(a) 4.73 min (b) 7.34 min
(c) 3.74 min (d) 3.47 min
30. 4 kg of oxygen gas is heated so as to raise its temperature from 20 to 120°C. If the heating is done at constant pressure, the external work done by the gas is ($C_p = 0.219 \text{ cal/g}^\circ\text{C}$ and $C_v = 0.157 \text{ cal/g}^\circ\text{C}$)
(a) 103 kJ (b) 628.8 kJ
(c) 206 kJ (d) 366 kJ
31. A system comprises 7.14 g of neon gas at 0°C and 1 atm. when 2025 J of heat is added to the system at constant pressure. The resultant expansion causes the system to perform 810 J of work. The final state of the system is [atomic weight of Ne = 20.2]
(a) $P = 1 \text{ atm}$, $T = 546 \text{ K}$, $V = 16 \text{ L}$
(b) $P = 1 \text{ atm}$, $T = 323 \text{ K}$, $V = 8 \text{ L}$
(c) $P = 2 \text{ atm}$, $T = 546 \text{ K}$, $V = 4 \text{ L}$
(d) $P = 1 \text{ atm}$, $T = 546 \text{ K}$, $V = 4 \text{ L}$
32. Two ideal gases A and B, each of n kmol, occupy two chambers of an isolated insulated cylinder shown in Fig. (a). The volume of chamber A is twice that of B and the partition between them is rigid and nonconducting. The pressures in A and B are equal. Thus the temperature of gas A is twice that of gas B. If the partition is made conducting, as shown in Fig. (b), the temperature, volume and pressure will be
(a) $T_A = 2T_B$, $V_A = 2V_B$ and $p_A = p_B$
(b) $T_A = T_B$, $V_A = 2V_B$ and $p_A = p_{B/2}$
(c) $T_A = 2T_B$, $V_A = 2V_B$ and $p_A = p_{B/2}$
(d) $T_A = T_B$, $V_A = 2V_B$ and $p_A = 2p_B$
33. 1 kmol. of an ideal gas is confined inside a cylinder of cross-section 0.1 m² such that the piston is at a distance of 0.5 m from the closed end. The gas is allowed to expand such that the piston moves 0.5 m further away from the closed end. If the initial temperature of the gas is 27°C, the work done due to expansion under isothermal conditions is
(a) $7.13 \times 10^6 \text{ J}$ (b) $5.76 \times 10^6 \text{ J}$
(c) $1.73 \times 10^6 \text{ J}$ (d) $3.71 \times 10^6 \text{ J}$

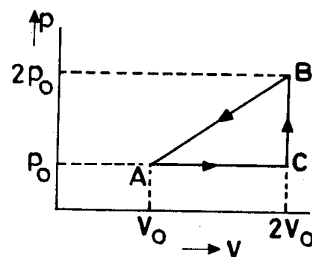
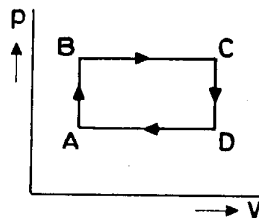


(a)

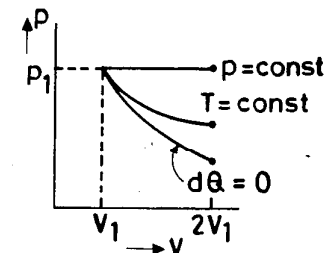
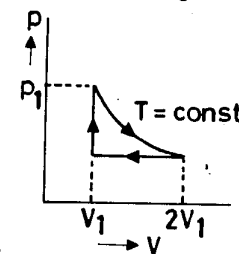


(b)

34. If, in the above problem, the pressure is kept constant, the work done in the process is
 (a) $4.98 \times 10^6 \text{ J}$ (b) $4.29 \times 10^6 \text{ J}$
 (c) $9.42 \times 10^6 \text{ J}$ (d) $2.449 \times 10^6 \text{ J}$
35. A thermodynamic transformation is shown in the p - V diagram. The system goes from state A to state C along the path ABC absorbing 50 J of heat and doing 30 J of work on the surroundings. On the return trip from C to A through CDA, an amount of 20 J of work is done on the system. The heat absorbed on the return trip is
 (a) 0 J (b) 70 J
 (c) 10 J (d) 50 J
36. 1 kmol. of a monoatomic gas is taken from a point A to another point B along the path ACB. The initial temperature at A is T_0 . The heat absorbed by the gas in the process $A \rightarrow C \rightarrow B$ is
 (a) $\frac{9}{2} RT_0$
 (b) $\frac{5}{2} RT_0$
 (c) $\frac{11}{2} RT_0$ (d) $\frac{15}{2} RT_0$
37. In the above problem, instead of following the path $A \rightarrow C \rightarrow B$, the gas is taken from A to B through the straight path AB, the heat absorbed would be
 (a) RT_0 (b) $6RT_0$
 (c) $4RT_0$ (d) $\frac{3}{2} RT_0$
38. A cylindrical aluminium bottle contained air at 10.33 bar and 300 K. When placed on a scale, it weighed 18 kg. Some air was let out and then the contents were at 5.165 bar and 250 K and the weight was 12 kg. The quantity of air released and volume of the cylinder are respectively
 (a) 6 kg and 1.25 m^3 (b) 4 kg and 1.25 m^3
 (c) 6 kg and 2.5 m^3 (d) 6 kg and 0.25 m^3
39. A closed system undergoes a process $1 \rightarrow 2$ for which the values W_{1-2} and Q_{1-2} are 50 kJ and -20 kJ respectively. If the system is returned to state 1 and $Q_{2 \rightarrow 1}$ is $+10 \text{ kJ}$, the work $W_{2 \rightarrow 1}$ is
 (a) 50 kJ (b) 40 kJ
 (c) -50 kJ (d) -60 kJ
40. A closed system undergoes a change of state by process $1 \rightarrow 2$ for which $Q_{12} = 10 \text{ J}$ and $W_{12} = -5 \text{ J}$. The system is now returned to its initial state by a different path $2 \rightarrow 1$ for which Q_{21} is -3 J . The total energy change for the cycle is
 (a) -8 J (b) zero
 (c) -2 J (d) $+5 \text{ J}$
41. Mark the wrong statement here.



- (a) 1 kg of air at a pressure of 2 bar and temperature 300 K is isothermally compressed until the volume is halved. The work required is 59.7 kJ.
 (b) 1 kg of air at a pressure of 4 bar and temperature 300 K expands isothermally to double the initial volume. The work input is -59.7 kJ .
 (c) One-tenth of 1 kg of air is contained in a piston-cylinder apparatus at 1 bar, 300 K. Heat is added to the air at constant pressure until the volume doubles. In the process, 10 kJ of work is performed. The quantity of heat transferred is 32.1 kJ.
 (d) When the valve of an evacuated bottle is opened, atmospheric air rushes into it. If the atmospheric pressure is 101.325 kPa and 0.6 m^3 of air enters into the bottle, the work done by air is 80.6 kJ.
42. A gas is compressed in a piston-cylinder device in such a manner that the relation between the pressure and the volume is $p = a + bV$ where a and b are constants. Initially, the gas is at 5 bar, 25 cm^3 , and the final state is 20 bar, 5 cm^3 . The work done on the system is
 (a) 0.0250 kJ (b) 250 kJ
 (c) 0.25 kJ (d) 25 kJ
43. A gas expands in a piston-cylinder device from V_1 to V_2 , the process being described by $p = aV^{-1} + b$ where p is in bars, and V in cubic metres. The work done in the process is
 (a) $a \ln \left(\frac{V_1}{V_2} \right) + b(V_2 - V_1)$ (b) $-a \ln \left(\frac{V_2}{V_1} \right) - b(V_2 - V_1)$
 (c) $-a \ln \frac{V_1}{V_2} - b(V_2 - V_1)$ (d) $a \ln \left(\frac{V_2}{V_1} \right) + b(V_2 - V_1)$
44. 8 g of helium in a container is initially at a temperature of 27°C . A movable piston forms the upper wall of the container. The gas is heated until the volume has increased to twice its initial volume. The total heat added to the gas is [p is given to be 1 atm. and the mass of the piston is negligible]
 (a) $1.2 \times 10^4 \text{ J}$ (b) $1.2 \times 10^3 \text{ J}$
 (c) $2.4 \times 10^4 \text{ J}$ (d) $2.4 \times 10^3 \text{ J}$
45. One mole of a gas is carried through the cycle shown in the figure. The gas expands at constant temperature from volume V_1 to $V_2 = 2V_1$. It is then compressed to the initial volume at constant pressure and is finally brought back to its original state by heating at constant volume. The work done by the gas per cycle in terms of initial temperature T_1 is
 (a) RT_1 (b) $2RT_1$
 (c) $0.38T_1$ (d) $0.19 RT_1$
46. The work is done by n moles of an ideal monoatomic gas which expands from a volume V_1 to $2V_1$. The initial temperature of the gas is T_1 and the expansion takes place at (a) constant pressure (b) constant temperature, and (c) adiabatically (as shown in the figure). The change in internal energy is ΔU .

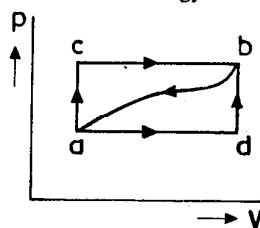
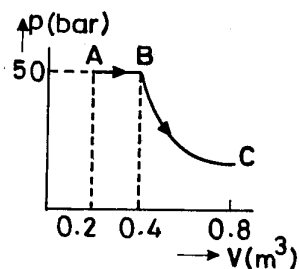
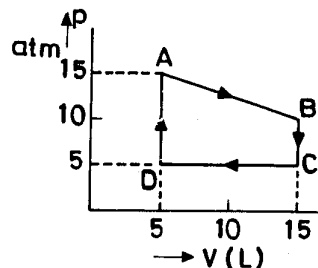


Corresponding to the three processes a, b and c, tick the statement which is wrong:

- (a) $W_a = nRT_1$ and $\Delta U_a = 1.5 nRT_1$
 (b) $W_b = \ln 2nRT_1$ and $\Delta U_b = 0$
 (c) $W_b = \ln 2nRT_1$ and $\Delta U_b = 1.5 nRT_1$
 (d) $W_c = 0.55 nRT_1$ and $\Delta U_c = -0.55 nRT_1$
47. When one gram of boiling water is converted into steam at a pressure of one atm; the steam occupies a volume of 1671 cm^3 (Latent heat of vaporization of water is 9.70 kcal/mol at 1 atm). The increase in its internal energy is
 (a) 2086 J (b) 4172 J
 (c) 1043 J (d) 286 J
48. During the charging of a storage battery, the current is 22 A and the voltage is 12 V. The rate of heat transfer from the battery is 12 W. The rate of change of internal energy is
 (a) 504 J/s (b) 252 J/s
 (c) 25.2 J/s (d) 126 J/s
49. A vessel containing a fluid is stirred by a paddle wheel. The work input to the paddle wheel is 4000 kJ. The heat transfer from the tank is 1200 kJ. Considering the vessel and the fluid as the system, the change in the internal energy of the system is
 (a) 5200 kJ (b) 1400 kJ
 (c) 5600 kJ (d) 2800 kJ
50. A cylinder fitted with a piston has an initial volume of 0.1 m^3 and contains nitrogen at 150 kPa and at 25°C . The piston is moved, compressing the nitrogen until the pressure is 1 MPa and the temperature is 150°C . During the compression process heat is transferred from the nitrogen and the work done on the nitrogen is 20 kJ. The amount of heat transfer is [$C_v = 0.745 \text{ kJ/kg}^\circ\text{C}$]
 (a) 15.8 kJ (b) -4.2 kJ
 (c) -35.8 kJ (d) 8.4 kJ
51. A system consists of 7.14 g of neon gas at 0°C and 1 atm. pressure when 2025 J of heat is added to the system at constant pressure the resultant expansion causes the system to perform 810 J of work. The change in its internal energy and enthalpy are respectively
 (a) 2430 J, 4050 J (b) 4050 J, 2430 J
 (c) 1215 J, 2025 J (d) 2025 J, 1215 J
52. The internal energy U is a unique function of any state because change ΔU
 (a) corresponds to an isothermal change
 (b) corresponds to an adiabatic change
 (c) depends only on the path
 (d) does not depend on the path.
53. The plot of the isotherms will not be a straight line when it is a plot between
 (a) p and T (b) V and p
 (c) S (entropy) and T (d) pV and V
54. The following is an incorrect statement based on the first law of thermodynamics:
 (a) The energy of an isolated system remains constant.
 (b) The heat transfer equals the work plus the energy change.
 (c) The heat transfer cannot exceed the work done.
 (d) The net heat transfer equals the net work done for a cycle.
55. The difference between C_p and C_v is equal to the universal gas constant when
 (a) one gram of gas is heated
 (b) any amount of gas is heated

- (c) one molecule of the gas is heated
 (d) one gram molecule is heated
56. A rigid 250-L tank contains methane gas at 500°C , 600 kPa. The tank is then cooled to -80°C . The final pressure and heat transfer for the process are (C_v for methane = 38 J/mol.K)
 (a) 150 kPa; -513 kJ (b) 15 kPa; 513 kJ
 (c) 15 kPa; 51.3 kJ (d) 150 kPa; 513 kJ
57. A cylinder with a piston restrained by a linear spring contains 1 kg of carbon dioxide at 400 kPa, 400°C . This system is now cooled at 40°C , at which point the pressure is 200 kPa. The heat transferred in the process is [C_v of $\text{CO}_2 = 0.653 \text{ kJ/kg.K}$]
 (a) 120.8 kJ (b) 241.7 kJ
 (c) 24.2 kJ (d) 72.6 kJ
58. The density of air is 1.3 kg/m^3 at NTP and $\gamma = 1.4$. If the pressure of the air is $1 \times 10^5 \text{ N/m}^2$, the velocity of sound waves in air in these conditions will be
 (a) 228 m/s (b) 250 m/s
 (c) 300 m/s (d) 328 m/s
59. 1 m^3 of an ideal gas is at a pressure of 10^5 N/m^2 and temperature 300 K. The gas is allowed to expand at constant pressure to twice its volume by supplying heat. If the change in the internal energy of the gas is 10^4 J , the heat supplied is
 (a) $1.1 \times 10^5 \text{ J}$ (b) $11 \times 10^5 \text{ J}$
 (c) $1.1 \times 10^4 \text{ J}$ (d) $1.1 \times 10^3 \text{ J}$
60. An electric storage battery is well insulated thermally while being recharged. The charging voltage is 12.0 V and the current is 25 A. The work done in a period of 30 minutes is
 (a) $5 \times 10^4 \text{ J}$ (b) $5.4 \times 10^3 \text{ J}$
 (c) $54 \times 10^4 \text{ J}$ (d) $4.5 \times 10^4 \text{ J}$
61. Among the following processes, only one is irreversible. Mark the one which is irreversible.
 (a) isothermal expansion or compression
 (b) electrolysis
 (c) free expansion
 (d) extension of a spring
62. A reversible engine converts one-fifth of the heat input into work. When the temperature of the sink is reduced by 50°C , its efficiency is doubled. The temperature of the source and the sink respectively are
 (a) 423 K and 373 K (b) 250 K and 200 K
 (c) 630 K and 440 K (d) 250 K and 200 K
63. During an expansion process, the volume of a gas changes from 4 to 6 m^3 while the pressure changes according to the relation $p = 25V + 75$ where pressure is in bars and volume is in cubic metres. The work done by the gas in joules is
 (a) 40×10^6 (b) 1×10^6
 (c) 28×10^6 (d) 21×10^6
64. The internal energy of an ideal gas decreases by the same amount as the work done by the system
 (a) the process must be isothermal
 (b) the process must be idiabatic
 (c) the process must be isobaric
 (d) the temperature of the system must increase

65. A cube of copper of side 10 cm is heated from 30 to 200°C. The change in its internal energy is (for copper, $\rho = 8.9 \text{ g/cc}$, sp. heat = $0.39 \text{ J/g}^\circ\text{C}$, coeff. of volume expansion = $5 \times 10^{-5}/^\circ\text{C}$)
 (a) 9370 kJ (b) 93.7 kJ
 (c) 590 kJ (d) 950 kJ
66. In an adiabatic expansion, a gas does 20 J of work while in an adiabatic compression, 100 J of work is done on a gas. The change of internal energy in the two processes respectively are
 (a) 20 J and -100 J (b) -20 J and 100 J
 (c) -20 J and -100 J (d) 20 J and 100 J
67. 5 g of a gas is carried through a cycle ABCDA in a piston cylinder assembly as shown. If in the portion from A to B, 5.5 kcal of heat flows into the gas and the temperature of the gas at A is 427°C, the C_V for the gas is
 (a) 0.49 kJ/kg°C
 (b) 0.298 kJ/kg°C
 (c) 3.03 kJ/kg°C
 (d) 0.98 kJ/kg°C
68. One kmol. of nitrogen gas at NTP expands isobarically to twice its initial volume. The work done by the gas in expansion is (atm. pressure = $1 \times 10^5 \text{ N/m}^2$)
 (a) 44.8 MJ (b) 4.48 MJ
 (c) 1.12 MJ (d) 2.24 MJ
69. 2 kmol. of hydrogen at NTP expands isobarically to twice its initial volume. The change in its internal energy is ($C_V = 10 \text{ kJ/kg.K}$ and atm pressure = $1 \times 10^5 \text{ N/m}^2$)
 (a) 10.9 MJ (b) 9.10 MJ
 (c) 109 MJ (d) 1.09 MJ
70. A gas system expands, following the process shown in the figure. For the process B to C, $pV^{1.3} = \text{const.}$ The total work done by the gas system is
 (a) 2.25 MJ
 (b) 1.475 MJ
 (c) 4.42 MJ
 (d) 0.74 MJ
71. A stationary mass of a gas is compressed without friction from an initial state of 0.3 m^3 and $0.105 \times 10^6 \text{ N/m}^2$ pressure to a final state of 0.15 m^3 and $0.105 \times 10^6 \text{ N/m}^2$ pressure. There is a transfer of 37.6 kJ of heat from the gas during the process. The change in the internal energy of the gas is
 (a) 218 kJ
 (b) 2.18 kJ
 (c) 21.85 kJ
 (d) 0.218 kJ
72. When a system is taken from state *a* to *b*, along a path *acb*, 84 kJ of heat flow into the system and the system does 32 kJ of



work. The following conclusions are drawn. Mark the one which is not correct.

- (a) if the work done along the path *adb* is 10.5 kJ, the heat that will flow into the system is 62.5 kJ.
 (b) when the system is returned from *b* to *a* along the curved path, the work done on the system is 21 kJ, and the system absorbs 73 kJ of heat
 (c) if $U_a = 0$, $U_d = 42 \text{ kJ}$, heat absorbed in the process *ad* is 52.5 kJ
 (d) if $U_a = 0$, $U_d = 42 \text{ kJ}$, heat absorbed in the process *db* is 10 kJ
73. A cyclic heat engine operates between a source temperature of 800°C and a sink temperature of 30°C. The least rate of rejection per kilowatt of net output of the engine is
 (a) 39 kW (b) 3.9 kW
 (c) 0.0392 kW (d) 0.392 kW
74. A certain gas has $C_p = 1.968$ and $C_V = 1.507 \text{ kJ/kg.K}$ and its molecular weight is 18.04 kg/mol. It is contained in a constant volume chamber of capacity 0.3 m^3 and its mass is 2 kg. The temperature is 5°C. Heat is transferred to the gas until the temperature is 100°C. Identify the statement which is wrong:
 (a) the work done in the process is zero
 (b) change in internal energy is 286.33 kJ
 (c) change in enthalpy is 173.92 kJ
 (d) change in entropy is 0.921 kJ/K
75. Two systems are in thermal equilibrium if
 (a) no heat flows between them
 (b) they are at rest or in motion with constant velocity
 (c) the average kinetic energy of their molecules is zero
 (d) they are each in thermal equilibrium with a common third system.
76. The state of a system is changed from that in which its thermodynamic variables have the values p_i, V_i, T_i to one described by the values p_f, V_f and T_f . If the equation of the state for the system is known, one should be able to calculate
 (a) the work done on the system
 (b) the amount of heat added to the system
 (c) the change in the internal energy of the system
 (d) the total heat content of the system.
77. In an adiabatic process
 (a) the temperature of the system remains constant
 (b) the temperature of the system must change
 (c) the internal energy of the system remains constant
 (d) no work is done.
78. The volume of an ideal gas is doubled in an adiabatic expansion. According to the second law of thermodynamics
 (a) its internal energy does not change
 (b) its temperature after the expansion is the same as before it
 (c) its entropy remains constant
 (d) its pressure afterwards is half its initial value.
79. A process is said to be reversible if
 (a) the net change in entropy of the system and the surroundings as the system passes from its initial state to the final state and then back to the initial state is zero
 (b) the system involved can be returned in any fashion to its initial state
 (c) there is no heat exchange between the system and the environment
 (d) it is cyclic.

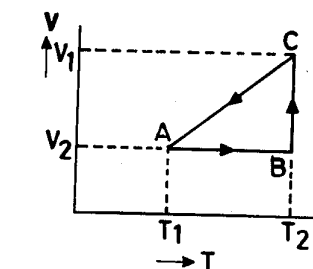
80. The Carnot cycle is an important conceptual device because
- it shows how to construct a perpetual motion device
 - it provides a means of determining the maximum possible efficiency of a heat engine
 - it is the design basis for most practical heat engines
 - it shows us why we should use internal combustion engines rather than steam engines.
81. One mole of an ideal gas expands at a constant temperature 300 K from an initial pressure of 300 atm to a final pressure of 5 atm. The volume of the initial and final states and the work done by the gas in expansion are respectively:
- 0.82 L, 4.92 L and 1.02×10^4 J
 - 0.082 L, 0.492 L and 1.02×10^3 J
 - 0.082 L, 4.92 L and 1.02×10^4 J
 - 0.82 L, 0.0492 L and 1.02×10^4 J
82. The equation of a state of a gas is given by $p(V - b) = nRT$. If 1 kmol of a gas is isothermally expanded from volume V to $2V$, the work done during the process is

$$(a) RT \ln \left[\frac{2V - b}{V - b} \right] \quad (b) RT \ln \left[\frac{V - b}{2V - b} \right]$$

$$(c) RT \ln \left[\frac{V}{V - b} \right] \quad (d) RT \ln \left[\frac{V - b}{V} \right]$$

83. A monoatomic ideal gas having an initial volume of 0.2 m^3 and a pressure of $5 \times 10^5 \text{ N/m}^2$ absorbs $5 \times 10^4 \text{ J}$ of heat in an isobaric process. The change in its internal energy is
- $2 \times 10^4 \text{ J}$
 - $5 \times 10^4 \text{ J}$
 - $2.5 \times 10^4 \text{ J}$
 - $3 \times 10^4 \text{ J}$

84. A cyclic process for 1 kmol of an ideal gas is shown in the V - T diagram. The work done in AB, BC and CA respectively is



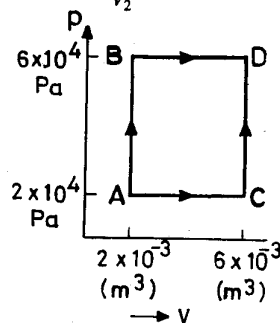
$$(a) 0, RT_2 \ln \frac{V_1}{V_2}, R[T_1 - T_2]$$

$$(b) R(T_1 - T_2), 0, RT_1 \ln \frac{V_1}{V_2}$$

$$(c) 0, RT_1 \ln \frac{V_1}{V_2}, R[T_1 - T_2]$$

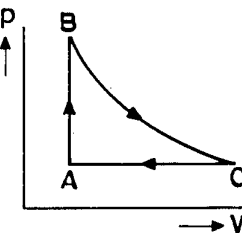
$$(d) 0, RT_2 \ln \frac{V_1}{V_2}, R[T_2 - T_1]$$

85. A thermodynamic process is shown in the figure. In the process AB, 500 J of heat are added and in process BD, 150 J of heat are added. The change in the internal energy in the process ABD is



- 890 J
- 410 J
- 650 J
- 240 J

86. In a diesel engine, air is compressed to $1/20$ of its initial volume. If the initial pressure is 1 atm. and the initial temperature is 27°C , the final pressure and temperature after the compression are [$\gamma_{\text{air}} = 1.40$]
- 50 atm. and 521°C
 - 72 atm. and 994°C
 - 66 atm. and 994°C
 - 66 atm. and 721°C
87. A gas is confined in a vertical cylinder fitted with a piston from above. The mass of the piston is 0.5 kg and the cross-sectional area is 20 cm^2 . The gas expands raising the piston by 4 cm, the external pressure being 1 atm. The work done by the gas is
- 8.16 J
 - 2.04 J
 - 0.816 J
 - 50 J
88. In the above problem, the system is supplied 50 J of heat. The change that takes place in the enthalpy of the gas is
- 40 J
 - 25 J
 - 100 J
 - 50 J

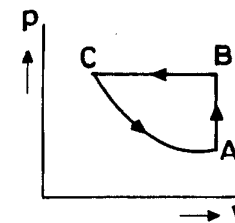


89. One mole of a monoatomic ideal gas is taken through a reversible cycle ABC as shown in the figure. The process BC is adiabatic. The work done per cycle is (given that temperatures of A, B and C are 300, 600 and 450 K)

- 75 R
- 150 R
- 112.5 R
- 45 R

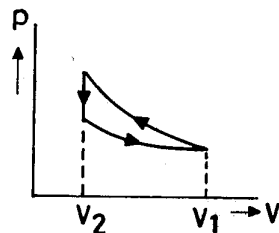
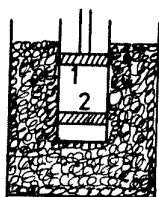
90. A certain mass of gas is taken through the cycle ABCA as shown in the figure.

- In $A \rightarrow B$, 10^6 J of heat is added to the system. In $B \rightarrow C$, an amount of work $2.4 \times 10^5 \text{ J}$ is done on it and the system gives out $1.1 \times 10^6 \text{ J}$ of heat. CA is an adiabatic process. The work done in CA is
- $14 \times 10^6 \text{ J}$
 - $1.4 \times 10^6 \text{ J}$
 - $0.14 \times 10^5 \text{ J}$
 - $0.14 \times 10^6 \text{ J}$

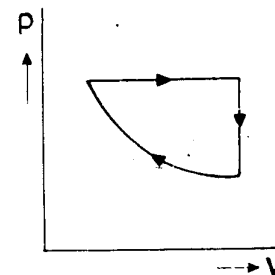
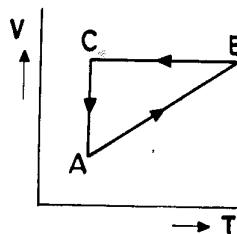


91. One mole of a gas is isothermally expanded at 27°C till its volume is doubled. Then it is adiabatically compressed to its original volume [$\gamma = 1.4$ and $R = 8.4 \text{ J/mol K}$]. The total amount of work done is:
- 270 J
 - 270 J
 - 540 J
 - 540 J
92. 2 m^3 volume of a gas at a pressure of $4 \times 10^5 \text{ N/m}^2$ is compressed adiabatically so that its volume becomes $1/4$ of its original volume. The resulting pressure is say, p_1 . The same process is repeated but isothermally. The resulting pressure is p_2 . The ratio of p_1 and p_2 is ($\gamma = 1.4$)
- 4.71
 - 1.47
 - 1.74
 - 2.94
93. A volume of air at 27°C expands adiabatically until its volume is increased four times. If $\gamma = 1.5$, the resulting temperature is
- -23°C
 - 12.3°C
 - 2.3°C
 - -123°C

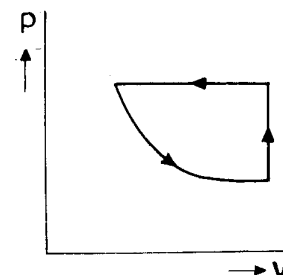
94. One mole of a perfect gas is compressed adiabatically. The initial pressure and the volume of the gas are 10^5 N/m^2 and 6 L respectively. The final volume of the gas is 2 L (molar specific heat at constant volume $= 3R/2$). The amount of work done in the process is
 (a) 972 J (b) 597 J
 (c) -597 J (d) 957 J
95. A metallic piston-cylinder system containing gas is immersed in an ice-water mixture as shown in the figure. The piston is quickly pushed down from position (1) to piston (2) and is held at (2) until the gas is again at 0°C . It is then slowly raised back to position (1) (the p - V diagram for the process is shown). If this results in melting 50 g of ice, the amount of work done on the gas is ($L = 80 \text{ cal/g}$)



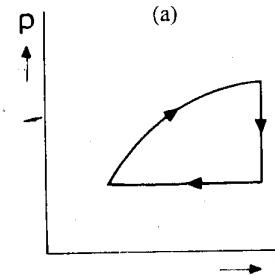
- (a) $1.7 \times 10^4 \text{ J}$ (b) $4 \times 10^4 \text{ J}$
 (c) $1.7 \times 10^3 \text{ J}$ (d) $0.17 \times 10^4 \text{ J}$
96. A Carnot engine is made to work between 200°C and 0°C first and then between 0°C and -200°C . The ratio of efficiencies (η_2/η_1) of the engine in the two cases is
 (a) 1 : 1.5 (b) 1 : 1
 (c) 1 : 2 (d) 1.73 : 1
97. One of the most efficient engines ever developed operates between 2100 K and 700 K. Its actual efficiency is 40%. The percentage fraction of its actual efficiency to its maximum efficiency is
 (a) 48% (b) 60%
 (c) 72% (d) 55%
98. A reversible engine takes in heat from a reservoir of heat at 527°C and rejects heat to the sink at 127°C . If it has to produce useful mechanical work at the rate of 750 W, it must take heat from the reservoir at the rate of
 (a) 557 cal/s (b) 753 cal/s
 (c) 177.5 cal/s (d) 357 cal/s
99. A cyclic process ABCA shown in the V - T diagram is performed with a constant mass of an ideal gas. If CA is parallel to the V -axis and BC is parallel to T -axis, the same process in the p - V diagram is



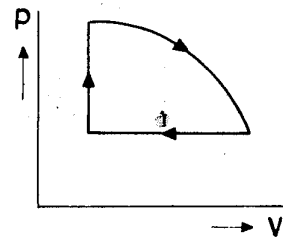
(a)



(b)



(c)



(d)

100. One mole of an ideal gas with heat capacity at constant pressure C_p undergoes the process $T = T_0 + \alpha V$ where T_0 and α are constants. If its volume increases from V_1 to V_2 , the amount of heat transferred to the gas is
 (a) $C_p \cdot RT_0 \ln (V_2/V_1)$ (b) $\alpha C_p (V_2 - V_1)/RT_0 \ln (V_2/V_1)$
 (c) $\alpha C_p (V_2 - V_1) + RT_0 \ln (V_2/V_1)$ (d) $RT_0 \ln (V_2/V_1) - \alpha C_p (V_2 - V_1)$

Answers

- | | | | | |
|---------|---------|---------|---------|----------|
| 1. (a) | 2. (c) | 3. (a) | 4. (d) | 5. (b) |
| 6. (b) | 7. (a) | 8. (d) | 9. (d) | 10. (c) |
| 11. (c) | 12. (d) | 13. (b) | 14. (d) | 15. (d) |
| 16. (b) | 17. (a) | 18. (b) | 19. (a) | 20. (d) |
| 21. (d) | 22. (b) | 23. (c) | 24. (d) | 25. (b) |
| 26. (a) | 27. (d) | 28. (d) | 29. (c) | 30. (a) |
| 31. (a) | 32. (b) | 33. (c) | 34. (d) | 35. (a) |
| 36. (c) | 37. (b) | 38. (a) | 39. (d) | 40. (b) |
| 41. (d) | 42. (a) | 43. (b) | 44. (a) | 45. (d) |
| 46. (c) | 47. (a) | 48. (b) | 49. (d) | 50. (b) |
| 51. (c) | 52. (d) | 53. (b) | 54. (c) | 55. (d) |
| 56. (a) | 57. (b) | 58. (d) | 59. (a) | 60. (c) |
| 61. (c) | 62. (d) | 63. (a) | 64. (b) | 65. (c) |
| 66. (b) | 67. (c) | 68. (d) | 69. (a) | 70. (a) |
| 71. (c) | 72. (b) | 73. (d) | 74. (c) | 75. (d) |
| 76. (c) | 77. (b) | 78. (c) | 79. (a) | 80. (b) |
| 81. (c) | 82. (a) | 83. (d) | 84. (c) | 85. (b) |
| 86. (d) | 87. (a) | 88. (d) | 89. (a) | 90. (d) |
| 91. (b) | 92. (c) | 93. (d) | 94. (a) | 95. (a) |
| 96. (d) | 97. (b) | 98. (d) | 99. (a) | 100. (c) |

Note: Almost all the problems in this chapter should be solved in five to seven minutes each except where no numerical calculations are involved. Problems free from numerical calculations should take two to three minutes.

15

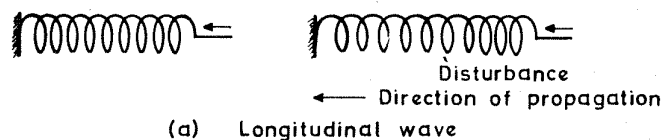
Wave Motion and Sound

Waves

A wave originates in the displacement of some portion of an elastic medium from its normal position, causing it to oscillate about an equilibrium position. Because of the elastic properties of the medium, the *disturbance* is transmitted from one layer to the next. Such a progressing disturbance is termed a wave.

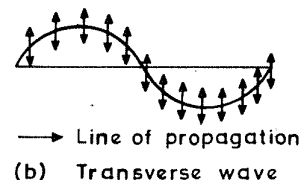
Longitudinal Waves

When the motion of particles of the medium conveying a mechanical wave is back and forth along the direction of propagation, this forms a *longitudinal* wave, shown in Figure (a).



Transverse Waves

When the motions of the matter particles conveying the wave are perpendicular to the direction of propagation of the wave itself, we then have a transverse wave (Figure (b)).



Equation of a Wave

A wave travelling to the right is given by $y = f(x - Vt)$ (increasing x as time goes on) and $y = f(x + Vt)$ represents a wave travelling to the left. For a particular phase of a wave travelling to the right, we require that $x - Vt = \text{constant}$ and $dx/dt - V = 0$ or $dx/dt = V$ so that V is really the *phase velocity* of the wave.

At $t = 0$, a waveform is given by

$$Y = Y_m \sin \frac{2\pi}{\lambda} \cdot x$$

The wave shape is a sine curve. The maximum displacement Y_m is the amplitude

of the sine curve. The symbol λ is called wavelength of the wave and represents the distance between two adjacent points in the wave having the same phase. With the passage of time, the wave travels to the right with a phase velocity V . The equation of the wave at time t is

$$Y = Y_m \sin \frac{2\pi}{\lambda} [x - Vt]$$

Time period T is the time required for the wave to travel a distance of one wavelength λ , so that

$$Y = Y_m \sin 2\pi \left[\frac{x}{\lambda} - \frac{t}{T} \right]$$

Wave number is defined as $K = \frac{2\pi}{\lambda}$

Angular frequency is defined as $\omega = \frac{2\pi}{T}$

The wave equation in terms of ω and k is

$$Y = Y_m \sin (Kx - \omega t)$$

Here, it has been assumed that the displacement Y is zero at the position $x = 0$ at time $t = 0$. The general expression for the sinusoidal wave travelling to the right is $Y = Y_m \sin (Kx - \omega t - \phi)$, where ϕ is called the phase constant.

Superposition Principle

Two or more waves can traverse the same space independently of one another. The displacement of any particle at a given time is simply the sum of displacements that the individual waves alone would give it. This process of vector addition of the displacements of a particle is called superposition.

Fourier Series

Any periodic motion of a particle can be represented as a combination of simple harmonic motions. If $Y(t)$ represents the motion of a source of waves having a period T , we can analyse $Y(t)$ as follows:

$$Y(t) = A_0 + A_1 \sin \omega t + A_2 \sin 2\omega t + \dots + A_n \sin n\omega t +$$

$$B_0 + B_1 \cos \omega t + B_2 \cos 2\omega t + \dots + B_n \cos n\omega t$$

Here A 's and B 's are constants which have definite values for any particular periodic motion $Y(t)$.

Speed of a Transverse Wave

Let us take the case of a transverse wave in a string of mass per unit length μ , which is subjected to a tension F .

$$V = \sqrt{\frac{\text{tension in the string}}{\text{mass per unit length of the string}}}$$

$$V = \sqrt{\frac{F}{\mu}}$$

Power and Intensity in Wave Motion

The transverse velocity of a particle of the string at x is

$$\frac{\partial Y}{\partial t} = -\omega Y_m \cos(kx - \omega t) \quad (x = \text{constant})$$

and the power transmitted through x is

$$P = Y_m^2 k \omega F \cos^2(kx - \omega t)$$

Intensity of the wave is defined as the power transmitted across a unit area normal to the direction in which the wave is travelling.

Intensity of a space wave is always proportional to the square of the amplitude or $I \propto Y_m^2$

Interference of Waves

Two waves of equal frequency and amplitude travelling with the same speed along the same direction (+ x) with a phase difference ϕ between them are represented by

$$Y_1 = Y_m \sin(kx - \omega t - \phi)$$

$$Y_2 = Y_m \sin(kx - \omega t)$$

The resultant wave of these two is represented by

$$Y = \left(2Y_m \cos \frac{\phi}{2}\right) \sin\left(kx - \omega t - \frac{\phi}{2}\right)$$

This resultant wave corresponds to a new wave having the same frequency but with an amplitude $2Y_m \cos \phi/2$.

Standing Waves

Two wavetrains of the same frequency, speed and amplitude which are travelling in opposite directions are represented by the equations

$$Y_1 = Y_m \sin(kx - \omega t) \quad \text{and} \quad Y_2 = Y_m \sin(kx + \omega t)$$

The resultant of these two waves when superimposed is given by

$$Y = 2Y_m \sin kx \cos \omega t$$

This is the equation of a *standing wave*.

Characteristics of a standing wave:

1. A particle at any particular point x executes simple harmonic motion and all particles vibrate with the same frequency.
2. The amplitude is not the same for different particles but varies with the location of x of the particle. The amplitude $2Y_m \sin kx$ has a *maximum* value of $2Y_m$ at positions where

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad \text{or} \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

These points are called *antinodes* and are spaced half a wavelength apart.

The amplitude has a *minimum* value of zero at positions where $kx = \pi, 2\pi, 3\pi, \dots$ or $x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$. These points are called *nodes*.

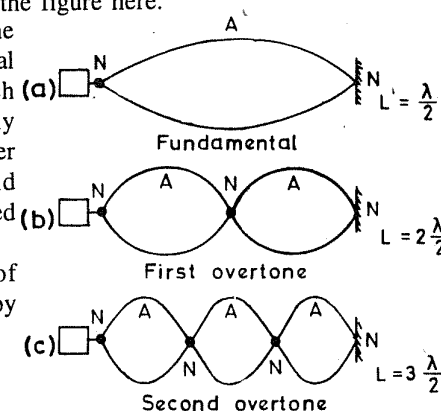
Resonance In general, whenever a system capable of oscillating is acted on by a periodic series of impulses having a frequency equal or nearly equal to one of the natural frequencies of oscillation of the system, the system is set into oscillations with a relatively large amplitude. This phenomenon is called *resonance* and the system is said to resonate with applied impulses.

A resonating string is shown in the figure here.

A string will resonate only if the vibration wavelength has certain special values: the wavelength must be such that the wave segments fit properly on the length of the string. A proper fit occurs only when nodes and antinodes exist at positions demanded by the constraints on the string.

The natural frequencies of oscillation of the system are given by

$$v = \frac{n}{2l} \sqrt{\frac{F}{\mu}}$$



where n is the number of half wavelengths or the number of loops formed in the string of length l and subjected to a tension F . μ is the mass per unit length of the string.

Speed of sound in a medium whose bulk modulus is B and density is ρ is given by

$$V = \sqrt{\frac{B}{\rho}}$$

Also,

$$V = \sqrt{\frac{\gamma P_0}{\rho}}$$

where γ is the ratio of specific heats for the gas, and p_0 is the undisturbed gas pressure. If the medium is solid, then the bulk modulus is replaced by Young's modulus.

Pressure Wave

$$p = (k \cdot \rho V^2 Y_m) \sin(kx - \omega t)$$

Here $(k \cdot \rho V^2 Y_m)$ is called the pressure amplitude and the equation can be written as

$$p = P \sin(kx - \omega t) \text{ where } P = k\rho V^2 Y_m$$

Here, a sound wave may be considered either as a displacement wave or as a pressure wave. If the displacement wave is expressed as a cosine function, the pressure wave will be expressed as a sine function and vice versa. Thus, the pressure wave and displacement wave are 90° out of phase.

Vibration in Air Column

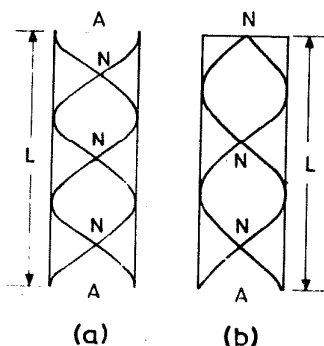
In an open pipe, the open ends are antinodes as shown in figure (a).

$$v_1 = \frac{V}{2l}, v_2 = \frac{V}{l},$$

$$v_3 = \frac{3V}{2l}, \dots$$

$$\lambda_1 = 2l, \lambda_2 = l,$$

$$\lambda_3 = \frac{2}{3}l, \dots$$



The first three modes of an open organ pipe are shown here. In an open pipe the fundamental frequency corresponds to a displacement antinode at each end and a displacement node in the middle. The succeeding nodes are the overtones, called the second and third harmonics. Hence, in an open pipe, the fundamental frequency is $V/2l$ and all harmonics are present.

In a closed pipe, as shown in figure (b), the closed end is a displacement node while the open end is a displacement antinode. The fundamental frequency is $V/4l$. The only overtones present are those that give a displacement node at the closed end and an antinode at the open end. Hence, even harmonics are missing and only odd harmonics are present.

Interference of Waves

When two wavetrains of the same frequency travel along the same line in opposite directions, standing waves are formed due to interference in space. When two wavetrains of slightly different frequency travel in the same direction, then interference in time takes place. In such a situation we have

$$Y_1 = Y_m \cos 2\pi v_1 t \quad \text{and} \quad Y_2 = Y_m \cos 2\pi v_2 t$$

and

$$y = \left[2Y_m \cos 2\pi \left(\frac{v_1 - v_2}{2} \right) t \right] \cos 2\pi \left(\frac{v_1 + v_2}{2} \right) t$$

The amplitude term given in parentheses here varies with time. In the case of sound, the varying amplitude gives rise to variations in loudness which are called *beats*.

The frequency of the resulting vibration is

$$\bar{v} = \left(\frac{v_1 + v_2}{2} \right)$$

$$\text{Number of beats per second} = v_1 - v_2$$

Sound Pressure Level and Sound Pressure Amplitude

The sound pressure L_1 in decibels is related to the sound pressure amplitude Δp according to the ratio

$$L_1 = 20 \log_{10} \frac{\Delta p}{\Delta p_0}$$

where Δp_0 is the sound pressure amplitude at a loudness level equal to zero.

The loudness level L_2 in phons is related to the sound intensity by the ratio

$$L_2 = 10 \log_{10} \frac{I}{I_0}$$

where I_0 is the zero loudness level. It is assumed that

$$I_0 = 10^{-12} \text{ W/m}^2 \text{ and } \Delta p_0 = 2 \times 10^{-5} \text{ N/m}^2$$

Doppler Effect

When there is a relative motion between a listener and the source of sound, the pitch of the sound as heard by the listener is not the same as when he is at rest with respect to the source. This is known as *Doppler effect*.

When the source is at rest with respect to the medium but the observer is moving through it, then the frequency of sound that he hears is given by

$$v' = v \left(\frac{V \pm V_0}{V} \right)$$

where v is the actual frequency of sound as emitted by the source, V is the speed of the sound in the medium and V_0 is the speed of the observer.

[Here, it is assumed that the source and the observer move along the line joining them.]

A positive sign indicates that the observer is moving towards the source

while a negative sign means that the observer is moving away from the source.

When the source is in motion and the observer is at rest, then the frequency of sound as heard by the observer is given by

$$\nu' = \nu \left[\frac{V}{V \mp V_s} \right]$$

Here, the negative sign indicates that the source is moving towards the observer and the positive sign means that the source is moving away from the observer.

If both source and observer move through the transmitting medium, then the frequency of sound as heard by the observer is given by

$$\nu' = \nu \left[\frac{V \pm V_o}{V \mp V_s} \right]$$

where the upper sign (+ numerator, - denominator) corresponds to the source and observer moving along the line joining the two in the direction towards each other and the lower signs in the direction away from each other.

Partial Differential Equation of a Wave

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

This means that y propagates as a travelling wave along the x -axis with a speed C .

Relationship between Waves and Oscillations

A wave is a disturbance travelling in a medium. The particles in a medium undergo changes in some physical variable [displacement, temperature, pressure are possible variables] in a coherent manner as a wave (disturbance) propagates. Consider a transverse wave travelling on the surface of water. Here each individual particle undergoes a displacement about a mean position and the displacement of particles at different space points have a definite phase relationship. Because of such a relationship, the properties of wave (wavelength, frequency, amplitude, etc.) are related to the properties of the oscillations of individual particles. We will assume the motion of each individual particle to be simple harmonic, which is an excellent approximation in most examples, when the amplitude of disturbance is not too large. Consider a wave travelling along the positive x -direction, whose displacement is given by

$$y(x, t) = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

where A is the amplitude, λ = wavelength, λ/T = velocity of the wave. If x is held fixed, we get the displacement of a single particle which is given by (for $x = a$)

$$y(a, t) = A \sin 2\pi \left(\frac{t}{T} - \frac{a}{\lambda} \right) = A \sin 2\pi (\nu t + \phi_0)$$

where $\nu = 1/T$ and $-a/\lambda = \phi_0$. This describes a simple harmonic motion with frequency ν and a phase of $2\pi\phi_0 = -2\pi a/\lambda$. Thus the frequency of the wave and that of the particle is the same. The wavelength is given by

$$\lambda = \frac{\text{velocity of wave}}{\text{frequency}}$$

The oscillations are usually caused by an external agency and hence ν is an independent parameter. Velocity is determined by the properties of the medium and so λ can be obtained once the velocity is known.

Some exercises based on the relationship between waves and oscillations are included in this chapter.

ILLUSTRATIONS

✓ 1. The equation of a transverse wave is given by

$$Y = 6.0 \sin (0.020\pi x + 4.0 \pi t)$$

where x and Y are expressed in centimetres and t in seconds. Calculate: (a) the amplitude, (b) the wavelength, (c) the frequency, (d) the speed, (e) the direction of propagation of the wave, and (f) the maximum transverse speed of a particle in the string.

Solution

The general form of the equation of the wave is

$$y = y_m \sin (kx + \omega t)$$

where $k = 2\pi/\lambda$, $\omega = 2\pi/T$ and y_m is the amplitude. Comparing this form with the given equation, we get

$$y_m = 6.0 \text{ cm}, 0.020\pi = k \text{ and } \omega = 4.0\pi$$

(a) The amplitude is 6.0 cm.

(b) The wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.020\pi} = \frac{2}{0.02} = 100 \text{ cm}$.

(c) The frequency, $2\pi\nu = \omega$ or $\nu = \frac{\omega}{2\pi} = \frac{4.0\pi}{2\pi} = 2 \text{ Hz}$.

(d) The speed $V = \frac{\omega}{k} = \frac{4.0\pi}{0.020\pi} = 200 \text{ cm/s}$.

(e) Since the wave travelling to the right is given by

$$y = y_m \sin (kx - \omega t)$$

the wave equation given in the problem shows that the term corresponding to ωt here is positive, and hence the wave is travelling in the negative x -direction (i.e. to the left).

$$(f) \quad V_{\max} = \left(\frac{dy}{dt} \right)_{\max} = y_m \cdot \omega = 6.0 \times 4.0\pi = 75 \text{ cm/s.}$$

2. A string is stretched between two ends with a force of 20 N and the linear density of the material of the string is 0.2 kg/m. One end of the string is given a sinusoidal motion of frequency 10 Hz and amplitude 0.02 m. At time $t = 0$, the end has zero displacement and is moving in the positive y direction.

(a) Find the wave speed, amplitude, angular frequency, period, wavelength and the wave number.

(b) Write the equation of the wave.

(c) Find the position of the point at $x = 0.2$ m at $t = 0.01$ s.

(d) Find the transverse velocity of the point at $x = 0.2$ m, at time $t = 0.2$ s.

(e) What is the slope of the string at the point $x = 0.2$ m at time $t = 0.2$ s?

Solution

$$(a) \quad V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{20}{0.2}} = 10 \text{ m/s}$$

Amplitude A is just the amplitude of motion of the end point.

$$\therefore A = 0.02 \text{ m}$$

$$\text{Angular frequency } \omega = 2\pi\nu = 2\pi \times 10 = 20\pi = 62.8 \text{ rad/s}$$

$$\text{Period } T = \frac{1}{\nu} = \frac{1}{10} \text{ s} = 0.1 \text{ s}$$

$$\text{Wavelength } \lambda = \frac{V}{\nu} = \frac{10}{10} = 1 \text{ m and Wave number } k = \frac{2\pi}{\lambda}$$

$$\therefore k = \frac{2\pi}{1} = 2\pi \text{ m}^{-1} = 6.28 \text{ m}^{-1}$$

(b) Equation of wave

$$y = 0.02 \sin 2\pi \left[\left(\frac{t}{0.1} - \frac{x}{1} \right) \right]$$

$$= 0.02 \sin 2\pi [10t - x]$$

$$(c) \quad y = 0.02 \sin 2\pi \left[\frac{0.01}{0.1} - \frac{0.2}{1} \right] = 0.02 \sin 2\pi (-0.1)$$

$$= -0.02 \sin (0.2\pi)$$

$$= -0.02 \times \sin 36^\circ$$

$$= -0.012 \text{ m}$$

(d) The transverse velocity at any point at any time is given by

$$V = \frac{\partial y}{\partial t} = \omega A \cos (\omega t - kx)$$

$$= 20\pi \times 0.02 \cos (20\pi \times 0.2 - 2\pi \times 0.2)$$

$$= 1.257 \cos (4\pi - 0.4\pi) = 1.257 \cos \pi (3.6)$$

$$= 1.257 \cos (648) = 1.257 \times \cos [4\pi - 72] = 1.257 \cos 72$$

$$\therefore V = 0.388 \text{ m/s}$$

(e) The slope at any point at any time is given by

$$\frac{\partial y}{\partial x} = -kA \cos (\omega t - kx)$$

$$= -2\pi \times 0.02 \cos (-72) = -2\pi \times 0.02 \times (0.3090)$$

$$= -0.039$$

3. A continuous sinusoidal longitudinal wave is sent along a coil spring from a vibrating source attached to it. The frequency of the source is 50 cycles/s and the distance between the successive rarefactions in the spring is 72 cm.

(a) Find the wave speed.

(b) If the maximum longitudinal displacement of a particle in the spring is 4 cm and the wave moves in the $-x$ direction, write the equation of the wave. Assume that the source is at $x = 0$ and that the displacement at $x = 0$ and $t = 0$ is zero.

Solution

$$\nu = 50 \text{ cycles/s and } \lambda/2 = 36 \text{ cm}$$

$$\therefore V = \nu\lambda = 50 \times 72 = 3600 \text{ cm/s} = 36 \text{ m/s}$$

General equation of wave travelling in the $-x$ direction is

$$y = y_m \sin (kx + \omega t)$$

$$\text{Here } k = 2\pi/\lambda = 2\pi/0.72 = 8.7 \text{ m}^{-1}, \omega = 2\pi\nu = 2\pi \times 50 = 100\pi$$

Amplitude is given to be $= 4 \text{ cm} = 0.04 \text{ m}$.

Therefore, the equation of the wave is

$$y = 0.04 \sin (8.7x + 100\pi t)$$

4. Find the amplitude of the resultant wave when two sinusoidal waves having the same frequency and travelling in the same direction are combined, if their amplitudes are 3.0 cm and 4.0 cm and they differ in phase by $\pi/2$ radians.

Solution

Let the two initial waves be represented by

$$y_1 = A_1 \sin (kx - \omega t) \quad \text{and} \quad y_2 = A_2 \sin (kx - \omega t - \pi/2)$$

$$\therefore y = y_1 + y_2 = A_1 \sin (kx - \omega t) - A_2 \cos (kx - \omega t)$$

$$= \sqrt{A_1^2 + A_2^2} \left[\frac{A_1}{\sqrt{A_1^2 + A_2^2}} \sin(kx - \omega t) - \frac{A_2}{\sqrt{A_1^2 + A_2^2}} \cos(kx - \omega t) \right]$$

Substitute $\frac{A_1}{\sqrt{A_1^2 + A_2^2}}$ by $\cos \alpha$ and $\frac{A_2}{\sqrt{A_1^2 + A_2^2}}$ by $\sin \alpha$

$$\therefore y = \sqrt{A_1^2 + A_2^2} [\cos \alpha \sin(kx - \omega t) - \sin \alpha \cos(kx - \omega t)]$$

$$= \sqrt{A_1^2 + A_2^2} \sin(kx - \omega t - \alpha)$$

Therefore, the amplitude of the resultant wave is $\sqrt{A_1^2 + A_2^2}$.

Substituting $A_1 = 3.0$ cm and $A_2 = 4.0$ cm, we get

$$A = 5 \text{ cm} = 0.05 \text{ m}$$

5. A string vibrates in four segments to a frequency of 400 Hz.

(a) What is its fundamental frequency?

(b) What frequency will cause it to vibrate into seven segments.

Solution

If the string has n segments, then

$$n \times \left(\frac{\lambda}{2}\right) = L$$

where L is the length of the string and we know that $V = v_n \lambda$. Therefore,

$$v_n = \frac{V}{\lambda} = \frac{V}{(2L/n)} = n \left(\frac{V}{2L}\right)$$

When the frequency $v_n = 400$ Hz, we have

$$400 = 4 \left(\frac{V}{2L}\right) \Rightarrow \frac{V}{2L} = 100 \text{ Hz}$$

(a) Fundamental frequency $v_1 = 1 \times 100 = 100$ Hz

(b) $v_7 = 7 \times 100 = 700$ Hz

6. A string vibrates according to the equation

$$y = 0.5 \sin \frac{\pi x}{3} \cos 40\pi t$$

where x and y are in centimetres and t is in seconds.

(a) What are the amplitude and velocity of the component waves whose superposition can give rise to this wave?

(b) What is the distance between the two nodes?

(c) What is the velocity of a particle of the string at the position $x = 1.5$ cm when $t = (9/8)$ s.

Solution

(a) When two waves travelling in opposite directions and having the same amplitude and frequency given by

$$y_1 = y_m \sin(kx - \omega t) \text{ and } y_2 = y_m \sin(kx + \omega t)$$

are superimposed, the resultant wave is given by

$$y = 2y_m \sin kx \cos \omega t$$

Comparing this equation with the given equation, we have

$$2y_m = 0.5 \text{ cm} \Rightarrow y_m = 0.25 \text{ cm}$$

$$V = \frac{\omega}{k} = \frac{40\pi}{\pi/3} = 120 \text{ cm/s}$$

$$(b) \text{ Distance between two nodes} = \frac{\lambda}{2} = \frac{1}{2} \times \frac{2\pi}{k} = \frac{\pi}{\pi/3} = 3.0 \text{ cm}$$

$$(c) V = \frac{dy}{dt} = -40\pi \times 0.5 \sin \frac{\pi x}{3} \sin 40\pi t$$

$$= -20\pi \sin \frac{\pi x}{3} \sin 40\pi t$$

At $x = 1.5$ cm and $t = 9/8$ s,

$$V = -20\pi \sin \frac{\pi}{3} \times 1.5 \sin 40\pi \times \frac{9}{8}$$

$$= -20\pi \sin 0.5\pi \sin 45\pi = 0$$

7. (a) A blast gives a sound of intensity of 0.80 W/m^2 and the frequency is measured as 1000 Hz. If the density of air is 1.29 kg/m^3 and the speed of sound is 330 m/s, find the amplitude of the sound wave.

Solution

$$\text{Intensity} = 2\pi^2 v^2 \rho V A^2$$

$$\therefore A = \frac{1}{\pi v} \sqrt{\frac{1}{2\rho V}} = \frac{1}{\pi \times 1000} \sqrt{\frac{0.80}{2 \times 1.29 \times 330}}$$

$$= \frac{1}{3.14 \times 1000} \sqrt{0.00093}$$

$$= 9.7 \times 10^{-6} \text{ m}$$

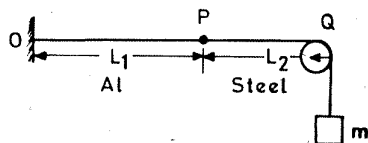
(b) A sound has an intensity of $5 \times 10^{-8} \text{ W/m}^2$. What is the sound level in decibels.

Solution

$$\text{Intensity in decibels} = 10 \log \left(\frac{I}{1 \times 10^{-12} \text{ W/m}^2} \right)$$

$$\begin{aligned}
 &= 10 \log \left(\frac{5 \times 10^{-8}}{1 \times 10^{-12}} \right) = 10 \log (5 \times 10^4) \\
 &= 10 (4 + \log 5) \\
 &= 10 (4 + 0.699) \\
 &= 10 (4.699) \\
 &= 46.99 \text{ dB}
 \end{aligned}$$

8. An aluminium wire of length $L_1 = 60.0$ cm and cross-sectional area $1.00 \times 10^{-2} \text{ cm}^2$ is connected to a steel wire of the same cross-sectional area. The compound wire is loaded with a block m of mass 10 kg. The length of the steel wire $L_2 = 86.6$ cm. Transverse waves are set up in the compound wire by means of a source of variable frequency.



(a) Find the lowest frequency of excitation for which standing waves are observed such that point P is a node.

(b) What is the total number of nodes observed at this frequency excluding the two at the ends of the wire?

(Given: density of aluminium is 2.60 g/cc and that of steel is 7.80 g/cc)

Solution

(a) Linear mass density $\mu = \rho \times A$ where ρ is the density of the material and A is the area of cross-section

$$v = \frac{n}{2L} \sqrt{\frac{F}{\mu}}$$

For aluminium,

$$v_1 = \frac{n_1}{2 \times 60} \sqrt{\frac{F}{A \times 2.6}}$$

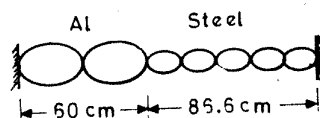
For steel,

$$v_2 = \frac{n_2}{2 \times 86.6} \sqrt{\frac{F}{A \times 7.8}}$$

But $v_1 = v_2$

$$\text{Hence } \frac{n_2}{n_1} = \frac{5}{2}$$

Thus there is one node in the aluminium wire and four nodes in the steel wire, with a node at the joint producing standing waves.



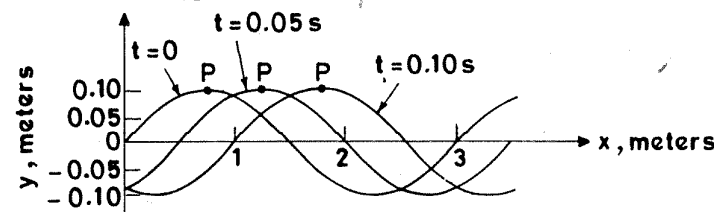
Therefore, $n_1 = 2$ or $n_2 = 5$ can give the frequency of excitation. Tension $F = 10 \text{ kg} \times 980 \text{ cm/s}^2 = 9.8 \times 10^6 \text{ dynes}$

$$A = 1 \times 10^{-2} \text{ cm}^2$$

$$\therefore v = \frac{2}{2 \times 60} \sqrt{\frac{9.8 \times 10^6}{1.0 \times 10^{-2} \times 2.6}} = 323 \text{ cps}$$

(b) The total number of nodes formed at this frequency excluding those at the ends of the wire is 6.

9. The figure here represents three flashes of a travelling wave on a spring of mass per unit length $\mu = 0.2$ kg/m. The first flash is labelled at $t = 0$ although the motion was initiated earlier. The second and the third flashes were recorded after 0.05 s and 0.10 s respectively. Determine: (i) the wave speed, (ii) wavelength and frequency and maximum transverse velocity, (iii) the tension, and (iv) the equation for the wave.



Solution

(i) From the figure, we see that the wavelength $\lambda = 3$ m because at $t = 0$, the spring shape goes through a complete cycle in 3 m. The wave clearly travels in the positive- x -direction and the wave speed is $1/0.1 = 10$ m/s because the point P , for example, travels a distance of 1 m in 0.1 s

$$(ii) \text{ Frequency } v = \frac{V}{\lambda} = \frac{10}{3} \text{ cycles/s}$$

$$\text{and Period } T = \frac{1}{v} = \frac{3}{10} = 0.3 \text{ s}$$

The displacement amplitude $y_m = 0.1$ m and the maximum transverse velocity is $y_m \omega = (2\pi \times 10/3) \times (0.1) = 2.1$ m/s.

$$(iii) \text{ The tension } = \mu V^2 = 0.2 \times 10^2 = 20 \text{ N.}$$

(iv) The shape of the spring at $t = 0$ is given by

$$y = y_m \sin \left(\frac{2\pi x}{\lambda} \right)$$

The general expression for a wave travelling in the positive x -direction is of the form

$$y(x, t) = y_m \sin \left[2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) + \phi \right]$$

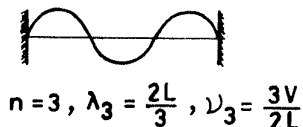
and if this wave is to have the shape $y = y_m \sin\left(\frac{2\pi x}{\lambda}\right)$ at $t = 0$, the phase angle must be such that

$$\sin\left(-\frac{2\pi x}{\lambda} + \phi\right) = \sin\left(\frac{2\pi x}{\lambda}\right)$$

It means that $\phi = \pi$. The equation then can be written as

$$y = -y_m \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

10. A string of mass 0.2 kg/m and length $L = 0.6$ m is fixed at both ends and stretched such that it has a tension of 80 N. Initially, the string, deformed to conform to the shape of its third normal mode, has an amplitude of 0.5 cm. What is the frequency of the oscillation? What is the maximum transverse velocity amplitude?



Solution

$$V = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{80 \text{ N}}{0.2 \text{ kg/m}}} = 20 \text{ m/s}$$

In the third normal mode, the string is $3/2$ wavelengths long

So that $\frac{3\lambda}{2} = L = 0.6$ m and $\lambda_3 = 0.4$ m and the frequency

$$\nu_3 = \frac{V}{\lambda_3} = \frac{20}{0.4} = 50 \text{ cycles/s and } T_3 = 0.02 \text{ s.}$$

The displacement of the string is

$$y(x, t) = A \sin \frac{3\pi x}{L} \cos \omega_3 t$$

and the transverse particle velocity is

$$V(x, t) = -\left(\omega_3 A \sin \frac{3\pi x}{L}\right) \sin \omega_3 t$$

The maximum amplitude of the velocity is therefore

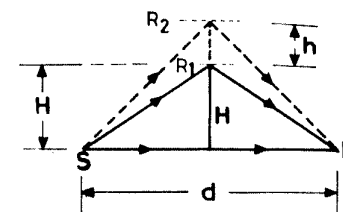
$$V_m = \omega_3 A = 2\pi \times 50 \times 0.5 = 50\pi \text{ cm/s}$$

Only the elements of string at $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ and $\frac{5\lambda}{4}$

have maximum velocity amplitude ($x = 0.1$ m, 0.3 m and 0.5 m).

11. A simple set-up to determine the velocity of sound is shown here in

the figure. This consists of a source of sound of frequency ν and a sensitive detector D set a distance d apart. The direct wave from S is found to be in phase at D with the wave from S that is reflected from a horizontal reflector R_1 at an altitude H from the source-detector line. The incident and reflected waves make the same angle with the reflecting surface. When the reflector is raised by a height h , no signal is detected at D. If absorption in the atmosphere is neglected, find the expression for the velocity of sound in terms of the known parameters d, h, H and ν .



Solution

From the relations

$$K = \frac{2\pi}{\lambda}, x = \frac{\phi}{K} \text{ and } x = d$$

we have

$$\text{Phase for the direct wave} = \frac{2\pi d}{\lambda}$$

$$\text{Phase } \phi_1 \text{ after reflection from } R_1 = \left(\frac{2\pi}{\lambda}\right) \sqrt{4H^2 + d^2}$$

$$\text{Total path} = 2 \sqrt{H^2 + \frac{d^2}{4}} = \sqrt{4H^2 + d^2}$$

Similarly, the phase ϕ_2 at D due to reflection from R_2 is

$$\phi_2 = \frac{2\pi}{\lambda} \sqrt{4(H+h)^2 + d^2}$$

For destructive interference, $\phi_2 - \phi_1 = \pi$. Hence,

$$\frac{2\pi}{\lambda} \sqrt{4(H+h)^2 + d^2} - \frac{2\pi}{\lambda} \sqrt{4H^2 + d^2} = \pi$$

$$\text{or } \lambda = 2 \sqrt{4(H+h)^2 + d^2} - 2 \sqrt{4H^2 + d^2}$$

$$\text{or } V = \nu \lambda = 2\nu \left[\sqrt{4(H+h)^2 + d^2} - \sqrt{4H^2 + d^2} \right]$$

12. Two trains are approaching each other, one moving at a speed of 30 miles/hr and the other at 60 miles/hr. The faster train blows a whistle producing a note of 2000 vib/s. Calculate the apparent frequency of the note heard by: (a) an observer at rest behind the faster train, and (b) an observer sitting in the slower train. (Assume the speed of sound to be 1100 ft/s.)

Solution

(a) The faster train is receding from the stationary observer with a speed of 60 miles/hr (or 88 ft/s)

$$v' = \left(\frac{V}{V + V_s} \right) \cdot v = \left(\frac{1100}{1100 + 88} \right) \times 2000 = 1852 \text{ Hz}$$

(b) Since the observer and the source are approaching each other, the apparent frequency is

$$v' = \frac{V + V_0}{V - V_s} \cdot v = \frac{1100 + 44}{1100 - 88} \times 2000 \quad [V_0 \text{ is the speed of the observer and } V_s \text{ is the speed of the source}]$$

$$= 2261 \text{ Hz}$$

13. A tuning fork when sounded over the open end of an air column produces 4 beats per second, the tuning fork giving the lower note. The temperature of air is 20°C. When the temperature falls to 5°C, the tuning fork produces three beats per second. Find the frequency of the tuning fork.

Solution

Let v be the frequency of the tuning fork.

$$\therefore \text{Frequency of air column at } 20^\circ\text{C} = v + 4$$

$$\text{and Frequency of air column at } 5^\circ\text{C} = v + 3$$

The velocities of sound at 20°C and 5°C are given by

$$\frac{V_{20}}{V_5} = \sqrt{\frac{273 + 20}{273 + 5}}$$

$$\therefore \frac{v + 4}{v + 3} = \sqrt{\frac{273 + 20}{273 + 5}} \Rightarrow v = 34 \text{ Hz}$$

14. A spectral line of wavelength 4000 Å in the spectrum of light from a star is found to be displaced from its normal position towards the red end of the spectrum by an amount 1 Å. If the velocity of light is 3×10^8 m/s, what is the velocity of the star in the line of sight?

Solution

$$\lambda = 4000 \times 10^{-10} \text{ m, shift of wavelength} = d\lambda = 10^{-10} \text{ m}$$

$$C = 3 \times 10^8 \text{ m/s is the velocity of light.}$$

Let V be the velocity of the star and λ' be the new wavelength of the spectral line when the star is approaching the earth.

$$\lambda' = \frac{C - V}{v} = \frac{(C - V)\lambda}{C} \quad (1)$$

When the star is receding from the earth,

$$\lambda' = \frac{C + V}{C} \lambda \quad (2)$$

$$\therefore \frac{\lambda'}{\lambda} = \left(\frac{C \pm V}{C} \right)$$

$$\text{or } \frac{\lambda + d\lambda}{\lambda} = \frac{C + V}{C}, [\lambda' - \lambda = d\lambda]$$

$$\text{or } 1 + \frac{d\lambda}{\lambda} = 1 + \frac{V}{C} \Rightarrow \frac{d\lambda}{\lambda} = \frac{V}{C} \quad \text{or } V = C \frac{d\lambda}{\lambda}$$

$$\therefore V = \frac{3 \times 10^8 \times 10^{-10}}{4000 \times 10^{-10}} \text{ m/s}$$

$$= \frac{3}{4} \times 10^5 = 0.75 \times 10^5 \text{ m/s} = 7.5 \times 10^4 \text{ m/s} = 75 \text{ km/s}$$

15. A sonometer wire fixed at one end has a solid mass M hanging from its other end to produce tension in it. It is found that a 70 cm length of the wire produces a certain fundamental frequency when plucked. When the same mass is hanging in water, completely submerged in it, it is found that the length of the wire has to be changed by 5 cm if it is to produce the same fundamental frequency. Calculate the density of the material of the mass M hanging from the wire.

Solution

$$v = \frac{n}{2l} \sqrt{\frac{T}{\mu}}$$

and if v is the fundamental frequency,

$$\text{then } v = \frac{1}{2 \times 70} \sqrt{\frac{Mg}{\mu}} \quad (i)$$

When M is submerged in water

$$v = \frac{1}{2 \times 65} \sqrt{\frac{M'g}{\mu}} \quad (ii)$$

Here M' is the value of mass in water.

$$\therefore \sqrt{\frac{M}{M'}} = \frac{14}{13} \quad (iii)$$

Let ρ be the density of the material of the mass M .

$$\therefore \text{Volume of mass} = \left(\frac{M}{\rho} \right) \text{ cc}$$

$$\therefore \text{Volume of water displaced} = \left(\frac{M}{\rho}\right) \text{ cc}$$

$$\therefore \text{Upthrust acting on the mass} = \left(\frac{M}{\rho}\right) \times 1 \times g$$

$$\therefore \text{Weight of mass } M \text{ in water} = M'g = Mg - \left(\frac{M}{\rho}\right)g$$

$$\therefore M' = M - \frac{M}{\rho} = M \left[1 - \frac{1}{\rho}\right]$$

Substituting this in Eq. (iii)

$$\sqrt{\frac{M}{M[1 - 1/\rho]}} = \frac{14}{13} \quad \text{or} \quad \frac{M}{M[1 - 1/\rho]} = \frac{196}{169}$$

$$1 - \frac{1}{\rho} = \frac{169}{196} \Rightarrow \rho = 7.26 \text{ g/cc}$$

16. Two wires of equal length and of radii r and $2r$ respectively are welded together end to end. The combination is used as a sonometer wire and is kept under a tension T . The welded point is midway between the two bridges. What would be the ratio of the number of loops formed in the wires such that the joint is a node when stationary waves are set up in the wires?

Solution

Let V_1 and V_2 be the velocities of sound in the thin and thick wires respectively. Therefore,

$$V_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{T}{\pi r^2 \rho}} \quad \text{and} \quad V_2 = \sqrt{\frac{T}{\pi (2r)^2 \rho}}$$

$$\therefore \frac{V_1}{V_2} = \sqrt{\frac{T}{\pi r^2 \rho} \times \frac{\pi 4r^2 \rho}{T}} = 2$$

The frequency remains the same on both the wires. (Let us call it ν .)

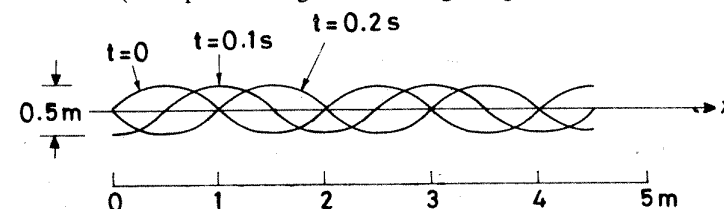
$$\therefore V_1 = \nu \lambda_1 \text{ and } V_2 = \nu \lambda_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{V_1}{V_2} = 2$$

Since the lengths of the two wires are equal and the joint point is a node, it follows that the number of loops will be inversely proportional to wavelengths

$$\therefore \frac{\text{Number of loops on the thinner wire}}{\text{Number of loops on the thicker wire}} = \frac{1}{2}$$

EXERCISES

- Mark the correct statement among the following:
 - the wave speed under isothermal conditions, V_{iso} , and adiabatic conditions, V_{ad} is the same
 - the wave speed under adiabatic conditions V_{ad} is $\sqrt{\frac{N_0 RT_0}{M}}$ (N_0 is Avogadro's number and M is gram molecular mass)
 - the wave speed under isothermal conditions V_{iso} is $\gamma \times V_{\text{ad}}$ (γ is the ratio of specific heats)
 - $V_{\text{ad}} = \sqrt{\frac{\gamma RT_0}{M}}$
- Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct. (Mass per unit length of the string = 3 g/cm .)



- displacement amplitude of the wave is 2.5 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
 - displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.4/s.
 - displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 Hz.
 - displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
- Which of the following is not a wave equation?
 - $y = y_m \sin K(x - Vt)$
 - $y = y_m \cos 2\pi \left(\frac{x}{\lambda} - vt\right)$
 - $y = y_m \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$
 - $y = 2y_m \sin Kx \cos \omega t$
 - A transverse sinusoidal wave is generated at one end of a long horizontal string by a bar which moves end up and down through a distance of 0.40 cm. The motion is continuous and is repeated 120 times per second. The string has linear density of 0.25 kg/m and is kept under a tension of 90 N. If the wave generated travels in positive- x -direction, its equation is
 - $y = 0.40 \sin [120x - 740t]$
 - $y = 0.20 \sin [0.39x - 740t]$
 - $y = 0.20 \sin [60x - 370t]$
 - $y = 0.40 \sin [0.39x - 120t]$
 - The rate of transfer of energy in a wave depends
 - directly on the square of the wave amplitude and square of the wave frequency

- (b) directly on the square of the wave amplitude and square root of the wave frequency
 (c) directly on the wave frequency and square of the wave amplitude
 (d) directly on the wave amplitude and square of the wave frequency.
6. When two waves of the same amplitude and frequency but having a phase difference of ϕ , travelling with the same speed in the same direction (positive x) interfere, then
 (a) their resultant amplitude will be twice that of a single wave but the frequency will be same
 (b) their resultant amplitude and frequency will both be twice that of a single wave
 (c) their resultant amplitude will depend on the phase angle while the frequency will be the same
 (d) the frequency and amplitude of the resultant wave will depend upon the phase angle.
7. The following are the equations of waves. Mark the one which represents a standing wave.
 (a) $y = (2y_m \cos \phi/2) \sin (kx - \omega t - \phi/2)$
 (b) $y = y_m \sin (kx - \omega t - \phi)$
 (c) $y = 2y_m \sin kx \cos \omega t$
 (d) $y = 2y_m \sin kx \cos (\omega t - \phi/2)$
8. In a demonstration experiment on standing waves, it is desired to show one loop, two loops and three loops on a string of length 7.3 m on which tensions of F_1 , F_2 and F_3 N respectively are applied. The linear density of the string is 7.47×10^{-3} kg/m and the frequency of the vibrator to which the string is attached is 20 Hz. The values of F_1 , F_2 and F_3 corresponding to one loop, two loops and three loops are
 (a) 320 N, 140 N and 80 N (b) 640 N, 240 N and 142 N
 (c) 640 N, 160 N and 71 N (d) 320 N, 80 N and 35 N
9. A transverse wave propagating in a string is described by the equation $y = 0.021 \sin (x + 30t)$ where x and y are measured in metres and t in seconds. If the linear density of the vibrating string is 1.3×10^{-4} kg/m, the tension in the string is
 (a) 0.21 N (b) 0.64 N
 (c) 2.1 N (d) 0.12 N
10. A continuous sinusoidal wave is travelling on a string with a velocity of 0.80 m/s. The displacement of the particles of the string at $x = 10$ cm is found to vary with time according to the equation $y = 8 \sin (0.5 - 4.0t)$ in centimetres. The linear density is 0.4 kg/m. The tension in the string is
 (a) 0.256 N (b) 2.56 N
 (c) 0.526 N (d) 1.28 N
11. In the above problem, the general equation of wave in the string is
 (a) $y = 8 \sin 2\pi \left(\frac{x}{10} - 4t \right)$ (b) $y = 8 \sin \pi \left(\frac{x}{4} - 2t \right)$
 (c) $y = 8 \sin \left(\frac{x}{10} - 4t \right)$ (d) $y = 8 \sin 2\pi(10x - 2t)$
12. A wave of frequency 400 Hz has a phase velocity of 300 m/s. The phase difference between two displacements at a certain point at times $t = 10^{-3}$ s apart is

- (a) 72° (b) 102°
 (c) 144° (d) 180°
13. A wave of frequency 400 Hz has a phase velocity of 300 m/s. Two points on this wave are out of phase by 60° . The separation between these two points is
 (a) 1.25 cm (b) 12.5 cm
 (c) 0.25 cm (d) 2.51 cm
14. Spherical waves are emitted from a 5.0 W source in an isotropic non-absorbing medium. The wave intensity at a distance of 2.0 m from the source is
 (a) 0.020 W/m^2 (b) 0.20 W/m^2
 (c) 0.10 W/m^2 (d) 0.01 W/m^2
15. Two sinusoidal waves travelling in the same direction have amplitudes of 5 and 8 cm and differ in phase by $\pi/2$. The resultant amplitude of these superimposed waves is
 (a) 9.4 cm (b) 8.9 cm
 (c) 13 cm (d) 89 cm
16. The temperature at which the velocity of sound will be double the velocity of sound in air at 0°C is
 (a) 1019°C (b) 721°C
 (c) 546°C (d) 819°C
17. Oxygen is 16 times heavier than hydrogen. Equal volumes of hydrogen and oxygen are mixed. The ratio of the velocity of sound in the mixture to that in oxygen is
 (a) $\sqrt{\frac{1}{8}}$ (b) $\sqrt{\frac{32}{17}}$
 (c) $\sqrt{\frac{17}{32}}$ (d) $\sqrt{8}$
18. When a plane wave train traverses a medium, individual particles execute a periodic motion given by the equation

$$y = 5 \sin \frac{\pi}{4} \left(4t + \frac{x}{16} \right)$$
 where the lengths are expressed in centimetres and time in seconds. The phase difference for two positions of the same particles which are occupied at a time interval 0.8 s apart is
 (a) 72° (b) 144°
 (c) 102° (d) 36°
19. The equation of a plane wave is

$$y = 4 \sin \frac{\pi}{4} \left(2t + \frac{x}{8} \right)$$
 The phase difference at any given instant of two particles 20 cm apart is
 (a) 112.5° (b) 130°
 (c) 65° (d) 135°
- 20.* A simple harmonic wavetrain of amplitude 5 cm and frequency 100 Hz is travelling in the positive x -direction with a velocity of 30 m/s. The displacement, velocity and acceleration at $t = 3$ s of a particle of the medium situated 100 cm from the origin are respectively
 (a) -3.44 cm, 1750 cm/s, 17×10^4 cm/s²
 (b) 4.33 cm, 1570 cm/s, 71×10^4 cm/s²

- (c) $-4.33 \text{ cm}, +1570 \text{ cm/s}, 171 \times 10^4 \text{ cm/s}^2$
 (d) $-3.44 \text{ cm}, -1750 \text{ cm/s}, 171 \times 10^4 \text{ cm/s}^2$
- 21.* A travelling wave in a gas along the positive x -direction has an amplitude of 2 cm, velocity 45 m/s and frequency 75 cps. Its particle acceleration after an interval of 3 s at a distance of 135 cm from the origin is
 (a) $0.44 \times 10^2 \text{ cm/s}^2$ (b) $4.4 \times 10^5 \text{ cm/s}^2$
 (c) $4.4 \times 10^3 \text{ cm/s}^2$ (d) $44 \times 10^5 \text{ cm/s}^2$
- 22.* A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. The wavelength of the pulse when it reaches the top of the rope is
 (a) 0.012 m (b) 0.06 m
 (c) 0.24 m (d) 0.12 m
- 23.* A steel wire of length 1 m and mass 0.1 kg and having a uniform cross-sectional area of 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C . If the transverse waves are set up by plucking the string in the middle, the frequency of the fundamental note of vibration is ($Y_{\text{steel}} = 2 \times 10^{11} \text{ N/m}^2$, $\alpha_{\text{steel}} = 1.21 \times 10^{-5}/^\circ\text{C}$)
 (a) 44 Hz (b) 88 Hz
 (c) 22 Hz (d) 11 Hz
24. One litre of hydrogen weighs 0.0896 g and one litre of air weighs 1.293 g. If the velocity of sound in air is 332 m/s, the velocity of sound in hydrogen gas is
 (a) 1261.6 m/s (b) 4120.0 m/s
 (c) 1100.0 m/s (d) 1200.0 m/s
- 25.* The specific gravity of oxygen and nitrogen are in the ratio of 16 : 14. The temperature at which the velocity of sound will be the same as that of nitrogen at 15°C will be
 (a) 112°C (b) 72°C
 (c) 48°C (d) 56°C
26. Two men are equidistant from the face of a plane vertical cliff and are 300 m apart. One of them fires a pistol, the other hears the echo one second after hearing the direct sound. The velocity of sound is 330 m/s. The distance of the men from the cliff is
 (a) 300 m (b) 350 m
 (c) 315 m (d) 415 m
27. A thin brass wire of density 8.5 g/cc is stretched between two clamps 90 cm apart while subjected to an extension of 0.05 cm per meter. If Young's modulus of brass is $10 \times 10^{10} \text{ N/m}^2$, the pitch of the lowest note emitted due to transverse vibration is
 (a) 85.2 cps (b) 42.6 cps
 (c) 62.6 cps (d) 24.6 cps
- 28.. The effects are produced at a given point in space by two waves described by the equations, $y_1 = y_m \sin \omega t$ and $y_2 = y_m \sin (\omega t + \phi)$ where y_m is the same for both the waves and ϕ is a phase angle. Tick the incorrect statement among the following:
 (a) the maximum intensity that can be achieved at a point is twice the intensity of either wave and occurs if $\phi = 0$
 (b) the maximum intensity that can be achieved at a point is four times the intensity of either wave and occurs if $\phi = 0$
 (c) the maximum amplitude that can be achieved at the point is twice the amplitude of either wave and occurs at $\phi = 0$
 (d) When the intensity is zero, the net amplitude is zero, and at this point $\phi = \pi$.
29. On a stretched string the waves of the form, $y_1 = A \sin (\omega t - kx)$ and $y_2 = -A \sin (\omega t + kx)$ are superimposed. The following conclusions are drawn about the resultant waveform. Mark the one which is incorrect.
 (a) the shape of the string at each point is a sine curve whose amplitude varies with time
 (b) the appearance is not that of a travelling wave shape but of a sinusoidal displacement in one position which grows larger and smaller with time
 (c) each point in the string still undergoes simple harmonic motion but instead of the progressively increasing phase difference between motions of adjacent points, all points move in phase or 180° out of phase
 (d) in the resultant wave each particle of the string vibrates with the same amplitude.
30. A wave-like disturbance is generated on a string under tension T and with a linear density ρ . We draw the following conclusions. Mark the one that is incorrect.
 (a) the disturbance must be transverse
 (b) the waveform must be sinusoidal
 (c) the disturbance can be either transverse or longitudinal
 (d) the frequency of the wave is given by $v = \frac{n}{2l} \sqrt{\frac{T}{\rho}}$.
31. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ . The angular frequency of the wave is ω . The following conclusions are drawn. Mark the one which is correct.
 (a) doubling the frequency doubles the rate at which energy is carried along the string
 (b) if the amplitude were doubled, the rate at which energy is carried would be doubled also
 (c) if the amplitude were doubled, the rate at which energy is carried would be halved
 (d) the rate at which energy is carried is directly proportional to the velocity of the wave.
32. Two waves may be represented at the same point by the expression $Y_1 = Y_m \sin \omega t$ and $Y_2 = Y_m \sin (\omega t + \phi)$. There will be destructive interference at this point if:
 (a) $\phi = 0$ (b) $\phi = \pi/2$
 (c) $\phi = \pi$ (d) $\phi = \pi/4$
33. A wire under tension vibrates with a frequency of 450 Hz. If the wire were half as long, twice as thick and under one-fourth the tension, its fundamental frequency would be
 (a) 225 Hz (b) 375 Hz
 (c) 112.5 Hz (d) 675 Hz
34. A wire of density 9 g/cc is stretched between two clamps 100 cm apart while subjected to an extension of 0.05 cm. If the Young's modulus of the material of the wire is $9 \times 10^{11} \text{ dynes/cm}^2$, the lowest frequency of transverse vibrations in the wire is
 (a) $50\sqrt{2} \text{ Hz}$ (b) $40\sqrt{2} \text{ Hz}$
 (c) $12\sqrt{2} \text{ Hz}$ (d) $25\sqrt{2} \text{ Hz}$

- 35.* A road runs between two parallel rows of buildings. A motorist moving just in the middle with a velocity of 30 km/h sounds the horn. He hears an echo one second after sounding the horn (velocity of sound = 330 m/s). The distance between the two rows of the buildings is
 (a) 289.3 m (b) 329.8 m
 (c) 114.9 m (d) 172.3 m
36. A sonometer wire under a tension of 10 kg weight is in unison with a tuning fork of frequency 320 Hz. In order that the wire may vibrate in unison with a tuning fork of frequency 256, the tension should be altered by
 (a) 2.6 kg wt (b) 1.2 kg wt
 (c) 3.6 kg wt (d) 4.2 kg wt
37. A sitar wire of length 0.9 m emits a fundamental note of 256 cps. In order to produce a note of 384 cps, the distance from top where finger should be pressed is
 (a) 20 cm (b) 35 cm
 (c) 15 cm (d) 30 cm
38. The lengths of two brass wires are in the ratio 1 : 2, their tensions are in the ratio 1 : 2 and their diameters are in the ratio 1 : 3. The ratio of the notes they emit when sounded together by the same source is
 (a) $2\sqrt{3}$ (b) $3\sqrt{2}$
 (c) $\sqrt{3}$ (d) $\sqrt{2}$
- 39.* A man standing in front of a mountain beats a drum at regular intervals. The rate of drumming is generally increased and he finds that the echo is not heard distinctly when the rate becomes 40 per minute. He then moves nearer to the mountain by 90 metres and finds that echo is again not heard when the drumming rate becomes 60 per minute. The distance between the mountain and the initial position of the man is
 (a) 205 m (b) 300 m
 (c) 180 m (d) 270 m
- 40.* A pipe of length 1.5 m closed at one end is filled with a gas, and resonates in its fundamental with a tuning fork. Another pipe of the same length but open at both ends is filled with air and it resonates in its fundamental with the same tuning fork. If the velocity of sound in air is 360 m/s at 30°C, the velocity of sound in gas at this temperature is
 (a) 720 m/s (b) 1040 m/s
 (c) 1100 m/s (d) 920 m/s
41. 50 tuning forks are arranged in order of increasing frequency and any two successive forks give 5 beats per second when sounded together. If the last fork gives the octave of the first, the frequency of the first tuning fork is
 (a) 490 cps (b) 375 cps
 (c) 245 cps (d) 255 cps
42. Two 80 cm long identical sonometer wires are stretched by the same tension to give a note of frequency 250 cps. In order to get 5 beats per second, the length of one of the wires should be altered by
 (a) 0.63 cm (b) 0.36 cm
 (c) 1.06 cm (d) 1.63 cm
- 43.* A locomotive engine approaches a railway station and whistles at a frequency of 400 cps. A stationary observer on the platform observes a change of 40 cps as the engine passes across him. If the velocity of sound is 330 m/s, the speed of the engine is
 (a) 33 m/s (b) 18 m/s
 (c) 16.5 m/s (d) 24 m/s
44. An engine approaches a bridge at a speed of 4 m/s while sounding a whistle of frequency 520 Hz. The sound is reflected from the bridge. If the velocity of sound is 330 m/s, the frequency of beats heard by a stationary observer behind the engine is
 (a) 13 (b) 9
 (c) 8 (d) 4
45. A wire is stretched as in a sonometer but a single bridge is placed near the middle point and the tension applied is 8 kg. On plucking the two parts of the wire, 3 beats per second are produced. The load is then doubled, and the two parts of the wire are plucked. The beats produced in this case are
 (a) 3/s (b) 4/s
 (c) 6/s (d) 5/s
- 46.* The frequency of a sonometer wire is v but when the weights producing the tension are completely immersed in water, the frequency becomes $v/2$ and on immersing the weights in a certain liquid, the frequency becomes $v/3$. The specific gravity of the liquid is
 (a) 1.320 (b) 1.032
 (c) 1.413 (d) 1.185
47. Bullets are fired at regular intervals of 20 s from an armoured car A, moving with a velocity of 40 km/h towards a car B which is approaching A with a speed of 60 km/h. The officer sitting in car B hears the report. If the velocity of sound is 330 m/s and that of the wind is 10 m/s in the direction BA, the interval at which the officer will hear the report is
 (a) 16.0 s (b) 19.2 s
 (c) 18.5 s (d) 14.5 s
48. A 25 cm long sonometer wire under a tension of 1.5 kg produces 5 beats per second. The length is decreased by 2 mm, and the number of beats reduces to 2 per second. In order that the tuning fork be in unison with the wire, its length should be reduced further by
 (a) 0.06 cm (b) 0.18 cm
 (c) 0.13 cm (d) 0.04 cm
49. A source of sound of frequency 256 Hz is moving rapidly towards a wall with a velocity of 5 m/s. If the sound travels at a speed of 330 m/s, the number of beats per second that will be heard is
 (a) 4 (b) 3
 (c) 5 (d) 2
- 50.* Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. The speed of the tuning fork is [velocity of sound = 340 m/s]
 (a) 0.5 m/s (b) 5 m/s
 (c) 3 m/s (d) 1.5 m/s
- 51.* Two wires of equal length, one of copper and the other of steel, are stretched side by side by the same tension. In order that they produce the same note, their diameters should bear a ratio ($\rho_{Cu} = 8.9 \text{ g/cc}$ and $\rho_{steel} = 7.7 \text{ g/cc}$)
 (a) 1.5 : 1 (b) 1.07 : 1
 (c) 1 : 1.2 (d) 1.2 : 1
- 52.* The displacement of the medium in a sound wave is given by the equation $Y_1 = A \cos(ax + bt)$ where A , a and b are positive constants. The wave is

reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave. Tick the statement among the following that is incorrect.

- (a) the wavelength and frequency of the wave are $2\pi/a$ and $b/2\pi$ respectively
 (b) the amplitude of the reflected wave is 0.8A
 (c) the resultant wave formed after reflection is
 $y = A \cos(ax + bt) + [-0.8A \cos(ax - bt)]$
 and $V_{\max}$ (maximum particle speed) is $-1.8ba$
 (d) the equation of the standing wave so formed is
 $y = 1.6A \sin ax \cos bt$
53. Two wires of different metals M_1 and M_2 but of the same length, radius and tension produce 5 beats per second when sounded together in their fundamental tones. Their frequencies of vibration, v_1 and v_2 , are ($\rho_{M_1} = 8.4 \text{ g/cc}$ and $\rho_{M_2} = 7.8 \text{ g/cc}$)
 (a) 132 and 137 (b) 176 and 181
 (c) 256 and 261 (d) 320 and 325
54. A wave is represented by the equation $y = A \sin(10\pi x + 15\pi t + \pi/3)$ where x is in metres and t is in seconds. The expression represents
 (a) a wave travelling in the positive- x -direction with a velocity of 1.5 m/s
 (b) a wave travelling in the positive- x -direction having a wavelength of 0.2 m
 (c) a wave travelling in the negative- x -direction having a wavelength of 0.2 m
 (d) a wave travelling in the negative- x -direction having a velocity of 3 m/s
55. A 110 cm long wire is to be divided into three segments by placing two bridges on a sonometer so that their fundamental frequencies are in the ratio 1 : 2 : 3. The positions of the two bridges should be
 (a) 40 and 50 cm from one end
 (b) 60 and 90 cm from one end
 (c) 30 and 50 cm from one end
 (d) 35 and 50 cm from one end
56. On increasing the tension of a stretched string by 2.5 N, the frequency is altered in the ratio of 3 : 2. The original stretching force is
 (a) 2 N (b) 5 N
 (c) 4 N (d) 6 N
57. The amplitude of a wave disturbance propagating in the positive x -direction is given by $y = 1/(1 + x^2)$ at time $t = 0$ and $y = 1/[1 + (x - 1)^2]$ at $t = 2$ seconds where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is
 (a) 2.5 m/s (b) 0.25 m/s
 (c) 0.5 m/s (d) 5 m/s
58. The velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency
 (a) 100 (b) 260
 (c) 400 (d) 320
- 59.* A train approaching a hill at a speed of 40 km/h sounds a whistle of frequency 580 Hz when it is at a distance 1 km from a hill. A wind with a speed of 40 km/h is blowing in the direction of motion of the train. If the velocity of sound in air is 1200 km/h, the frequency of the whistle as heard by an observer on the hill is

- (a) 602 Hz (b) 599 Hz
 (c) 640 Hz (d) 615 Hz
60. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is
 (a) $3/8$ (b) $1/3$
 (c) $1/2$ (d) $8/3$
61. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation $y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$ where x and y are in centimetres and t is in seconds. The equations of the component waves whose superposition gives the above equation are
 (a) $y_1 = 2 \sin(\pi x/15 - 96\pi t)$ and $y_2 = 2 \sin(\pi x/15 + 96\pi t)$
 (b) $y_1 = 2 \sin \pi x/15 \cos 96\pi t$ and $y_2 = 2 \sin \pi x/15 \sin 96\pi t$
 (c) $y_1 = 4 \sin(2\pi x/15 - 96\pi t)$ and $y_2 = 4 \sin(2\pi x/15 + 96\pi t)$
 (d) $y_1 = 4 \cos(2\pi x/15 - 48\pi t)$ and $y_2 = 4 \sin(2\pi x/15 + 48\pi t)$
62. Two sound waves of length 1 m and 1.01 m in a gas produce 10 beats in 3 s. The velocity of sound in gas is
 (a) 360 m/s (b) 337 m/s
 (c) 330 m/s (d) 300 m/s
63. Two open pipes A and B are sounded together such that beats are heard between the first harmonic of A and second harmonic of B. If the fundamental frequency of A is 256 Hz and that of B is 170 Hz, then the beat frequency between the harmonics is
 (a) 4/s (b) 3/s
 (c) 2/s (d) 5/s
64. A wave is represented by the equation $y = 10 \sin 2\pi(100t - 0.02x) + 10 \sin 2\pi(100t + 0.02x)$. The maximum amplitude and loop length are respectively
 (a) 20 units and 30 units (b) 20 units and 25 units
 (c) 30 units and 20 units (d) 25 units and 20 units
65. The equation of a standing wave set up in a medium is given by $y = 4 \cos \frac{\pi x}{5} \sin 40\pi t$ where x and y are in centimetres and t is in seconds. The amplitude and the velocity of the two component waves are respectively
 (a) 4 cm and 60 cm/s (b) 8 cm and 60 cm/s
 (c) 2 cm and 60 cm/s (d) 2 cm and 120 cm/s
66. A tuning fork and an air column at 51°C produce 4 beats per second when sounded together. When the temperature of the air column is decreased to 16°C , the same tuning fork produces two beats per second? The frequency of the tuning fork is
 (a) 49.8 Hz (b) 57.8 Hz
 (c) 35.7 Hz (d) 60.8 Hz
67. If $\lambda_1, \lambda_2, \lambda_3$ are the wavelengths of the waves giving resonance in the fundamental, first and second harmonic modes respectively within a pipe closed at one end, the ratio of the wavelengths, $\lambda_1 : \lambda_2 : \lambda_3$, is
 (a) 1 : 2 : 3 (b) 1 : 3 : 5
 (c) 1 : $1/2$: $1/3$ (d) 1 : $1/3$: $1/5$
68. An open pipe has a fundamental frequency of 300 Hz. The first overtone of this organ pipe is the same as the first overtone of a closed organ pipe. The length of the closed organ pipe is
 (a) 82 cm (b) 90 cm
 (c) 41 cm (d) 78 cm

69. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound is 320 m/s. Its estimated diameter is
 (a) 3.3 cm (b) 0.33 cm
 (c) 2.4 cm (d) 4.4 cm
70. A string of length 25 cm and mass 2.5 g is under tension. A closed pipe at one end is 40 cm long. When the string is vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. The beat frequency decreases on decreasing the tension. The speed of sound is 320 m/s. The tension in the string is
 (a) 14 N (b) 24 N
 (c) 35 N (d) 27 N
71. The equation of harmonic oscillatory motion with an amplitude of 8 cm if 150 oscillations are performed during one minute and the initial oscillation phase is 45° is
 (a) $8 \sin (10\pi t + \pi/4)$ (b) $8 \sin (5\pi t + \pi/4)$
 (c) $8 \sin (4\pi t + \pi/4)$ (d) $8 \cos (4\pi t - \pi/4)$
72. The equation of motion of a point is given by $x = \sin \left(\frac{\pi}{6} \right) t$. The moments of time at which the maximum velocity and acceleration respectively are attained are
 (a) 0, 6, 12 s, etc. and 3, 9, 15 s, etc.
 (b) 0, 4, 8 s, etc. and 3, 6, 9, 12 s, etc.
 (c) 0, 2, 6, 8 s, etc. and 0, 3, 6, 9 s, etc.
 (d) 2, 4, 6 s, etc. and 0, 4, 8, 12 s, etc.
73. An equation of undamped oscillations is $x = 4 \sin 600\pi t$ cm. The displacement from the position of equilibrium of a point 75 cm away from the source of oscillations 0.01 s after they begin is (speed of propagation of oscillation is 300 m/s)
 (a) 0.02 m (b) 0.2 m
 (c) 0.04 m (d) 0.4 m
74. The displacement from the position of equilibrium of a point 4 cm from a source of oscillations is half the amplitude at the moment $t = T/6$ (T is the time period). The length of the running wave is
 (a) 0.96 m (b) 0.48 m
 (c) 0.24 m (d) 0.12 m
75. The period of oscillation of a point is 0.04 s and the velocity of propagation of oscillation is 300 m/s. The difference of phases between the oscillations of two points at distances 10 and 16 m respectively from the source of oscillation is
 (a) 2π (b) $\pi/2$
 (c) $\pi/4$ (d) π

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (c) | 3. (d) | 4. (b) | 5. (a) |
| 6. (c) | 7. (c) | 8. (c) | 9. (d) | 10. (a) |
| 11. (c) | 12. (c) | 13. (b) | 14. (c) | 15. (a) |
| 16. (d) | 17. (c) | 18. (b) | 19. (a) | 20. (c) |
| 21. (b) | 22. (d) | 23. (d) | 24. (a) | 25. (d) |
| 26. (c) | 27. (b) | 28. (a) | 29. (d) | 30. (c) |

- | | | | | |
|---------|---------|---------|----------|---------|
| 31. (d) | 32. (c) | 33. (a) | 34. (d) | 35. (b) |
| 36. (c) | 37. (d) | 38. (b) | 39. (d) | 40. (a) |
| 41. (c) | 42. (d) | 43. (c) | 44. (a) | 45. (b) |
| 46. (d) | 47. (c) | 48. (c) | 49. (a) | 50. (d) |
| 51. (b) | 52. (d) | 53. (a) | 54. (c) | 55. (b) |
| 56. (a) | 57. (c) | 58. (c) | 59. (b) | 60. (a) |
| 61. (a) | 62. (b) | 63. (c) | 64. ((b) | 65. (d) |
| 66. (a) | 67. (d) | 68. (c) | 69. (a) | 70. (d) |
| 71. (b) | 72. (a) | 73. (c) | 74. (b) | 75. (d) |

Note: Problems marked with a star should take about five minutes. The rest should be solved in about three minutes each.

16

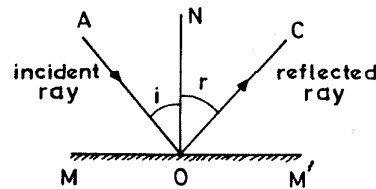
Reflection, Refraction and Optical Instruments

Laws of Reflection

When a ray falls on a smooth polished surface (called a mirror), it is reflected from the surface. The two laws of reflection are as follows:

1. The incident ray AO , normal NO and the reflected ray OC , all lie in one plane.
2. The angle of incidence i and the angle of reflection r are equal.

If the angle of incidence is zero, angle of reflection is zero and in the condition of normal incidence, light retraces its path after reflection.



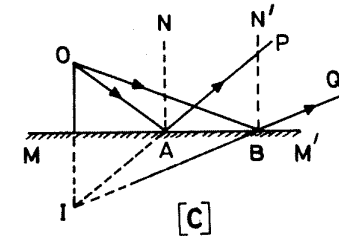
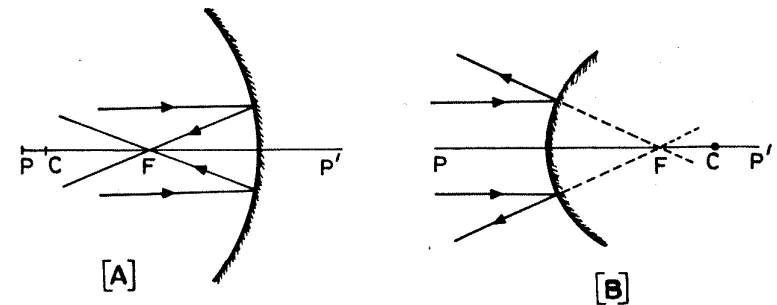
Principle of Reversibility of Light

From the laws stated above, one observes that AO or CO can be considered to be the incident ray. In this way, if a light-ray is reversed, it always travels along the original path.

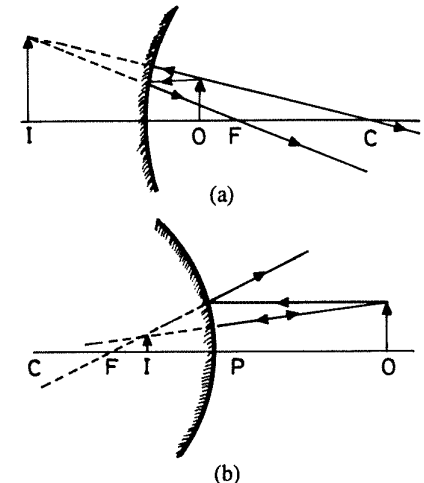
Real and Virtual Images

An optical image is a point where rays of light either intersect or appear to intersect. Thus the image of an object is simply an assemblage of image points corresponding to the various points of the object. If the rays, after reflection or refraction, actually converge at a point, then the image is said to be *real*. It can be formed on a screen suitably placed. If the rays do not actually converge but appear to do so and actually diverge, the image is said to be *virtual*.

In Figs (a) and (b), PP' are marked as the principal axes of the concave and convex mirror. In (a), F is the real focus of the concave mirror while in (b), F is the virtual focus of the convex mirror. C is marked as the centre of curvature of the mirror and the real or virtual focus is located on the principal axis, midway between the centre of curvature and the mirror.



Concave mirrors form inverted real images of the object placed beyond the principal focus F . If the object is between the principal focus and the mirror, the image is virtual, erect and enlarged as shown in Fig. (a) here. Convex mirrors form only erect virtual images of the object placed in front of them. The images are always smaller in size than the object. This is shown in Fig. (b).



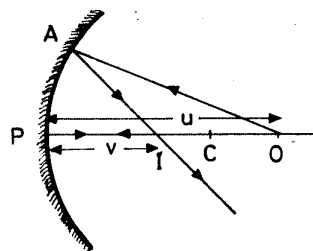
Relation between Focal Length and Radius of Curvature

If f is the distance of the focus F from the pole P of the mirror, then PF is the focal length f of the mirror. If R is the radius of curvature of the mirror, i.e. if the distance from C to P is R then $f = R/2$.

Relation between u , v and f for Mirrors

Let us take a point object situated at O on the principal axis of a concave mirror, I is the position of the image and C is the centre of curvature. For small apertures, one can assume that A is close to P and so let $PI = v$ and $PO = u$. The relation between u , v and f is

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$



The same formula is valid for concave as well as convex mirrors.

The Sign Convention

1. The object distance is positive for real objects and negative for virtual objects.
2. The image distance is positive for real images and negative for virtual images.
3. The radius of curvature and focal length are positive for converging mirrors and negative for diverging mirrors.

According to this convention; the focal lengths of the concave mirrors are taken as positive and those of convex mirrors as negative.

*See footnote

Linear Magnification

For a thin long object situated vertically on the axis, a thin long image is formed vertically on the axis and the ratio of the length of the image to that of the object is called linear magnification (m) and $m = -v/u$. (If the image is inverted, the magnification is represented as negative.)

Superficial Magnification

When a plane object of finite size is placed in front of a mirror perpendicular to its principal axis, it will have its length and breadth both magnified in the ratio of v/u and therefore,

$$\text{Superficial magnification} = \frac{v^2}{u^2}$$

Conjugate Foci

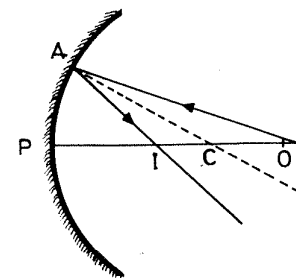
In the figure, OA is the incident ray while AI is the reflected ray. If IA were

* An alternative sign convention known as coordinate sign convention is as follows

(i) Take the pole or the optical centre as the origin and the principal axis as the X-axis. Then the coordinates u, v, R and f on the axis of x denote the coordinates of object, image, centre of curvature and the focus respectively.

(ii) Any of these quantities is positive if the corresponding point lies on the positive side of the origin and if it lies on the negative side, it is taken as negative.

the incident ray, it is evident that AO would be the corresponding reflected ray. Thus the positions of the object and the image are interchangeable. These two points are said to be conjugate foci.



Rotation of Reflecting Surface

When a plane mirror rotates through a certain angle, the reflected ray moves through double the angle.

Formation of Images by Two Plane Mirrors

(a) Plane mirror at right angles

When two mirrors are placed at right angles and an object is placed between them, three images are seen in any position of the eye.

(b) Plane mirrors at 72°

When two plane mirrors are placed at angle of 72° and an object is placed between them, five images are seen.

(c) Plane mirrors parallel to each other

When two plane mirrors are placed parallel to each other and an object is placed between them, an infinite number of images are formed.

(d) Two plane mirrors inclined at an angle θ

(i) Number of images seen = $\frac{360}{\theta}$, if $\frac{360}{\theta}$ is odd

(ii) Number of images seen = $\left(\frac{360}{\theta} - 1\right)$, if $\frac{360}{\theta}$ is even.

Smallest Size of Plane Mirror Required to See the Full Image of an Object

If the size of the object, say the face of a person is a cm, then the minimum size of the mirror to see the full face must be $(a/3)$ cm.

Images Due to Two Spherical Mirrors

An image of an object obtained by the first mirror can serve as an object for the second mirror. Such an image which acts as an object may be real or virtual object. The nature and position of the final image may be determined in the same manner as for a single mirror.

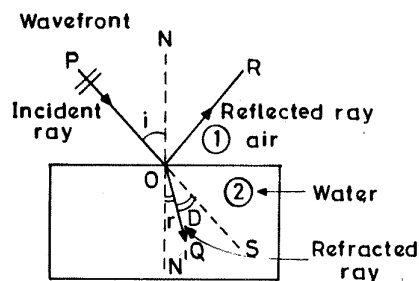
Spherical Aberrations

The relatively simple relations that are derived between object and image distances, focal lengths, radii of curvature, etc. were derived by making the approximation $\sin \theta = \theta$, i.e. they apply only to rays originating at axial points and making small angles with the axis. But actually, the rays originate not only from axial points but from points which lie off the axis as well. The consequence of this is that the image is not sharp. Furthermore, dependence of refractive index on wavelength gives rise to number of images corresponding to each wavelength (if the light is not monochromatic). All these aspects when taken into consideration suggest that there is a departure of the actual image from the predictions of first-order theory. These departures are known as aberrations.

Spherical aberrations cannot be completely removed in a spherical mirror: however, they can be minimised to a great extent. One way is to minimise the width of the beam of light by using a "stop" at the centre of curvature of the mirror. These can also be minimised by using parabolic or ellipsoidal mirrors.

Refraction and its Laws

An incident ray of light travelling from some source in air travels and enters another medium, say water. It bends in the denser medium toward the normal (NN'). This bending of light is called refraction. OQ is the refracted ray. OS is the direction in which the incident ray PO would have travelled in absence of the refracting medium.



Angle of deviation (D) = angle of incidence (i) – angle of refraction (r)
If medium (air) here is designated as 1 and water as 2 then

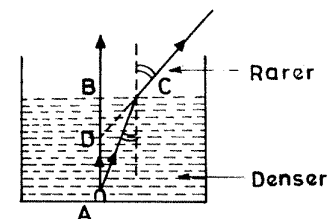
$$\frac{\sin i_1}{\sin r_2} = {}_1\mu_2 \text{ [here suffixes 1 and 2 refer to the medium].}$$

${}_1\mu_2$ is called the refractive index of medium 2 with respect to medium 1. Thus the law of refraction states that the ratio of the sine of the angle of incidence in medium 1 and the sine of the angle of refraction in medium 2 is a constant. Also, the incident ray, the refracted ray and the normal lie in one plane.

$$\frac{\sin i_1}{\sin r_2} = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}$$

Refraction from Denser to a Rarer Medium

In the figure here, A is an object in the denser medium. When this object is viewed from a rarer medium, its virtual image is seen at D. BA is the real depth of the object while BD is its apparent depth. If $AB = u$ and $BD = v$, then $v = u/\mu$ where μ is the refractive index of the denser medium with reference to the rarer medium.



As μ increases, v decreases, i.e. the object appears to be raised more and more.

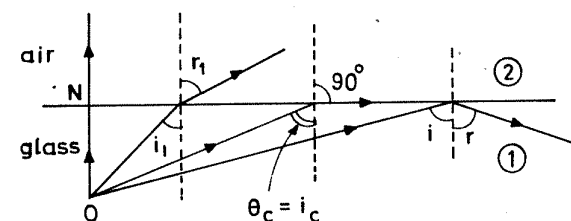
Similarly, if an object placed in air is seen by an eye placed within the liquid (denser medium), rays from the object are refracted into the liquid and u and v are related by the equation $v = \mu u$, i.e. as μ increases, v increases.

If there are several media denser than air of heights $h_1, h_2, h_3, \dots$ and of refractive indices $\mu_1, \mu_2, \mu_3, \dots$ respectively, and an object at the bottom of all these media is viewed from the rarer medium, say air, then

$$\text{Apparent depth of the object} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} + \frac{h_3}{\mu_3} + \dots$$

Total Internal Reflection

Object O is placed in the denser medium. Hence, angle of refraction r_1 is greater than angle of incidence i_1 . As i increases, r also increases. When $i = i_c$, $r = 90^\circ$ (i_c is called the critical angle). If $i > i_c$, the incident ray gets reflected in the same medium. This is total internal reflection ($\angle i = \angle r$)



$$\mu_1 \sin \theta_c = \mu_2 \sin 90^\circ \Rightarrow \sin \theta_c = \frac{\mu_2}{\mu_1}$$

$$\therefore \frac{\text{Refractive index of denser medium}}{\text{Refractive index of rarer medium}} = \frac{1}{\sin \theta_c}$$

or $\sin \theta_c$ = refractive index of rarer medium with respect to denser medium.

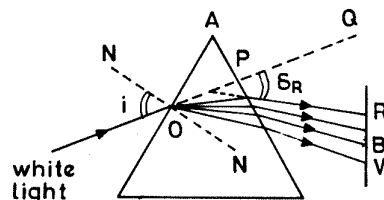
If air is the rarer medium, and glass is the denser medium, then

$${}_a\mu_g = \frac{1}{\sin \theta_c} \text{ or } \theta_c = \sin^{-1} \left(\frac{1}{{}_a\mu_g} \right) = \sin^{-1} {}_g\mu_a$$

$$\left[{}_a\mu_g = \frac{1}{{}_g\mu_a} \right]$$

Refraction through a Prism (Dispersion)

Due to fact that the refractive index of any medium with respect to another is dependent upon the wavelength of light, a ray of light when passed through



a prism gets refracted through different angles corresponding to the wavelengths constituting white light. Thus, white light gets separated into its constituent colours or wavelengths. This phenomenon is called dispersion. It is observed that $\mu_{\text{blue}} > \mu_{\text{red}}$, i.e. blue light is refracted more than red. Hence δ_R is the angle of deviation for red colour with respect to the undeviated path OPQ.

Angular dispersion is the difference of deviations for the colours under consideration.

Dispersive Power

$$\omega = \frac{\delta_V - \delta_R}{\delta}$$

where δ_V and δ_R are the deviations for violet and red rays respectively and δ is the mean deviation (deviation for the mean ray).

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)}$$

[μ is the refractive index for the mean light (yellow).]

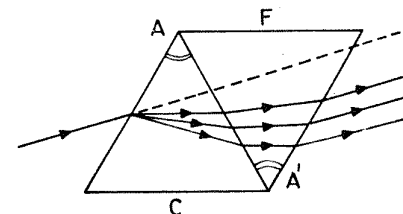
Deviation without Dispersion

Take two prisms, one of crown glass and the other of flint glass, with their angles equal to A and A' respectively. (Remember that the dispersive power of a flint glass prism is higher than that of a crown glass prism.)

Angular separation for crown glass prism

$$\theta_C = (\mu_V - \mu_R)A$$

and $\theta_F = (\mu'_V - \mu'_R)A'$



(μ_V and μ_R are refractive indices for the crown glass prism and μ'_V and μ'_R for the flint glass prism.)

For no net angular separation, $\theta_C + \theta_F = 0$

$$A' = - \frac{(\mu_V - \mu_R)}{(\mu'_V - \mu'_R)} A \quad (1)$$

Two prisms which satisfy the condition given in Eq. (1) are said to be achromatic. It is found that if the angle A of the crown glass prism is 60° , then the angle A' of the flint glass prism is $29^\circ 17'$ for an achromatic combination.

Equation (1) can also have the following form

$$\omega\delta + \omega'\delta' = 0$$

where ω and ω' are the dispersive powers of the two prisms and δ and δ' are their mean deviations.

Dispersion without Deviation

If the angles of the crown and flint glass prisms are so adjusted that the deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second, then the emergent beam will be dispersed but its mean position will be parallel to the incident beam. Such a combination of prisms which produces dispersion without deviation is called a *direct vision prism*.

If A and A' be the angles of two prisms and δ and δ' be the deviations of the mean rays of light, then

$\delta = (\mu - 1)A$ and $\delta' = (\mu' - 1)A'$, where μ and μ' are the refractive indices.

For deviation to be zero, $\delta + \delta' = 0$

$$\text{Hence the condition is } A' = - \frac{(\mu - 1)}{(\mu' - 1)} A$$

The net dispersion $(D_V - D_R) = (\mu - 1)(\omega - \omega')$ where μ is the refractive index for the mean ray and ω and ω' are the dispersive powers of flint and crown glass prisms.

Fermat's Principle

A light ray travelling from one point to another will follow a path such that, compared with nearby paths, the time required is either a minimum or a

maximum or will remain unchanged (that is, it will be stationary). The laws of reflection and refraction can be readily derived from this principle.

Relation between Minimum Deviation and Refractive Index

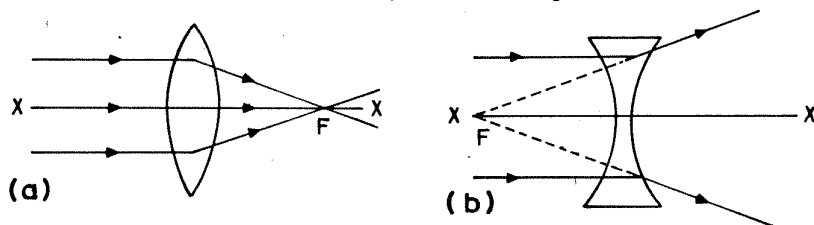
The angle of deviation δ is minimum for a particular value of angle of incidence when a ray of light is allowed to pass through a prism. If μ is the refractive index of the material of the prism and A is the refracting angle of the prism,

$$\mu = \frac{\sin \frac{\delta + A}{2}}{\sin \frac{A}{2}}$$

Here δ is the angle of minimum deviation.

Refraction Through Lenses

A converging lens is shown in Fig. (a) here. The principal focus or focal point of a thin lens with spherical surfaces is the point F where rays parallel to the principal axis XX are brought to a focus. This is a real focus for a converging lens but for a diverging lens [Fig. (b)], it is a virtual focus. The distance between F and the lens is the focal length of the lens. If the two spherical surfaces of a lens are identical, two symmetric focal points exist for each lens.



Relation between Object and Image Distance

If u is the object distance from the lens, v is the image distance from the lens and f is the focal length of the lens, the relation is

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Here, the lens is assumed to be thin and this relation is true for converging as well as diverging lenses.

Sign Convention

- (i) u is positive for a real object and negative for a virtual object.
- (ii) v is positive for a real image and negative for a virtual image.
- (iii) f is positive for a converging lens and negative for a diverging lens.

If the two surfaces of a lens have radii of curvature r_1 and r_2 , μ is the refractive index of glass, then the focal length of the lens may be calculated from the relation:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

This equation holds good for all types of thin lenses. The radii of curvature are considered to be positive for convex surfaces and negative for concave surfaces. If a lens with refractive index μ_1 is immersed in a material with index μ_2 , then μ is replaced by (μ_1/μ_2) in the above general equation and it becomes

$$\frac{1}{f} = \left(\frac{\mu_1}{\mu_2} - 1 \right) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Power of a Lens

The power of a lens is expressed as the reciprocal of its focal length, measured in metres. The unit of power is the dioptre (m^{-1}). The power of a convex lens is positive while that of a concave lens is negative.

If two lenses of focal length f_1 and f_2 are in contact, then the equivalent power P is equal to the sum of the individual powers of the lenses, i.e.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \text{or} \quad P = P_1 + P_2$$

Two Convex Lenses in Contact

The focal length of the combination of two convex lenses of focal length f_1 and f_2 is given as

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

If one of the lenses is replaced by a concave lens, then

$$\frac{1}{F} = \frac{1}{f_1} - \frac{1}{f_2} \Rightarrow F = \frac{f_1 f_2}{f_2 - f_1}$$

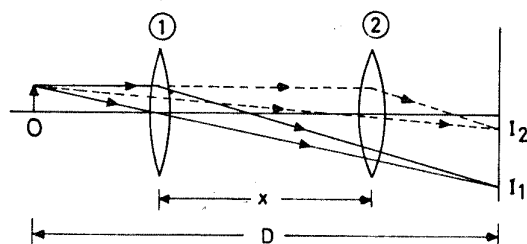
If $f_2 > f_1$, then F is positive and the combination will behave like a convex lens and if $f_1 > f_2$, then F is negative and the combination will behave like a concave lens.

Displacement of the Lens with Object and Screen Fixed

An object and the screen are fixed such that their separation D is greater than $4f$. Position (1) of the lens gives an image I_1 on the screen. When the lens is shifted by a distance x to position (2), a clear image I_2 is formed on the screen. The focal length of the lens is given by

$$f = \frac{D^2 - x^2}{4D}$$

If I_1 and I_2 are the sizes of the images corresponding to the two positions of the lens, then the size of the object is



$$O = \sqrt{I_1 \times I_2}$$

This shows that the size of the object is equal to the geometric mean of the sizes of two images.

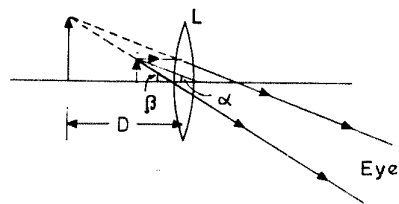
Optical Instruments

(a) The magnifying glass or a simple microscope

The magnifying power of a microscope is defined as

$$M = \frac{\text{angle subtended by the image at the eye}}{\text{angle subtended by the object at the eye}} \\ = \frac{\beta}{\alpha} = \left(1 + \frac{D}{f}\right)$$

where f is the focal length of the lens.



(b) Compound microscope

Such a microscope consists of two converging lenses, an objective lens of focal length f_o and an eyepiece lens of focal length f_e . The magnification

$$M = \left[\frac{d}{f_e} + 1\right] \left[\frac{V_o}{f_o} - 1\right]$$

where d is the least distance of distinct vision (for the normal eye, it is about 25 cm), f_e is the focal length of the eyepiece lens and f_o is the focal length of the objective lens. V_o is the distance from the objective lens to the image it forms. It can alternatively be expressed as

M = linear magnification (m) for the objective lens $\times$ angular magnification (m_θ) for the eyepiece

$$= m \times m_\theta = -\left(\frac{S}{f_o}\right) \left(\frac{d}{f_e}\right)$$

d is the least distance of distinct vision and the distance S is so chosen that the image from the objective lens is formed at the focal point of the eyepiece (S is sometimes called the tube length). In such a situation, parallel rays enter the eye and a final image is formed at infinity.

(c) A telescope

The magnifying power of a telescope is given by

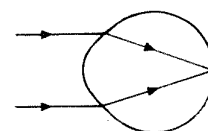
$$M = -\frac{f_o}{f_e} \left(1 + \frac{f_e}{d}\right)$$

d is the least distance of distinct vision.

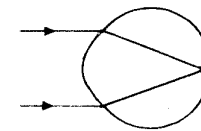
Alternatively,

$$M = -\frac{f_o}{f_e}$$

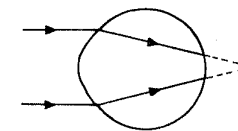
(d) Defects of vision and its correction



(a) Normal eye



(b) Myopic eye



(c) Hyperopic eye

Corrections

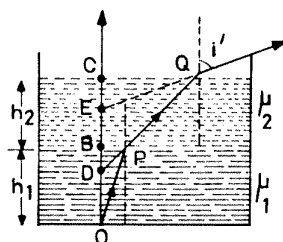
- For a myopic eye, a divergent lens is used so that the image is shifted to the retina.
- For hyperopia, a convergent lens is used so that the image is formed at the retina.

ILLUSTRATIONS

1. An object O is located at the bottom of a tank containing two different liquids (which do not mix with each other), and is seen vertically from above. The lower and upper liquids are of depths h_1 and h_2 respectively and of refractive indices μ_1 and μ_2 . Locate the position of the image of the object O as seen from above.

Solution

$$\angle BOP = \angle i, \angle BDP = \angle r, \angle CEQ = \angle i'$$



For refraction at P

$$\mu_1 \sin i = \mu_2 \sin r \Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sin r}{\sin i} = \frac{\tan r}{\tan i} = \frac{h_1}{BD} \Rightarrow BD = h_1 \frac{\mu_2}{\mu_1}$$

For refraction at Q

$$\mu_2 \sin r = \sin i' \text{ (final medium is air)}$$

$$\text{or } \mu_2 = \frac{\sin i'}{\sin r} = \frac{\tan i'}{\tan r} = \frac{BD + BC}{CE} \quad \text{or} \quad CE = \frac{BD + BC}{\mu_2}$$

$$\therefore CE = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \quad (\text{CE is the apparent depth of the object})$$

2. A ray of light enters the face AB of a glass prism of refractive index μ at an angle of incidence i . Find the value of i such that no ray emerges from the face AC of the prism (given that the angle of the prism is A .)

Solution

The critical value of r' is given by

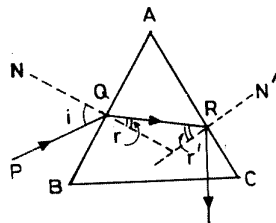
$$\sin r' = \frac{1}{\mu}$$

$$\text{or } \mu \sin r' = 1 \text{ and } \sin i = \mu \sin r$$

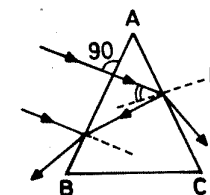
$$\begin{aligned} &= \mu \sin (A - r') = \mu (\sin A \cos r' - \cos A \sin r') \\ &= \mu \sin A \cos r' - \cos A \\ &= \mu \sin A \sqrt{1 - \frac{1}{\mu^2}} - \cos A \\ &= \sqrt{(\mu^2 - 1)} \sin A - \cos A \end{aligned}$$

For this value of i , the value of r' is equal to the critical angle and the emergent ray grazes along the face AC.

3. A parallel beam of light falls normally on face AB of a thin prism and undergoes a deviation of $1^\circ 15'$ on passing through the prism. The part of the



beam which is reflected internally at the face AC, on emergence from the face AB, makes an angle of $6^\circ 30'$ with the incident beam. Calculate the angle of the prism and the refractive index of the material of the prism.



Solution

$$\text{Deviation } D = 1^\circ 15' = \left(\frac{5}{4}\right)^\circ$$

$$\therefore (\mu - 1) A = \frac{5}{4} \quad (i)$$

When the ray emerges from the first face (AB), the angle of emergence,

$$i = 6^\circ 30' = \left(\frac{13}{2}\right)^\circ$$

$$\text{and hence } 2A \times \mu = \frac{13}{2}$$

$$\text{or } \mu A = \frac{13}{4} \quad (ii)$$

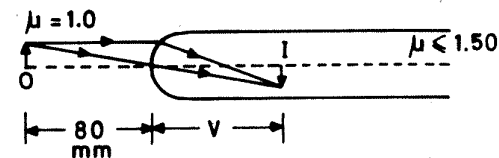
Combining Eqs (i) and (ii), we get

$$A = 2^\circ \text{ and } \mu = 1.625$$

4. One end of a cylindrical glass rod of refractive index 1.50 is ground and polished to a hemispherical surface of radius $R = 20$ mm. A linear object 1 mm in length is placed on the axis of the cylinder vertically as shown in the figure. The object is 80 mm to the left of the vertex of the surface.

(a) Find the position and magnification of the image.

(b) If the same rod along with the object is immersed in water of refractive index 1.33, what will be the position and magnification of the object.



Solution

(a) The given data is

$$\mu = 1.00, \mu' = 1.50, R = +20 \text{ mm and } u = +80 \text{ mm}$$

From the relation

$$\frac{\mu}{u} + \frac{\mu'}{v} = \frac{\mu' - \mu}{R}, \text{ we have}$$

$$\frac{1}{80} + \frac{1.5}{v} = \frac{1.5 - 1}{20} \Rightarrow v = +120 \text{ mm}$$

The image is therefore at the right of the surface and at a distance of 120 mm from it. It is a real image as the refracted rays converge at it.

$$m = -\left(\frac{\mu v}{\mu' u}\right) = -\frac{1 \times 120}{1.5 \times 80} = -1$$

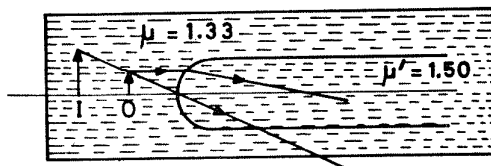
This means that the image is of the same height but is inverted.

(b) When the object and the rod are immersed in water, we have

$$\frac{\mu}{u} + \frac{\mu'}{v} = \frac{\mu' - \mu}{R} \quad \text{or} \quad \frac{1.33}{80} + \frac{1.5}{v} = \frac{1.5 - 1.33}{20}$$

Therefore,

$$v = -180 \text{ mm}$$



The fact that v is negative means that the rays do not converge after refraction but appear to diverge from a point 180 mm to the left of the vertex. One may infer that the image is virtual.

The lateral magnification is

$$m = -\left(\frac{\mu v}{\mu' u}\right) = -\frac{1.33 \times (-180)}{1.5 \times 80} = +2$$

Since the magnification is positive, the image is erect and its height is twice that of the object.

5. It is desired to design: (i) a direct-vision prism and (ii) an achromatic prism, using flint and crown glass of dispersive powers $\omega_F = 0.031$ and $\omega_C = 0.018$, respectively.

(a) If the mean refractive indices are $\mu'_D = 1.620$ and $\mu_D = 1.508$, and the angle of the flint glass prism is 10° , find the angle of the crown glass prism and the net dispersion for the direct vision set up.

(b) If the compound prism has to be achromatic, find the angle of the crown glass prism (given that $\mu_F = 1.513$, $\mu_C = 1.504$ and $\mu'_F = 1.632$ and $\mu'_C = 1.613$). Note that these values of refractive indices are obtained from the plots of variation of index with wavelength for various types of glass.

Solution

(a) Direct vision prism:

$$\delta_D = (\mu_D - 1)A \text{ and } \delta'_D = (\mu'_D - 1)A'$$

If now the two prisms are combined apex to base, and the angles A and A' are chosen so that $\delta_D = \delta'_D$, the mean deviation of the first prism is offset by that of the second. The required ratio of the angles is

$$\frac{A}{A'} = \frac{\mu'_D - 1}{\mu_D - 1} = \frac{1.620 - 1}{1.508 - 1} = 1.22$$

If the angle A' of the flint glass is 10.0° , the angle A of the crown glass should be $A = 10 \times 1.22 = 12.2^\circ$.

To find net dispersion, we have

$$\begin{aligned} \text{Dispersion produced by the flint glass prism} &= \delta'_F - \delta'_C \\ &= (\mu'_F - \mu'_C)A' \end{aligned}$$

Similarly, for the crown glass prism,

$$\text{Dispersion produced} = (\mu_F - \mu_C)A$$

$$\therefore \text{Net dispersion} = (\mu'_F - \mu'_C)A' - (\mu_F - \mu_C)A \quad (i)$$

Expressing A' in terms of A and multiplying the numerator and denominator of the second term of Eq. (i), we get

$$\begin{aligned} \text{Net dispersion} &= A(\mu_D - 1)(\omega' - \omega) \\ &= 12.2 \times (1.508 - 1)(0.031 - 0.018) = 0.079^\circ \end{aligned}$$

(b) An achromatic prism: If the compound prism is to be achromatic, the dispersion of the two prisms must be equal. That is,

$$(\mu'_F - \mu'_C)A' = (\mu_F - \mu_C)A$$

or

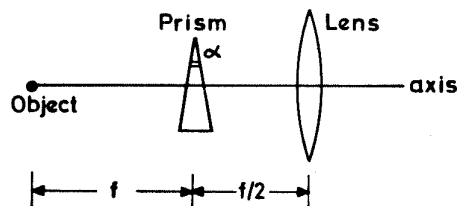
$$\frac{A}{A'} = \frac{\mu'_F - \mu'_C}{\mu_F - \mu_C} = \frac{1.632 - 1.613}{1.513 - 1.504} = \frac{0.019}{0.009} = 2.111$$

$$\therefore A = 21.1^\circ$$

6. A small angle prism (refractive index μ and angle α) and a convex lens are arranged as shown in the figure. A point object O is placed as shown.

(a) Calculate the angle of deviation of the rays hitting the prism at nearly normal incidence.

(b) If the distances between the object, prism and the lens are as shown in the figure, locate the position of the image both along and transverse to the axis.



Solution

(a) Deviation $\delta = (\mu - 1)\alpha$

This can be derived using Snell's Law. We can observe that deviation is independent of the angle of incidence.

(b) Let O' be the virtual position of the object viewed from behind the prism. We may observe from the expression obtained in (a) that deviation δ is same for each ray. The object distance does not change when object moves from O to O' .

To locate the position of the image of the object O formed by the lens, we use the formula

$$u = \frac{3}{2}f$$

where f is the focal length of the object.

$$\frac{1}{v} = \frac{1}{f} - \frac{2}{3f} \Rightarrow v = 3f$$

Thus image position is at $3f$ on the right side of the lens along the axis. The transverse object distance is OO' and is related to angle δ by $OO' = f\delta$. Image transverse distance is II' and the ratio

$$\frac{II'}{OO'} = \frac{-v}{u} \quad \text{or} \quad II' = \frac{-v}{u} OO' = \frac{-v}{u} f(\mu - 1)\alpha$$

Here,

$$u = \frac{3f}{2}$$

$$\begin{aligned} \therefore II' &= \frac{-3f}{3f/2} f(u - 1)\alpha \\ &= -2f(u - 1)\alpha \end{aligned}$$

The negative sign implies that the image is below the axis.

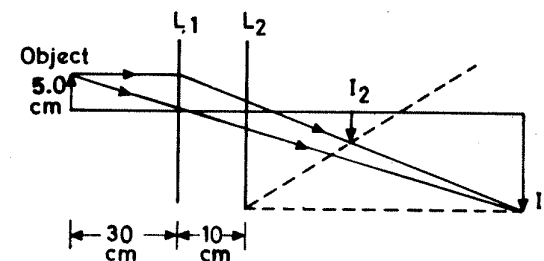
7. Two lenses, with focal lengths 20 and 30 cm respectively, are separated by a distance of 10 cm. An object of length 5 cm is put in front of the first lens perpendicular to the optical axis. The distance of the object from the first

lens is 30 cm. Find the size of the image behind the lens system and state its nature.

Solution

Image from lens L_1 :

$$\frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f_1}$$



$$\text{or} \quad \frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}$$

Therefore, the image from lens L_1 is formed at a distance of 60 cm from it on the right side and is real. This image I_1 serves as an object for lens L_2 and is situated at a distance of $(60 - 10) = 50$ cm from L_2 and its size is 10 cm.

Image from lens L_2 :

$$u_2 = -50 \text{ cm}, f = 30 \text{ cm}$$

$$\therefore \frac{1}{v_2} - \frac{1}{50} = \frac{1}{30} \Rightarrow v_2 = 18.8 \text{ cm}$$

Size of final image = $m_2 \times$ size of object (1st image in this case)

$$\begin{aligned} &= \frac{-v_2}{u_2} \times -10 \text{ cm} = \frac{-18.8}{-50} \times -10 \\ &= -3.76 \text{ cm} \end{aligned}$$

Thus, the final image is 3.76 cm high and is inverted.

8. The radius of curvature of the convex face of a planoconvex lens is 12 cm and its refractive index is 1.5.

(a) Find the focal length of the lens. The plane surface of the lens is now silvered.

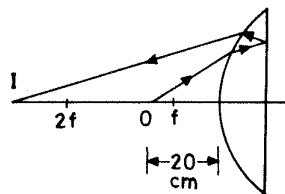
(b) At what distance from this lens will parallel rays incident on the convex face converge?

(c) Sketch a ray diagram to locate the image when a point object is placed on the axis 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c).

Solution

$$\begin{aligned}
 (a) \quad \frac{1}{f} &= (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \\
 &= (1.5 - 1) \left(\frac{1}{12} - \frac{1}{\infty} \right) \\
 &= \frac{1}{24} \Rightarrow f = 24 \text{ cm}
 \end{aligned}$$



(b) This is equivalent to an equiconvex lens. Hence

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{12} + \frac{1}{12} \right) = \frac{1}{12} \Rightarrow f = 12 \text{ cm}$$

The rays are focussed at a distance of 12 cm on the convex side of the lens.

(c) The object is at O (20 cm from the lens), and the image is formed at I.

(d) To calculate the image distance from the lens, we proceed as follows.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{12} - \frac{1}{20} = \frac{1}{30} \Rightarrow v = 30 \text{ cm}$$

9. An object of height 4 cm is placed 20 cm in front of a convex lens of focal length 40 cm. A plane mirror is placed 20 cm behind the lens. Find the position of the final image and its magnification.

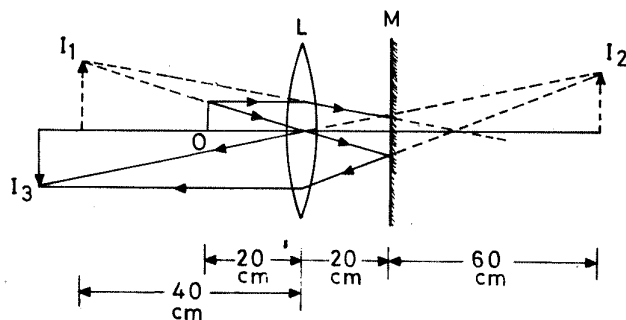
Solution

For the formation of the first image from the lens, it is observed that a virtual, erect and magnified image is formed on the same side where the object is

$$f = 40 \text{ cm}, u = 20 \text{ cm}$$

$$\therefore \frac{1}{v_1} = \frac{-1}{40}$$

$$v_1 = -40 \text{ cm}$$



Thus, the first image I_1 formed by the lens is erect, virtual and has a magnification 2. The second virtual image I_2 of the first image I_1 is formed in the plane mirror, 60 cm behind the mirror. This image I_2 acts as an object for the final image I_3 formed by the lens. Here $f = 40 \text{ cm}$, $u = 80 \text{ cm}$. Therefore, the distance of image I_3 from the lens (v_3) may be worked out as follows.

$$\frac{1}{v_3} + \frac{1}{80} = + \frac{1}{40} \quad \text{or} \quad \frac{1}{v_3} = \frac{1}{40} - \frac{1}{80} = \frac{1}{80} \Rightarrow v_3 = 80 \text{ cm}$$

The magnification in this case is 1 ($I_2 = I_3$).

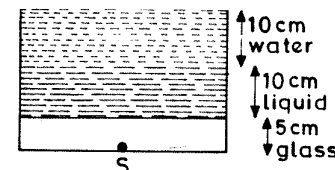
Hence the total magnification is 2.

The position of the final image is 80 cm from the lens on the side where the object is situated.

10. The base of a tank is a horizontal plate of glass 5 cm thick and its refractive index is 1.6. On this, there is a layer of liquid whose refractive index is 1.5 and thickness 10 cm. On this liquid floats a layer of water 10 cm thick ($\mu_w = 4/3$). An observer looking vertically downwards observes a spot on the lower surface of the base. What is the apparent position of the spot? Where would it be if the observing eye were under the surface of water?

Solution

When the eye is in air,



$$\begin{aligned}
 \text{Apparent depth} &= \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} + \frac{h_3}{\mu_3} \\
 &= \frac{5}{1.6} + \frac{10}{1.5} + \frac{10}{4/3} = 17.29 \text{ cm}
 \end{aligned}$$

Hence, the apparent position of the spot is

$$(5 + 10 + 10) - 17.29 = 7.71 \text{ cm}$$

from the lowest surface of the glass.

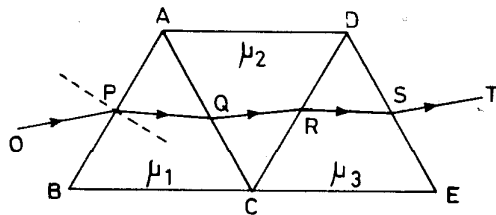
When the eye is under water,

$$\text{Apparent depth} = \mu_w \left(\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} \right) = \frac{4}{3} \left(\frac{5}{1.6} + \frac{10}{1.5} \right) = 13.05 \text{ cm}$$

Hence Position of spot = 15 - 13.05 = 1.95 cm

11. Three right-angled prisms of refractive indices μ_1 , μ_2 and μ_3 are fitted together as shown in the figure. If a ray passes through the prisms without

suffering any deviation, then find the relation between the three refractive indices of the prisms.



Solution

Let OPQRST be the ray through the prisms. According to the requirement of the problem, there should be no deviation and therefore OP and ST should be parallel. Let the angles of incidence and of refraction at P be i and r_1 . The angle of incidence at Q is $(90 - r_1)$. Let the angle of refraction at Q be r_2 . The angle of incidence at R is $(90 - r_2)$. Let the angle of refraction at R be r_3 . The angle of incidence at S is $(90 - r_3)$. Since the normal ray at S is parallel to AB and the emergent ray ST is parallel to OP, the angle of emergence = $(90 - i)$.

For refraction at P:

$$\sin i = \mu_1 \sin r_1$$

or

$$\sin^2 i = \mu_1^2 \sin^2 r_1 \quad (a)$$

For refraction at Q:

$$\mu_2 \sin r_2 = \mu_1 \sin (90 - r_1)$$

$$\therefore \mu_2^2 \sin^2 r_2 = \mu_1^2 \cos^2 r_1 \quad (b)$$

For refraction at R:

$$\mu_2 \sin (90 - r_2) = \mu_3 \sin r_3$$

$$\mu_2^2 \cos^2 r_2 = \mu_3^2 \sin^2 r_3 \quad (c)$$

For refraction at S:

$$\sin (90 - i) = \mu_3 \sin (90 - r_3)$$

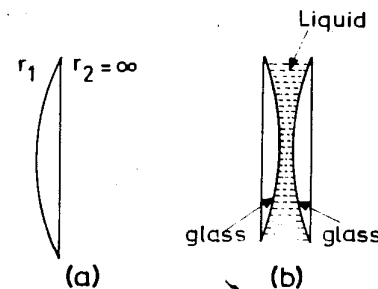
$$\cos^2 i = \mu_3^2 \cos^2 r_3 \quad (d)$$

On adding (a), (b), (c) and (d), we get

$$1 + \mu_2^2 = \mu_1^2 + \mu_3^2$$

This is the required condition for the ray passing through this combination of three prisms so as not to suffer any deviation.

12. A planoconvex lens has a focal length of 25 cm and is made of glass of refractive index 1.5. Find the radius of curvature of its curved surface. Now two such lenses are placed with their convex surfaces in contact and the space between them is filled with a liquid of refractive index 1.7. Find the focal length of this combination.



Solution

Here, $\mu = 1.5$, $r_1 = ?$, $r_2 = \infty$ [see Fig. (a)]

$$\begin{aligned} \therefore \frac{1}{25} &= (1.5 - 1) \left(\frac{1}{r_1} - \frac{1}{\infty} \right) \\ &= 0.5 \left(\frac{1}{r_1} \right) = \frac{0.5}{r_1} \Rightarrow r_1 = 0.5 \times 25 = 12.5 \text{ cm} \end{aligned}$$

For the combination [Fig. (b)]:

It consists of two planoconvex lenses of glass and one equiconcave lens of liquid of radii of curvature of 12.5 cm. Obviously, it is a diverging lens. Let its focal length be f .

$$\begin{aligned} \frac{1}{f} &= (1.7 - 1) \left(-\frac{1}{12.5} - \frac{1}{12.5} \right) = 0.7 \times \left(-\frac{2}{12.5} \right) \\ &= -0.112 \Rightarrow f = -8.93 \text{ cm} \end{aligned}$$

The focal length of the combination is given by the relation

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{25} + \frac{1}{25} + (-0.112)$$

$$\frac{1}{f} = \frac{2}{25} - 0.112 = 0.08 - 0.112 = -0.032$$

$$\text{or } f = -31.25 \text{ cm}$$

13. The refractive indices of crown and flint glasses for C, D and F lines are given below

Glass	C	D	F
Crown	1.527	1.530	1.535
Flint	1.790	1.795	1.806

Calculate the dispersive powers of the two glasses. If the angle of the crown glass prism is 9° , calculate the angle of the flint glass prism which combines with the crown glass prism to produce dispersion without deviation.

Solution

For crown glass:

$$\mu_C = 1.527, \mu_F = 1.535 \text{ and mean } \mu_D = 1.530$$

$$\omega_1 = \frac{1.535 - 1.527}{1.530 - 1} = \frac{0.008}{0.530} = 0.015$$

For flint glass:

$$\mu_C = 1.790, \mu_F = 1.806 \text{ and mean } \mu_D = 1.795$$

$$\omega_2 = \frac{1.806 - 1.790}{1.795 - 1} = 0.02$$

The condition for zero mean deviation is

$$(\mu - 1)A + (\mu' - 1)A' = 0$$

where μ and μ' are the mean refractive indices of the two glasses with respect to D lines.

$$A' = -\frac{(\mu - 1)}{(\mu' - 1)} A = -\frac{1.530 - 1}{1.795 - 1} \times 9^\circ = -6^\circ$$

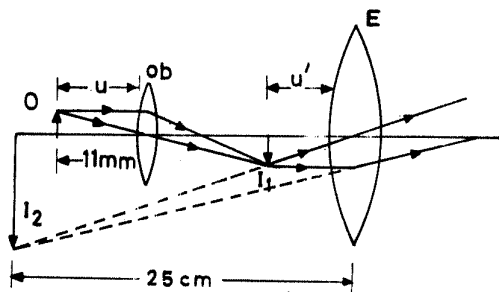
The negative sign here signifies that the prism is inverted with respect to the first prism.

14. Using the data given in Illustration 13, and given that the refracting angle of a crown glass prism is 9° , find the angular dispersion between C and F lines.

Solution

$$\begin{aligned} \text{Net dispersion} &= (\mu'_F - \mu'_C) A' - (\mu_F - \mu_C) A \\ &= (1.806 - 1.790) 6^\circ - (1.527 - 1.527) 9^\circ \\ &= 0.018^\circ \end{aligned}$$

15. The focal lengths of the objective and eyepiece of a compound microscope are 1 and 5 cm respectively. An object is placed 11 mm from the



objective and the final image is 25 cm from the eye. Find the magnification and the separation between the objective and the eyepiece.

Solution

Let the focal length of the objective be f_1 and that of the eyepiece be f_2 .

Image formation by objective:

$$u_1 = 1.1 \text{ cm}, f_1 = 1 \text{ cm}, v = ?$$

$$\frac{1}{1} = \frac{1}{1.1} + \frac{1}{v} \Rightarrow v = 11 \text{ cm}$$

It means that the image I_1 is real, inverted and is situated at a distance of 11 cm from the objective. Its magnification,

$$m_1 = \frac{v}{u} = \frac{11}{1.1} = 10$$

Image formation by eyepiece: The first image formed by the objective serves as an object (a virtual object) for the eyepiece. The final image I_2 is 25 cm from the eyepiece (the eye is very close to the lens).

$$\therefore v' = -25 \text{ cm}, f_2 = 5 \text{ cm}, u' = ?$$

$$\frac{1}{u'} - \frac{1}{25} = \frac{1}{5} \Rightarrow u' = \frac{25}{6} \text{ cm}$$

As I_1 , the object for the eyepiece, lies within the focal length of the eyepiece ($f_2 > u'$), a virtual, enlarged and erect image I_2 is formed.

Magnification produced by the eyepiece,

$$m_2 = \frac{25}{25/6} = 6$$

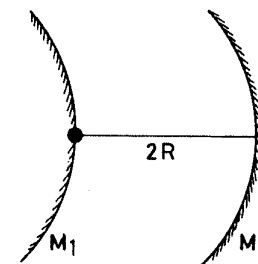
Therefore, total magnification of the object is

$$M = m_1 \times m_2 = 10 \times 6 = 60$$

Separation between objective and eyepiece = $v + u'$

$$= 11 \text{ cm} + \frac{25}{6} \text{ cm} = \frac{91}{6} \text{ cm} = 15.17 \text{ cm}$$

16. Two spherical mirrors, one convex and the other concave, each of same radius of curvature R are arranged coaxially at a distance of $2R$ from each other. A small circle of radius a is drawn on the convex mirror near the pole as shown in the figure. Find the radii of the first three images of the circle.



Solution

First Image: The first image is formed by reflection from the concave mirror (M_2).

$$\frac{1}{v_1} + \frac{1}{2R} = \frac{2}{R} \Rightarrow v_1 = \frac{2R}{3}$$

$$m_1 = \frac{2R/3}{2R} = \frac{1}{3}$$

$$\therefore \text{Radius of image circle} = \frac{a}{3}$$

Second Image: The second image is formed by the convex mirror (M_1). The image formed by the concave mirror serves as the object for the convex mirror

$$\therefore \text{Object distance} = 2R - \frac{2R}{3} = \frac{4R}{3}$$

$$\frac{1}{v_2} + \frac{3}{4R} = -\frac{2}{R} \Rightarrow v_2 = -\frac{4R}{11}$$

$$m_2 = \frac{4R/11}{4R/3} = \frac{3}{11}$$

$$\therefore \text{Radius of image circle} = \frac{a}{3} \times \frac{3}{11} = \frac{a}{11}$$

Third Image: The third image is formed again by the concave mirror (M_2).

$$\text{Object distance} = 2R - \left(-\frac{4R}{11}\right) = \frac{26R}{11}$$

$$\frac{1}{v_3} + \frac{11}{26R} = \frac{2}{R} \Rightarrow v_3 = \frac{26R}{41}$$

$$m_3 = \frac{26R/41}{26R/11} = \frac{11}{41}$$

$$\text{Radius of third image} = \frac{a}{11} \times \frac{11}{41} = \frac{a}{41}$$

17. A convex lens of refractive index 1.52 is placed in (i) water ($\mu_w = 1.33$) and (ii) in carbon disulphide ($\mu_c = 1.68$). Determine in each case whether the lens behaves as a converging lens or a diverging lens and find its focal length in each case. Take the focal length of lens in air as 10 cm.

Solution

$${}_a\mu_g = 1.52, {}_a\mu_w = 1.33 \text{ and } {}_a\mu_c = 1.68$$

$$\therefore {}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w} = \frac{1.52}{1.33} \quad \text{and} \quad {}_c\mu_g = \frac{{}_a\mu_g}{{}_a\mu_c} = \frac{1.52}{1.68}$$

Let f_a , f_c and f_w be the focal lengths of the lens in air, carbon disulphide and water respectively. We have

$$\frac{1}{f_a} = -({}_a\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (1)$$

$$\frac{1}{f_w} = -({}_w\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (2)$$

$$\frac{1}{f_c} = -({}_c\mu_g - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (3)$$

We have,

$$\frac{f_w}{f_a} = \frac{({}_a\mu_g - 1)}{({}_w\mu_g - 1)} = \frac{1.52 - 1}{\left(\frac{1.52}{1.33}\right) - 1}$$

and as $f_a = -10$ cm,

$$f_w = -36.4 \text{ cm}$$

Hence, the lens behaves as a convergent lens in this case. Similarly,

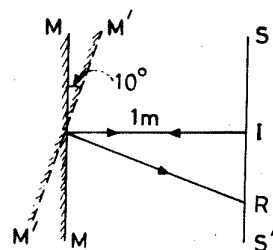
$$\frac{f_c}{f_a} = \frac{1.52 - 1}{\left(\frac{1.52}{1.68}\right) - 1} \Rightarrow f_c = 54.6 \text{ cm}$$

This shows that the lens in carbon disulphide behaves as a diverging lens.

EXERCISES

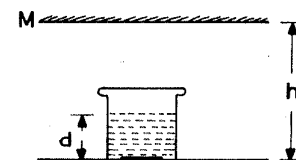
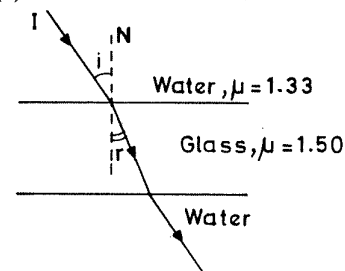
- The minimum size of a plane mirror hung on a wall of a room in which an observer standing at the centre can see the total height of the wall behind him is (take the height of the wall to be H)
(a) $H/2$ (b) $H/3$
(c) $H/4$ (d) H
- Two plane mirrors are inclined at an angle of 30° with each other. A luminous object is placed between the two mirrors. The number of images formed is
(a) three (b) five
(c) four (d) eleven
- A small girl of height 1 m can just see her image in a vertical plane mirror 4 m away from her. Her eyes are 0.92 m from the floor. In order that she sees her full image in the mirror, the shortest vertical dimension of the mirror is
(a) 0.50 m (b) 0.70 m
(c) 0.46 m (d) 0.36 m
- Images formed by an object placed between two plane mirrors whose reflecting surfaces make an angle of 90° with one another lie on a
(a) straight line (b) zig-zag curve
(c) circle (d) ellipse
- * In a lamp-and-scale arrangement to measure small deflection, the arrangement is shown in the figure. SS' is the glass scale placed at a distance of 1 m from the plane mirror MM and I is the position of the light spot formed after

reflection from the undeflected mirror MM. The mirror is deflected by 10° and comes to the deflected position $M'M'$. The distance moved by the spot on the scale (IR) is



- (a) 24.6 cm
(b) 36.4 cm
(c) 46.4 cm
(d) 34.6 cm
6. When an object lies at the focus of a concave mirror, the position of the image formed and its magnification are respectively
(a) infinity and zero
(b) centre of curvature and infinity
(c) infinity and infinity
(d) infinity and unity
7. A convex mirror has a radius of curvature of 20 cm. An object is placed at a distance of 6 cm from the mirror. The position of the image and its magnification respectively are
(a) -3.75 cm and 0.63
(b) 3.3 cm and 0.33
(c) 0.66 cm and 3.3
(d) 0.66 cm and 0.33
8. A convex mirror has a radius of curvature of 60 cm. The position of an object is so fixed that the size of the image is exactly half that of the object. The distance between the object and the mirror is
(a) 20 cm
(b) -20 cm
(c) 25 cm
(d) -30 cm
9. It is desired to see one's face five times as large by keeping a mirror in front of the face at a distance of 20 cm. The radius of the curvature of the mirror should be
(a) -17.0 cm
(b) -33.3 cm
(c) 17.0 cm
(d) 33.3 cm
10. An object is 3 cm high and is placed at a distance of 120 cm from a convex mirror made from a sphere of radius 60 cm. The size of the image is
(a) 1 cm
(b) 0.5 cm
(c) 0.2 cm
(d) 0.6 cm
11. In the headlights of automobiles, the reflectors employed are parabolic because
(a) this helps in providing a wide beam of light
(b) it increases the intensity of light
(c) it minimizes spherical aberration and provides a sharp image of the source
(d) it eliminates all colour effects in the beam of light.
12. An erect image four times magnified is formed when an object is placed in front of a converging mirror. If the radius of curvature of the mirror is 50 cm, the position of the object is
(a) 18.75 cm
(b) 36.37 cm
(c) 9.09 cm
(d) 3.75 cm
13. When an object is placed at a distance of 25 cm from a mirror, the magnification is m_1 . The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes m_2 . If $m_1/m_2 = 4$, the focal length of the mirror is
(a) 10 cm
(b) 30 cm
(c) 15 cm
(d) 20 cm

14. For a crown glass prism of 60° , the refractive index is 1.50. The critical value of the angle of incidence for transmission through the prism is
(a) $31^\circ 22'$
(b) $27^\circ 55'$
(c) $23^\circ 40'$
(d) $41^\circ 22'$
15. The refractive index of the material of a prism is 1.54 and the refracting angle is 60° . The angle of minimum deviation is
(a) $40^\circ 42'$
(b) $22^\circ 20'$
(c) $31^\circ 30'$
(d) $27^\circ 20'$
16. In the above problem, the deviation when the light is incident on one face of this prism at glancing incidence is
(a) $40^\circ 30'$
(b) $30^\circ 28'$
(c) $60^\circ 56'$
(d) $15^\circ 14'$
17. The velocity of light in air is 3×10^8 m/s. The velocity and wavelength of sodium light ($\lambda = 5893 \text{ \AA}$) in glass of refractive index 1.658 are respectively
(a) 3×10^8 m/s and 2500 $\text{\AA}$
(b) 1.8×10^8 m/s and 2500 $\text{\AA}$
(c) 1.8×10^8 m/s and 3000 $\text{\AA}$
(d) 1.8×10^8 m/s and 3554 $\text{\AA}$
18. The medium above a glass slab is water instead of air and an incident beam I makes an angle of 45° with the normal N. The angle of refraction (r) is
(a) 41°
(b) 37°
(c) 45°
(d) 38.5°
19. Suppose light is incident from below on the same boundary surface as in the preceding example at an angle of incidence of 38.5° . The angle of refraction is
(a) 45°
(b) 38.5°
(c) 41°
(d) 34°
20. The refractive index for the mean ray in a spectrum is 1.620 and a ray of light passes through a prism whose refracting angle is 10° and dispersive power is 0.031. The mean deviation and dispersion respectively are
(a) 6.20° and 0.192°
(b) 3.10° and 0.096°
(c) 4.65° and 1.44°
(d) 2.32° and 0.77°
21. A glass plate one inch thick, of refractive index 1.50, having plane parallel faces, is held with its faces horizontal and its lower end 4 inches above a printed page. If the rays make a small angle with the normal to the plate, the position of the image of the page with respect to the top face of the plate is
(a) 0.5 inches below it
(b) 4.67 inches below it
(c) 2.35 inches below it
(d) 3.0 inches below it
22. A plane mirror is held at a height h above the bottom of an empty beaker. The beaker is now filled with water up to depth d . The general expression for the distance from a scratch at the bottom of the beaker to its image in terms of h and the depth d of water in the beaker is



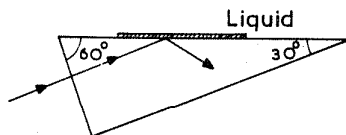
- (a) $2h - d \left(\frac{\mu}{\mu - 1} \right)$ (b) $2h - \frac{d}{2} \left(\frac{\mu - 1}{\mu} \right)$
 (c) $2h - d \left(\frac{\mu - 1}{\mu} \right)$ (d) $2h - d \left(\frac{2\mu - 1}{\mu} \right)$

23. A ray of light is incident on the left vertical face of a glass cube of index 1.5 as shown in the figure. The plane of incidence is in the plane of the paper, and the cube is surrounded by water. If total internal reflection is to occur at the top surface of the glass cube, the maximum angle at which the ray should be incident on the left vertical surface is



- (a) 17° (b) 11°
 (c) 37° (d) 32°

24. On the hypotenuse of a right prism ($30^\circ - 60^\circ - 90^\circ$), of refractive index 1.50, a drop of liquid is placed as shown in the figure. Light is allowed to fall normally on the short face of the prism. In order that the ray of light may get totally reflected, the maximum refractive index of the liquid should be



- (a) 1.30 (b) 1.47
 (c) 1.02 (d) 1.23

25. A projector makes an image of a slide on a screen 5 m from the lens. If a 5 cm dimension of the slide is magnified to 1 m, the focal length of the lens is
 (a) 0.42 m (b) 0.24 m
 (c) 4.2 m (d) 0.82 m

26. While an aquarium is being filled with water, a motionless fish looks up vertically through the surface of the water at a monochromatic plane-wave source of frequency ν . If the index of refraction of water is μ and water level rises at a rate of dh/dt , the shift in the frequency ($d\nu/\nu$) that the fish observes is [velocity of light is C]

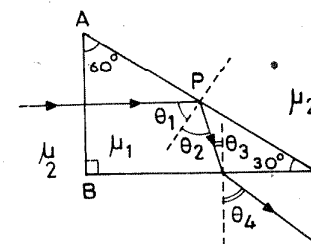
- (a) $\frac{(\mu - 1)}{C} \frac{dh}{dt}$ (b) $\frac{\mu}{C} \frac{dh}{dt}$
 (c) $\frac{C}{(\mu - 1)} \frac{dh}{dt}$ (d) $\frac{C}{\mu} \frac{dh}{dt}$

27. A glass paperweight in the form of a hemisphere is placed, flat surface down, on a page of a book. (Choose a letter of the print at centre of the flat base). If the refractive index of the glass is 1.50 and the radius of the hemisphere is R , the position of the image of the print and its magnification respectively are

- (a) along the vertical axis at the top of the hemisphere and magnification unity
 (b) along the horizontal plane in which the print was and magnification unity
 (c) along the horizontal plane in which the print was and magnification 1.50 times the size of the print

- (d) along the vertical axis at the top of the hemisphere and magnification 1.50 times the size of the print.
 28. A narrow beam of light passes through a plate of glass of thickness 4 mm and of refractive index 1.52. The beam enters from air at an angle of 30° with normal at the surface. The ray will get deviated from its initial path by
 (a) 0.40 mm (b) 0.53 mm
 (c) 0.25 mm (d) 0.92 mm

29. A $30^\circ - 60^\circ - 90^\circ$ flint glass prism is immersed in water and a ray of light is incident on the face AB as shown ($\mu_g = 1.655$ and $\mu_w = 1.333$). The angle of emergence θ_4 is



- (a) 19.2°
 (b) 30.4°
 (c) 38.4°
 (d) 24.2°

30. In the above problem, a substance is dissolved so as to increase the refractive index of water. The value of refractive index of this impure water when the total internal reflection ceases at point P is
 (a) 1.433 (b) 1.201
 (c) 1.613 (d) 1.113

- 31.* A rod 1.5 m long is placed along the principal axis of a convex mirror of focal length 0.6 m. The near end of the rod is 1 m from the pole of the convex mirror. The length of the image of the rod is
 (a) 15.00 cm (b) 14.44 cm
 (c) 6.24 cm (d) 10.88 cm

32. An object is placed 25 cm from the surface of a convex mirror and a plane mirror is set so that the images formed in the two mirrors lie adjacent to each other in the same plane. The plane mirror is then 20 cm from the object. The radius of curvature of the convex mirror is
 (a) -50 cm (b) -125 cm
 (c) -45 cm (d) -75 cm

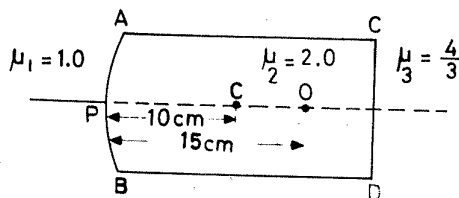
33. A concave mirror forms a real image of twice the linear dimensions of the object on the screen. The object and the screen are then moved until the image is three times the size of the object. If the shift of the screen is 25 cm, the shift of the object and the focal length of the mirror respectively are
 (a) 4.2 cm and 25.0 cm (b) 24.0 cm and 10.0 cm
 (c) 8.4 cm and 50.0 cm (d) 12.0 cm and 20.0 cm

34. A convex mirror of focal length f produces an image $1/n$ th of the size of the object. The distance of the object from the mirror is

- (a) $(1 - n) \frac{1}{f}$ (b) $(n - 1)f$
 (c) $(nf - 1)$ (d) $(n - 1) \frac{1}{f}$

35. A rectangular block of glass is placed on a printed page lying on a horizontal surface. The minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of block is
 (a) $\geq \sqrt{3}$ (b) $\geq \sqrt{2}$
 (c) ≥ 1.55 (d) ≥ 1.38

36. A thin rod of length $f/3$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated just touches the rod. The magnification is
 (a) 2 (b) 4
 (c) 2.5 (d) 1.5
- 37.* A point object is placed at a distance of 12 cm from a convex lens on the axis of focal length 10 cm, on the axis of the letter. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is
 (a) 20 cm (b) 25 cm
 (c) 15 cm (d) 30 cm
- 38.* A ray of light strikes a plane mirror M at an angle of 45° as shown in the figure. After reflection, the ray passes through a prism of index 1.50 whose apex angle is 4° . In order to have the total deviation of the ray equal to 90° , the angle through which the mirror should be rotated is
 (a) 1°
 (b) 4°
 (c) 30°
 (d) 3°
- 39.* A horizontal ray of light passes through a prism P of index 1.50 whose apex angle is 4° and then strikes a vertical mirror M as shown. For the ray after reflection to become horizontal, the mirror must be rotated through an angle of
 (a) 6° (b) 4°
 (c) 1° (d) 18°
- 40.* A glass slab of refractive index (μ_2) 2.0 has a curved surface APB of radius of curvature 10 cm and a plane surface CD. On the left of APB is air and on the right of CD is water ($\mu_3 = 4/3$). An object O is placed at a distance of 15 cm from the pole P as shown. The distance of the final image of O from P as viewed from the left will be

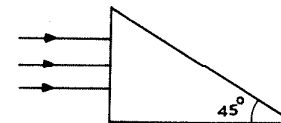


(Given that BD is 20 cm. Consider reflection and refraction both at each surface.)

- (a) - 30 cm or - 50 cm (b) 35 cm or 50 cm
 (c) - 35 cm or 50 cm (d) - 30 cm or 50 cm

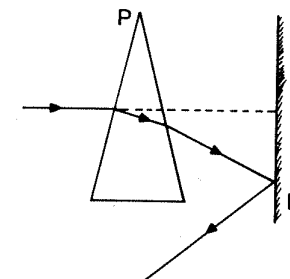
41. A thin prism P_1 with angle 4° made of glass of refractive index 1.54 is combined with another thin prism P_2 made of glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is
 (a) 3° (b) 2.6°
 (c) 4° (d) 5.33°

42. A beam of light consisting of red, green and blue colours is incident on a right angled prism as shown in the figure. The refractive indices of the material of prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively.



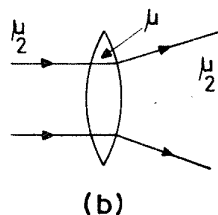
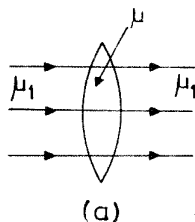
The prism will pass and

- (a) separate part of the red colour from the green and blue colours
 (b) separate part of the blue colour from the red and green colours
 (c) separate all the three colours from one another
 (d) not separate even partially any colour from the other two colours.
- 43.* An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eye piece is 36 cm and the final image is formed at infinity. The focal lengths of the objective (f_o) and that of the eye piece (f_e) respectively are
 (a) $f_o = 45$ cm and $f_e = 10$ cm (b) $f_o = 50$ cm and $f_e = 10$ cm
 (c) $f_o = 7.2$ cm and $f_e = 5$ cm (d) $f_o = 30$ cm and $f_e = 6$ cm
44. A prism having an apex angle of 4° and refractive index of 1.50 is located in front of a vertical plane mirror as shown. A horizontal ray of light is incident on the prism. The total angle through which the ray is deviated is
 (a) 4°
 (b) 178°
 (c) 2°
 (d) 8°



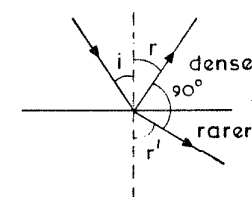
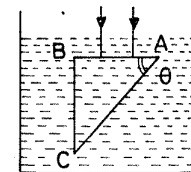
- 45.* A silicate crown prism of apex angle 15° is to be combined with a prism of silicate flint glass so as to result in no net deviation of light of wavelength 550 $m\mu$. The angle of the flint glass prism is ($\mu_F = 1.620$ and $\mu_C = 1.505$)
 (a) 8.40° (b) 4.60°
 (c) 12.20° (d) 6.40°
- 46.* Refer to problem 45. The angular dispersion of two rays of wavelengths 400 and 700 $m\mu$ is
 (Given: For $\lambda = 400$ $m\mu$, $\mu_C = 1.520$, $\mu_F = 1.66$
 and For $\lambda = 700$ $m\mu$, $\mu_C = 1.505$, $\mu_F = 1.61$)
 (a) 1.17° (b) 0.835°
 (c) 2.34° (d) 0.68°
47. A silicate crown prism of apex 15° is to be combined with a prism of silicate flint so as to be achromatized for rays of wavelengths 400 and 700 $m\mu$. The angle of the flint glass prism is (Take the values of refractive indices from the above problem)
 (a) 4.5° (b) 10°
 (c) 54° (d) 9°

48. A microscope is focussed on a small scratch on the top surface of a glass slide. When a cover slip with thickness $500\text{ }\mu\text{m}$ is placed on the slide, the microscope must be raised by $123\text{ }\mu\text{m}$ to bring the scratch into focus. The index of refraction of the cover slip is
- (a) 1.292 (b) 1.431
(c) 1.246 (d) 1.513
49. A cylindrical tank with an open top has a diameter of 3 m and is completely filled with water. When the setting sun reaches an angle of 28° above the horizon, sunlight ceases to illuminate the bottom of the tank. The depth of the tank is
- (a) 5.30 cm (b) 3.38 cm
(c) 2.36 cm (d) 4.20 cm
50. Light is incident normally on a 1 cm layer of water that lies on top of a flat plastic plate of thickness 0.5 cm. Light traverses through this double layer in t_1 s, and takes t_2 s to traverse an equal distance in air. The difference between t_1 and t_2 is ($\mu_{\text{plastic}} = 1.491$ and $\mu_{\text{water}} = 1.333$)
- (a) 1.93×10^{-10} s (b) 3.91×10^{-11} s
(c) 3.91×10^{-10} s (d) 1.93×10^{-11} s
51. The refractive index of the material of a prism of refracting angle 45° is 1.6 for a certain monochromatic ray. The minimum angle of incidence of a ray on the prism so that no total internal reflection takes place as the ray comes out of the prism is
- (a) $10^\circ 6'$ (b) $12^\circ 8'$
(c) $15^\circ 5'$ (d) $5^\circ 12'$
52. A ray of light is travelling from diamond to glass. If no light is to refract into glass, the minimum angle of incidence of the ray on the diamond-glass interface is ($\mu_D = 2.47$, $\mu_g = 1.51$)
- (a) $39^\circ 32'$ (b) $41^\circ 42'$
(c) $37^\circ 41'$ (d) $33^\circ 47'$
53. A glass lens has a focal length of 5 cm in air. Its focal length in water will be (refractive index of glass is 1.51 and that of water is 1.33)
- (a) 7.55 cm (b) 15.1 cm
(c) 12.6 cm (d) 18.2 cm
54. Two lenses of same material and focal length are placed in two different media as shown in Figs (a) and (b). In Fig. (a), the lens has a medium of refractive index μ_1 on both sides of it. In Fig. (b) the medium is of refractive index μ_2 on both sides of the lens. The refractive index of the material of the lens is μ in both the cases. The correct conclusion that can be drawn from these figures is

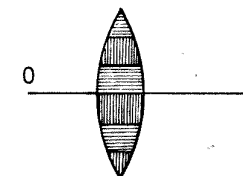


- (a) $\mu_1 < \mu$ but $\mu < \mu_2$ (b) $\mu_1 > \mu$ but $\mu < \mu_2$
(c) $\mu_1 = \mu$ but $\mu < \mu_2$ (d) $\mu_1 = \mu$ but $\mu_2 < \mu$

55. Mark the correct statement amongst the following: When a ray of light enters a glass slab from air,
- (a) its wavelength decreases
(b) its wavelength increases
(c) its frequency increases
(d) neither its frequency nor its wavelength changes.
56. A glass prism of refractive index 1.5 is immersed in water of refractive index $4/3$. A light beam incident normally on the face AB is totally reflected to reach the face BC if
- (a) $\sin \theta \leq 8/9$
(b) $2/3 < \sin \theta$
(c) $\sin \theta \leq 2/3$
(d) $\sin \theta \geq 8/9$
57. A ray of light from a denser medium strikes a rarer medium at an angle of incidence i as shown in the figure. The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r' . The critical angle is
- (a) $\sin^{-1}(\cos r)$
(b) $\sin^{-1}(\tan i)$
(c) $\sin^{-1}(\tan r')$ (d) $\sin^{-1}(\sin i)$



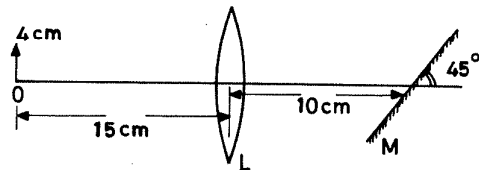
- 58.* Fill in the blanks:
- (a) A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is _____ and its wavelength is _____
- (b) A beam of white light passing through a hollow prism gives no _____
- (c) A layered lens as shown in the figure here is made of two different transparent materials. A point object O is placed on its axis. The number of images formed will be _____
- (d) A convex lens A of focal length 20 cm and a concave lens B of focal length 5 cm are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam, then d is equal to _____ cm.
- (e) A projector lens has a focal length of 10 cm. It throws an image of a $2\text{ cm} \times 2\text{ cm}$ slide on a screen 5 m away from the lens. The size of the picture on the screen is _____
- (f) An object is placed 20 cm to the left of a convex lens of focal length 10 cm. If a concave mirror of focal length 5 cm is placed 30 cm to the right of the lens, the magnification will be _____ and the nature of the image will be _____



332 Problems in Physics

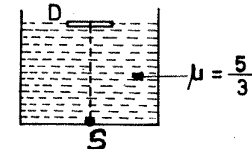
- (g) The focal length of a convex lens is measured using blue light. This blue light is replaced by monochromatic red light. The focal length of the lens will _____
- (h) The angle of a prism is 60° and its refractive index is 1.50. In order that there is no emergent ray at the second face of the prism, the angle of incidence on the first face should be equal to or greater than _____

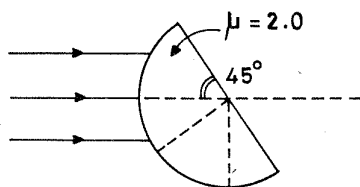
- 59.* An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. The radius of curvature of the convex mirror is
- (a) 50 cm (b) 12.5 cm
(c) 25 cm (d) 100 cm
- 60.* An object of height 4 cm is kept to the left end on the axis of a converging lens of focal length 10 cm as shown in the figure. A plane mirror M is placed inclined at 45° to the lens axis at a distance of 10 cm to the right of the lens. The location and size of the image formed by this combination are (take the centre of the lens as the origin and the lens axis as the axis of x)



- (a) (8 cm, 20 cm) to (20 cm, 20 cm), 4 cm
(b) (10 cm, 20 cm) to (4 cm, 20 cm), 8 cm
(c) (8 cm, 20 cm) to (4 cm, 20 cm), 4 cm
(d) (10 cm, 20 cm) to (2 cm, 20 cm), 8 cm.
- 61.* An object is placed at a distance of 10 cm to the left and on the axis of a convex lens A of focal length 20 cm. A second convex lens of focal length 10 cm is placed coaxially to the right of lens A at a distance of 5 cm from A. The position of the final image is
- (a) 16.67 cm right of B (b) 8.33 cm right of B
(c) 25.0 cm left of B (d) 12.5 cm left of B
- 62.* A vertical microscope is focussed on a point at the bottom of an empty tank. Water ($\mu = 4/3$) is then poured into the tank. The height of the water column is 4 cm. Another liquid which does not mix with water and which has a refractive index of $3/2$ is then poured over the water. The height of the liquid column is 2 cm. The vertical distance through which the microscope must be moved to bring the point into focus again is
- (a) $5/3$ cm (b) $4/3$ cm
(c) 1.5 cm (d) 3.5 cm
63. A convex lens of focal length 15 cm is placed in front of a convex mirror. Both are coaxial and the lens is 5 cm from the apex of the mirror. When an object is placed on the axis at a distance of 20 cm from the lens, it is found that the image coincides with the object. The radius of curvature of the mirror is

- (a) 60 cm (b) 55 cm
(c) 65 cm (d) 30 cm
- 64.* A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index $5/3$. A person is viewing the source from above the surface. There is an opaque disc D of radius 1 cm floating on the surface of the liquid. The centre of the disc lies vertically above the source S. The liquid from the vessel is gradually drained out through a tap. The maximum height of the liquid for which the source cannot be seen at all from above is
- (a) 1.50 cm (b) 1.64 cm
(c) 1.33 cm (d) 1.86 cm
65. A ray of light makes an angle of incidence of 60° at one of the faces of a prism and suffers a total deviation of 30° on emergence from the other face. If the angle of the prism is 30° , the refractive index of the prism is
- (a) 1.481 (b) 1.612
(c) 1.550 (d) 1.732
66. A six-foot tall man wants to see his full image in a plane mirror. The shortest length of the mirror should be
- (a) 3 feet (b) 2 feet
(c) 4 feet (d) 3.5 feet
- 67.** An optical system consists of a thin convex lens of focal length 30 cm and a plane mirror 15 cm behind the lens. An object is placed 15 cm in front of the lens. The distance of the final image from the lens is
- (a) 60 cm (b) 30 cm
(c) 35 cm (d) 40 cm
68. Two plane mirrors are inclined to each other at a certain angle. A ray of light first incident on one of them at an inclination of 10° with the mirror retraces its path after five reflections. The angle between the mirrors is
- (a) 12° (b) 22°
(c) 30° (d) 20°
69. The refractive angle of a prism is 60° and the index of refraction is $\sqrt{7/3}$. The limiting angle of incidence of a ray that will be transmitted through the prism is
- (a) 30° (b) 45°
(c) 15° (d) 50°
70. The refractive index of the material of a prism is 1.54. If the limiting angle of incidence of a ray that is transmitted through the prism is 30° , the angle of the prism is
- (a) 60° (b) $59^\circ 27'$
(c) 30° (d) 54°
71. The refractive index of the material of a prism ($A = 60^\circ$) is 1.54. The angular extent of the field of view through the prism is
- (a) $44^\circ 6'$ (b) $51^\circ 6'$
(c) $59^\circ 4'$ (d) $32^\circ 8'$
72. A parallel beam of light is incident on the surface of a transparent hemisphere of radius R and refractive index 2.0 as shown in the figure. The position of the image formed by refraction at the first surface is





- (a) $R/2$ (b) $3R$
(c) $3R$ (d) $2R$
73. A concave mirror is to form an image of the filament of a headlight lamp on a screen 4 m from the mirror. The filament is of height 5 mm and the image is to be 40 cm high. The focal length of the mirror should be
(a) 8.8 cm (b) 12.0 cm
(c) 4.94 cm (d) 6.32 cm
74. Light passes symmetrically through a 60° prism. After emergence, it is incident on a plane mirror fixed to the base of the prism extending beyond it. (μ of the prism = 1.54.) The final deviation produced is
(a) zero (b) $5^\circ 6'$
(c) $4^\circ 2'$ (d) $2^\circ 3'$
75. The relative refractive index of a material with respect to glass is 0.9 and the absolute refractive index of glass is 1.512. The refractive index of the material is
(a) 1.42 (b) 1.58
(c) 1.68 (d) 1.72
- 76.* Two mirrors, one concave and the other convex, are placed 60 cm apart with their reflecting surfaces facing each other. An object is placed 30 cm from the pole of either of them on their axis. If the focal lengths of both the mirrors are 15 cm, the position of the image formed by reflection, first at the convex and then at the concave mirror, is
(a) 19.09 cm from the pole of the concave mirror
(b) 19.09 cm from the pole of the convex mirror
(c) 11.09 cm from the pole of the concave mirror
(d) 11.09 cm from the pole of the convex mirror
77. The critical angle for a ray of light passing from a medium of refractive index $3/2$ into a medium of refractive index $4/3$ is
(a) $37^\circ 32'$ (b) $44^\circ 36'$
(c) $40^\circ 24'$ (d) $62^\circ 44'$
78. The indices of refraction of an equiangular prism for wavelengths λ_D and λ_C are $\mu_D = 1.620$ and $\mu_C = 1.613$. A ray of wavelength λ_D passes through the prism at minimum deviation. The angle of incidence is
(a) $54^\circ 6'$ (b) $39^\circ 4'$
(c) $46^\circ 6'$ (d) $50^\circ 8'$
79. In the above problem, if the angle of incidence is $54^\circ 6'$ for the ray corresponding to λ_C , the angle that the refracted ray inside the prism makes with the base of the prism is
(a) $27^\circ 3'$ (b) $48^\circ 12'$
(c) zero (d) $4^\circ 6'$
80. A glass prism of angle 60° and index of refraction 1.50 is immersed in a liquid of refractive index 1.40. The angle of minimum deviation for a parallel beam of light passing through the prism is

- (a) 22.4° (b) 4.78°
(c) 32.5° (d) 28.6°
81. Mark the correct statement among the following:
In order that there is no emergent ray in a prism,
(a) the angle of the prism should be equal to the angle of incidence
(b) the angle of the prism should be equal to half the angle of incidence
(c) the angle of the prism should be greater than twice the critical angle of the glass of which it is made
(d) the angle of the prism should be half of the critical angle of the glass of which it is made.
82. The maximum refractive index of a prism which permits the passage of light through it when the refracting angle of the prism is 90° , is
(a) 1.500 (b) 1.380
(c) 1.660 (d) 1.414
83. A ray of white light passes through a prism whose refracting angle is 10° at approximately minimum deviation. If $\mu_g = 1.513$ for red and $\mu_g = 1.532$ for violet, the angular dispersion produced by the prism is
(a) 0.19° (b) 3.0°
(c) 2.14° (d) 3.80°
84. If the radii of curvature of a double convex lens be 30 cm and 50 cm, its focal length in water will be ($\mu_g = 1.5$ and $\mu_w = 1.33$)
(a) 37.5 cm (b) 146 cm
(c) 75 cm (d) 100 cm
85. A concavo-convex lens is made of glass of refractive index 1.50. The radii of curvature of its two surfaces are 30 and 50 cm. Its focal length when placed in a liquid of refractive index 1.40 is
(a) 200 cm (b) 500 cm
(c) 1050 cm (d) 800 cm
- 86.* A compound microscope consists of an objective of focal length 1 cm and eye-piece of focal length 5 cm. The observer, whose distance of distinct vision at which the final image is formed is 25 cm from the eye, needs an overall magnification of 15. The distance between the objective and the eye-piece is
(a) 5.54 cm (b) 9.26 cm
(c) 3.54 cm (d) 7.67 cm
- 87.* The image of an object when placed at a distance of 20 cm from the pole of a concave mirror is formed at a distance of 30 cm. A transparent material slab with parallel sides of thickness 4 cm and of refractive index 1.60 is introduced between the object and the mirror perpendicular to the axis. The consequent shift in the position of the image is
(a) 4.15 cm (b) 6.20 cm
(c) 2.40 cm (d) 3.64 cm
- 88.* A slab of glass, of thickness 6 cm and refractive index 1.5, is placed in front of a concave mirror, the faces of the slab being perpendicular to the principal axis of the mirror. If the radius of curvature of the mirror is 40 cm and the reflected image coincides with the object, then the distance of the object from the mirror is
(a) 30 cm (b) 22 cm
(c) 42 cm (d) 28 cm
89. A ray of light traverses a prism of refractive index 1.6 and just undergoes total reflection at the second face. If the refracting angle of the prism is 60° , the angle of incidence at the first face is

- (a) $42^{\circ}6'$ (b) $35^{\circ}35'$
(c) $32^{\circ}40'$ (d) $47^{\circ}20'$
90. If m_1 and m_2 be the magnifications for two positions of the lens between the object and the screen, and d the distance between the two positions of the lens, the focal length of the lens is
- (a) $\left(\frac{m_1 - m_2}{d}\right)$ (b) $\frac{d}{(m_1 - m_2)}$
(c) $(m_1 - m_2)d$ (d) $\frac{m_1 m_2}{d^2}$
91. A small object is enclosed in a sphere of solid glass 10 cm in radius. It is situated at a distance of 2 cm from the centre and is viewed from the side to which it is the nearest. If the refractive index of glass is 1.5, the object will appear to be at (from the centre along the diameter)
- (a) 5.5 cm (b) 3.5 cm
(c) 3.0 cm (d) 2.73 cm
92. Refer to Problem 91. The object is now viewed from the farthest point on the sphere along the diameter, the apparent position of the object will be at (from the centre along the diameter)
- (a) 3.3 cm behind the object
(b) 3.3 cm in front of the object
(c) 6.6 cm behind the object
(d) 6.6 cm in front of the object
93. A convex lens forms a real image of a bright point object at a distance of 50 cm behind the lens. A concave lens is placed 10 cm behind the convex lens on the image side. On placing a plane mirror on the image side and facing the concave lens, it is observed that the image now coincides with the object. The focal length of the concave lens is
- (a) 20 cm (b) 40 cm
(c) 60 cm (d) 30 cm
94. A convex lens of focal length 30 cm has refractive index 1.5. When the lens is immersed in a liquid of refractive index 1.47, its focal length will be
- (a) 1.75 m (b) 0.80 m
(c) 2.85 m (d) 7.35 m
95. A convex lens of focal length 25 cm is placed 12 cm in front of a convex mirror of radius of curvature 20 cm. In order that the positions of an object pin and its image coincide, the pin must be placed
- (a) 114.3 cm in front of the lens
(b) 70.6 cm in front of the lens
(c) 40.8 cm in front of the lens
(d) 80.8 cm in front of the lens
96. A person cannot see objects nearer than 75 cm. The power of the lens he should use to see the objects at 25 cm should be
- (a) + 3.75 dioptres (b) - 3.75 dioptres
(c) + 2.67 dioptres (d) - 2.67 dioptres
97. The minimum value of the refractive index for a 45° prism which is used to turn a beam of light by total internal reflection through a right angle is
- (a) 1.54 (b) 1.49
(c) 1.41 (d) 1.62
- 98.* The dispersive powers of crown and flint glass prisms are 0.03 and 0.05 respectively. The difference in the refractive indices of blue and red colours is

0.014 for crown glass and 0.023 for flint glass. A deviation of 10° is produced without any dispersion. The angle of the crown glass prism and that of flint glass prism are respectively

- (a) 42.6° and 30.4° (b) 53.6° and 32.6°
(c) 36.4° and 48.2° (d) 10.4° and 18.2°
- 99.* A pin is placed 10 cm in front of a convex lens of focal length 20 cm and refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature of 25 cm. The position of the final image is
- (a) 8.5 cm (b) 10.5 cm
(c) 15.5 cm (d) 12.5 cm
- 100.* The focal length of the objective and the eye-piece of a microscope are 2 cm and 5 cm respectively and the distance between them is 30 cm. If the image seen by the eye is 25 cm from the eye-piece, the distance of the object from the objective is
- (a) 0.8 cm (b) 2.3 cm
(c) 0.4 cm (d) 1.2 cm

Answers

- | | | | | |
|---------|---------|---------|---------|----------|
| 1. (b) | 2. (d) | 3. (a) | 4. (c) | 5. (b) |
| 6. (c) | 7. (a) | 8. (c) | 9. (d) | 10. (d) |
| 11. (c) | 12. (a) | 13. (d) | 14. (b) | 15. (a) |
| 16. (c) | 17. (d) | 18. (d) | 19. (a) | 20. (a) |
| 21. (b) | 22. (c) | 23. (d) | 24. (a) | 25. (b) |
| 26. (a) | 27. (c) | 28. (d) | 29. (c) | 30. (a) |
| 31. (d) | 32. (d) | 33. (a) | 34. (b) | 35. (b) |
| 36. (d) | 37. (b) | 38. (a) | 39. (c) | 40. (d) |
| 41. (a) | 42. (a) | 43. (d) | 44. (b) | 45. (c) |
| 46. (b) | 47. (a) | 48. (c) | 49. (b) | 50. (d) |
| 51. (a) | 52. (c) | 53. (d) | 54. (c) | 55. (a) |
| 56. (d) | 57. (b) | 58. * | 59. (c) | 60. (d) |
| 61. (a) | 62. (a) | 63. (b) | 64. (c) | 65. (d) |
| 66. (b) | 67. (a) | 68. (d) | 69. (a) | 70. (b) |
| 71. (c) | 72. (d) | 73. (c) | 74. (a) | 75. (c) |
| 76. (a) | 77. (d) | 78. (a) | 79. (c) | 80. (b) |
| 81. (c) | 82. (d) | 83. (a) | 84. (b) | 85. (c) |
| 86. (d) | 87. (a) | 88. (c) | 89. (b) | 90. (b) |
| 91. (d) | 92. (a) | 93. (b) | 94. (d) | 95. (a) |
| 96. (c) | 97. (c) | 98. (b) | 99. (d) | 100. (b) |

Note: Problems marked with a star should take five to seven minutes each. All the rest should not take more than five minutes each.

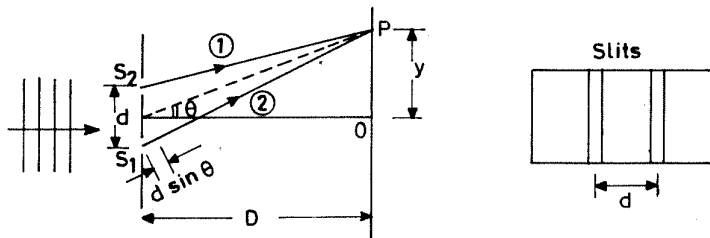
- *58. (a) 2×10^8 m/s and $4000 \text{ \AA}$ (b) Dispersion
(c) Two (d) 15 cm
(e) $48 \text{ cm} \times 48 \text{ cm}$ (f) real, erect, $m = 1$
(g) increase (f) 26.9°

17

Interference and Diffraction of Light

Double-Slit Interference

Young's double slit set-up for observing interference fringes is shown in the figure. Two slits have a separation d and the distance between the slits and the screen on which the interference pattern is observed is D . The path difference between ray 1 and 2 is $d \sin \theta$. When the path difference is one wavelength, two wavelengths or any integral number of wavelengths, the two waves will be in phase and will add together. They are said to interfere *constructively*. The condition may be expressed as



$$d \sin \theta = m\lambda, \quad \text{where } m = 0, 1, 2, \dots \text{ maxima}$$

If, however, the path difference is half a wavelength, one-and-a-half wavelengths, and so forth, the two waves will be exactly out of phase and will cancel. This results in a minimum in the pattern. The expression for the minima is

$$d \sin \theta = (m + 1/2)\lambda, \quad \text{where } m = 0, 1, 2, \dots \text{ minima}$$

Since angle θ may be very small,

$$\sin \theta = \frac{y}{D}$$

For a maxima:

$$y = m \frac{\lambda D}{d}, \quad m = 0, 1, 2, \dots$$

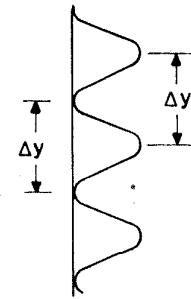
Positions for any two adjacent maxima are

$$y_m = m \frac{\lambda D}{d}$$

$$\text{and } y_{m+1} = (m + 1) \frac{\lambda D}{d}$$

Their separation

$$\Delta y = y_{m+1} - y_m = \frac{\lambda D}{d}$$



This is the separation between two adjacent bright fringes. The same expression holds good for two adjacent dark fringes.

Intensity in the Double Slit Interference Pattern

The amplitude of the light from both the slits is almost exactly the same. Because of the path difference, there is a phase difference. The resultant wave is given by

$$E = \underbrace{\left(2E_0 \cos \frac{\phi}{2} \right)}_1 \underbrace{\sin \left(\omega t + \frac{\phi}{2} \right)}_2$$

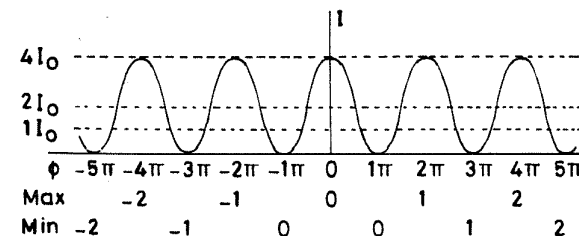
$$\begin{aligned} E_1 &= E_0 \sin(\omega t) \\ E_2 &= E_0 \sin(\omega t + \phi) \\ E &= E_1 + E_2 = E_0 [\sin \omega t + \sin(\omega t + \phi)] \\ &= 2E_0 \cos \frac{\phi}{2} \sin \left(\omega t + \frac{\phi}{2} \right) \end{aligned}$$

The resultant wave consists of the product of two terms. The first term is independent of time and the second term is time-dependent. The amplitude of the wave depends on the phase difference and hence on the position of the observation point. Let I_0 be the intensity corresponding to the amplitude E_0 and I corresponding to E at the point P defined by the angle θ as given in the figure for the double-slit set up, then

$$I = 4I_0 \cos^2 \frac{\phi}{2}$$

$$\text{and the phase difference } \phi = \frac{2\pi d \sin \theta}{\lambda}$$

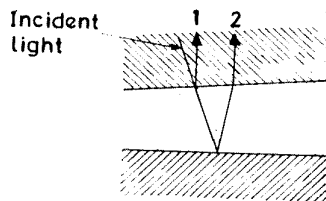
The intensity has maxima if $\phi = 2\pi m$ where m is an integer. The intensity pattern is shown in the figure here. The intensity due to one slit alone is I_0 , but the intensity due to both the slits is 4 times that for one slit alone. The average intensity is equal to twice that for one slit. The horizontal axis gives the order number for maxima and minima as well as the phase difference.



Thin Film Interference

This kind of interference occurs between the light reflected from the top of a thin film and that which is reflected from the bottom of the film. This film might typically be an air film between two glass plates, an oil film on the surface of water or a magnesium fluoride film on the surface of a lens.

When light is incident on an interface from the side with the lower refractive index, the part that is reflected undergoes a *phase reversal*. Hence, ray 2 in the given figure undergoes a phase shift and ray 1 does not.



Interference Fringes at Wedge-shaped Film

For normal incidence, the total path difference between the rays of light reflected at the upper and the lower surfaces of the film is

$$2\mu t + \frac{\lambda}{2}$$

If $2\mu t + (\lambda/2) = (2m \pm 1)(\lambda/2)$ or $2\mu t = m\lambda$ where $m = 0, 1, 2$, etc., the point P will appear dark, and if the path difference is $2\mu t + (\lambda/2) = 2m(\lambda/2)$ or $2\mu t = (2m - 1)\lambda/2$, then the point P will appear bright.

Separation between two consecutive dark or bright fringes is given by

$$w = \frac{\lambda}{2\mu\theta} \text{ where } \theta = t/x$$

Here t is the thickness of the air and x is the distance of the point P from the edge where the two plates touch each other (μ is the refractive index of the material of the film; here it is air and so $\mu = 1$).

Newton's Rings

When the convex surface of a lens of large radius of curvature is placed in contact with a plane glass surface and the incident light and direction of view are practically normal to the surface, a series of concentric rings can be seen surrounding the point of contact. If the incident light is monochromatic, many distinct rings are visible. These are called Newton's rings.

If D_n is the diameter of the n th bright ring, D_m that of the m th bright ring, and R the radius of curvature of the planoconvex lens used to form the air film, then the wavelength of light is given by

$$\lambda = \frac{D_n^2 - D_m^2}{4R(n - m)}$$

If the film is made of a transparent liquid of refractive index μ instead of air, then

$$\mu = \frac{(n - m)\lambda 4R}{(D_n^2 - D_m^2)}$$

When the wavelength of light used is not known, then we first find the diameter of the n th dark ring with an air film and then with the liquid whose refractive index is to be determined. Thus $(D_n)_{\text{air}}$ and $(D_n)_{\text{liquid}}$ are known.

$$\mu_{\text{liquid}} = \left(\frac{D_{\text{air}}}{D_{\text{liquid}}} \right)^2$$

The diameter of a bright ring is proportional to $\sqrt{2n + 1}$

$$\text{or } D_n \propto \sqrt{2n + 1}$$

(The diameter is proportional to square root of odd natural numbers.)

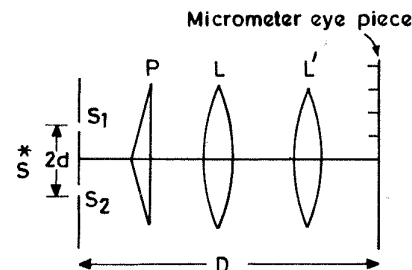
The diameter of a dark ring is proportional to $\sqrt{n}$

$$\text{or } D_n \propto \sqrt{n}$$

(The diameter is proportional to the square root of a natural number.)

Fresnel's Biprism Experiment

S_1 and S_2 are two virtual sources produced by the biprism and these are separated by a distance $2d$. The fringe width, i.e. the separation between two consecutive bright fringes, is given by W and is equal to



$$W = \frac{\lambda D}{2d}$$

where λ is the wavelength employed, D is the distance between the virtual source positions and the micrometer eye-piece.

Determination of $2d$

The separation between the two virtual sources is measured, using a convex lens whose focal length is less than $D/4$. The lens is placed between the micrometer eye-piece and the biprism as shown in the above figure, without disturbing their positions. The separation between the virtual images is measured in position L of the lens. Let this be d_1 . Similarly, in the second position of the lens L' the separation is measured as d_2 . The value of $2d = \sqrt{d_1 d_2}$.

If fringe width is measured, one gets the value of the wavelength of light

$$\lambda = \frac{2d \cdot W}{D}$$

Diffraction of Light

This refers to the bending or spreading of waves around the edges of apertures and opaque obstacles. The deviation of light from a straight-line path gives rise to interference patterns that blur the edges of shadows. It also places a limitation on the size of the details that can be observed and limits the precision of even the best measurements.

Single-slit Diffraction

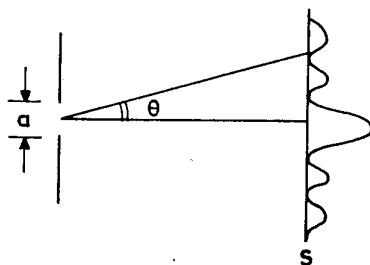
When parallel rays of light of wavelength λ are incident normally upon a slit of width a , a diffraction pattern is observed beyond the slit. Complete darkness is observed at angle θ_n to the straight-through beam where

$$a \sin \theta_n = n\lambda$$

here $n = 1, 2, 3, \dots$ is the order number of the diffraction dark band.

Limit of Resolution of Two Objects

If two objects are viewed through an optical instrument, the diffraction patterns caused by the aperture of the instrument limit our ability to distinguish the objects from each other. For two objects to be distinguished from each other, the angle θ subtended at the aperture by the objects must be larger than a critical value θ_c given by



$$\sin \theta_c = (1.22) \cdot \frac{\lambda}{D}$$

where D is the diameter of the aperture. As a rough rule of thumb, it is impossible to observe detail that is smaller than the wavelength of the radiation used for the observation.

Diffraction Grating

A diffraction grating consists of equally spaced, parallel slits; the distance between slits is the grating spacing d . When light of wavelength λ is incident normally upon a grating with spacing d , brightness is observed beyond the grating at an angle θ_n to the normal and the relation between λ , d and θ_n is

$$d \sin \theta_n = n\lambda$$

where $n = 1, 2, 3, \dots$ is called the order of the diffracted image.

This relation is applicable to the major maxima in the interference patterns of even two or three slits. The only difference that is observed in these cases is that the maxima are not so sharply defined as for a grating consisting of a very large number of slits.

Grating Element

If a is the width of the slit and b is the width of the opaque portion between two adjacent slits, then the sum $(a + b)$ is the distance between the corresponding points of adjacent slits. This is known as the grating element. The grating equation can be written as

$$(a + b) \sin \theta_n = n\lambda$$

$$a + b = d \quad \begin{matrix} a \rightarrow \text{slit} \\ b \rightarrow \text{opaque} \end{matrix}$$

Dispersive Power of a Grating

$$\frac{d\theta}{d\lambda} = \frac{n}{(a + b) \cos \theta}$$

here n is the order of the spectrum and $d\theta/d\lambda$ is the change of angle of diffraction corresponding to a change of wavelength from λ to $\lambda + d\lambda$. For normal incidence, $\theta = 0$ and dispersive power is $n/(a + b)$ and the dispersion is minimum.

Resolving Power of a Grating

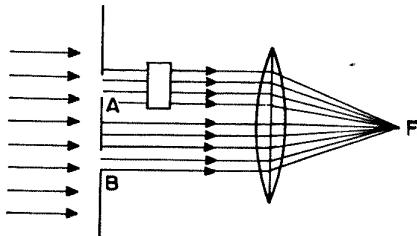
$$\frac{\lambda}{d\lambda} = nN$$

where n is the order of the spectrum and N is the number of lines on the grating.

ILLUSTRATIONS

1. In a modified Young's double slit experiment, a monochromatic, uniform and parallel beam of light of wavelength $6000 \text{ \AA}$ and intensity $(10/\pi) \text{ W/m}^2$

is incident normally on two circular apertures A and B of radii 0.001 m and 0.002 m respectively. A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for wavelength 6000 Å is placed in front of aperture A as shown in the figure. Calculate the power (in watts) received at the focal spot of the lens, F. The lens is symmetrically placed with respect to the apertures. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.



Solution

Data given:

$$\lambda = 6000 \text{ Å}, I = (10/\pi) \text{ W/m}^2$$

$$r_A = 0.001 \text{ m}, r_B = 0.002 \text{ m}, t = 2000 \text{ Å}$$

$$\mu = 1.5, \text{ transparent for } \lambda = 6000 \text{ Å}$$

10% power goes through the apertures.

$$\begin{aligned} \text{Path difference between two beams at } F &= (\mu - 1)t \\ &= 0.5 \times 2000 \text{ Å} \\ &= 10^{-7} \text{ m} \end{aligned}$$

∴ Phase difference between the two beams reaching F

$$= \frac{2\pi}{\lambda} \times 10^{-7} = \frac{2\pi \times 10^{-7}}{6000 \times 10^{-10}} = \frac{\pi}{3} \text{ radians}$$

$$\text{Power} = \text{intensity} \times \text{area} = (10/\pi)\pi r_A^2 = 10(0.001)^2 = 10^{-5} \text{ W}$$

$$\text{Power transmitted is 10\%} = 10^{-6} \text{ W}$$

Similarly,

$$\begin{aligned} \text{Power incident on slit B} &= \frac{10}{\pi} \times \pi (0.002)^2 \\ &= 4 \times 10^{-5} \text{ W} \end{aligned}$$

$$\text{Power transmitted is 10\%} = 4 \times 10^{-6} \text{ W}$$

$$\text{Intensity, } I \propto (\text{amp})^2 \text{ or amp.} \propto \sqrt{\text{power}} \propto \sqrt{\text{intensity}}$$

Therefore, resultant amplitude A_R at F is

$$\sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \frac{\pi}{3}}$$

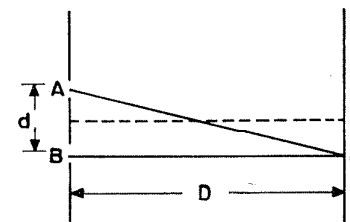
or

$$\begin{aligned} \text{Resultant Power, } P_R &= P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \frac{\pi}{3} \\ &= 10^{-6} + 4 \times 10^{-6} + 2 \times 2 \times 10^{-6} \times 1/2 \\ &= 7 \times 10^{-6} \text{ W} \end{aligned}$$

2. In a Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

Solution

The geometric path difference for two waves coming from A and B at C is xd/D . When a mica sheet is introduced in the path of one of the interfering rays, the optical path difference at C is



$$\frac{xd}{D} - (\mu - 1)t$$

where t is the thickness of the mica sheet and μ is its refractive index.

$$\text{Therefore, } x = (\mu - 1)t \cdot \frac{D}{d} \quad (i)$$

When the distance is doubled, the fringe width is

$$W = \frac{2D}{d} \lambda \quad (ii)$$

But $x = W$. Therefore,

$$\begin{aligned} \frac{2D}{d} \lambda &= (\mu - 1)t \cdot \frac{D}{d} \Rightarrow \lambda = (\mu - 1) \frac{t}{2} \\ &= (0.6) \times 1.964 \times 10^{-6} \times \frac{1}{2} \\ &= 5892 \times 10^{-10} \text{ m} \end{aligned}$$

3. A beam of light consisting of two wavelengths 6500 Å and 5200 Å, is used to obtain interference fringes in a Young's double slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maxima for $\lambda = 6500 \text{ Å}$.

(b) What is the least distance from the central maxima where the bright fringes due to both the wavelengths coincide. The distance between the slits is 2 mm and that between the plane of the slits and the screen is 120 cm.

Solution

(a) The distance of the m th fringe from the central maxima is given by

$$y_m = m \frac{\lambda D}{d}$$

$$\therefore y_3 = \frac{3 \times 6.5 \times 10^{-5} \times 120}{0.2} = 0.117 \text{ cm}$$

$$(b) \quad y = \frac{m \lambda_m D}{d} = \frac{n \lambda_n D}{d}$$

$$\text{or} \quad \frac{\lambda_m}{\lambda_n} = \frac{n}{m} = \frac{6500}{5200} = \frac{5}{4}$$

$$y_{m=4} = y_{n=5} = \frac{4 \times 6.5 \times 10^{-5} \times 120}{0.2} \\ = 0.156 \text{ cm}$$

4. A plane transmission grating when set for normal incidence in front of a mercury source of light gives an angular deviation of 13.5° for violet and 25.25° for red in the first order spectrum. If the wavelength of violet colour is $4000 \text{ \AA}$, find the grating element of the grating and the wavelength of red colour.

Solution

The angles of deviation for which the maxima occur are given by

$$\sin \theta = m \cdot \frac{\lambda}{d} \quad (m = 1, 2, \dots)$$

$$\sin \theta_V = m \cdot \frac{\lambda_V}{d} \quad (i)$$

(Here, the order of the spectrum is 1.)

$$\sin \theta_R = m \cdot \frac{\lambda_R}{d} \quad (ii)$$

From (i),

$$\sin 13.5 = \frac{4000 \times 10^{-10}}{d}$$

$$\text{or} \quad d = \frac{4000 \times 10^{-10}}{0.233} = 1.7 \times 10^{-6} \text{ m}$$

$$\sin 25.25 = 1 \times \frac{\lambda_R}{1.7 \times 10^{-6}}$$

$$\text{or} \quad \lambda_R = 0.43 \times 1.7 \times 10^{-6} = 0.7310 \times 10^{-6} = 7310 \text{ \AA}$$

5. A wedge is formed using two glass plates of length 8 cm each by putting one end in contact and separating the other end by placing a spacer of thickness 0.03 mm . A light of wavelength of $5000 \text{ \AA}$ is employed to observe interference fringes.

(a) Find the spacing between two adjacent dark fringes when the wedge contains air.

(b) Is the fringe adjacent to the line of contact bright or dark?

(c) Now air is replaced by water of refractive index 1.40 . What is the new spacing between two consecutive dark fringes?

(d) If the top plate is of glass ($\mu_g = 1.40$) and the wedge is filled with a silicon grease ($\mu_s = 1.5$) and the bottom plate has a refractive index of 1.6 , what is the fringe separation now?

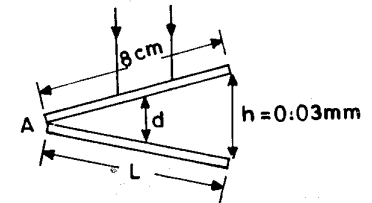
Solution

$$(a) \quad 2d = m\lambda \quad (1)$$

$$\text{and} \quad \frac{d}{x} = \frac{h}{L} \quad (2)$$

(where x is the separation between two consecutive fringes.) Combining (1) and (2),

$$\frac{2xh}{L} = m\lambda \text{ or } x = m \cdot \frac{L\lambda}{2h} = \frac{8 \text{ cm} \times 5000 \times 10^{-8} \text{ cm}}{2 \times 0.003 \text{ cm}} \\ = \frac{8 \times 5 \times 10^{-2} \text{ cm}}{2 \times 3} \\ = 6.6 \times 10^{-2} \text{ cm} \\ = 0.66 \text{ mm}$$



(Here m is of no relevance as we take successive fringes.)

(b) The fringe at the line of contact is dark because the wave reflected from the lower surface of the air wedge undergoes a phase shift of half a cycle while that from the upper surface does not. For this reason, the condition of destructive interference (a dark fringe) is that the path difference ($2d$) be an integral multiple of the wavelength.

(c) When air is replaced by water, the phase changes are the same but the wavelength λ changes to

$$\frac{\lambda_0}{1.4} = \frac{5000}{1.4} \times 10^{-8} \text{ cm} \\ = 3571 \times 10^{-8} \text{ cm}$$

Therefore, the fringe spacing becomes

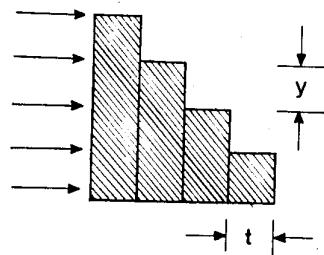
$$x' = \frac{L \cdot \lambda_0 / 1.40}{2 \times 0.003} = \frac{0.66}{1.40} \text{ mm} = 0.47 \text{ mm}$$

(d) In this case, there are half-cycle phase shifts at both the surfaces bounding the wedge and the line of contact corresponds to a *bright* fringe, not a dark one. The fringe spacing is obtained in the same way, using $\lambda = \lambda_0/1.5$, i.e.

$$\lambda = \frac{5000 \times 10^{-8}}{1.5} = 3333 \text{ \AA}$$

$$x'' = \frac{0.66}{1.5} \text{ mm} = 0.44 \text{ mm}$$

6. A stack of N glass plates of refractive index μ and thickness t are assembled with each plate projecting a distance y beyond the one following it. The stack is illuminated from the left by a monochromatic light of wavelength λ . Considering the light coming from each step in the assembly,



(a) What is the condition for constructive interference beyond the stack for small angles with respect to the incident beam?

(b) What is the order of interference for $\mu = 1.5$, $t = 0.5 \text{ cm}$ and $\lambda = 5000 \text{ \AA}$?

(c) Calculate the angular dispersion and resolving power for a stack of 50 such plates. (Assume that the refractive index varies very slightly.)

Solution

(a) Let δ be the phase change between the light emerging from each set of two consecutive steps. We find that this phase change

$$\delta = \frac{2\pi[\mu - \cos \theta)t + y \sin \theta]}{\lambda}$$

and for small angles,

$$\delta = \frac{2\pi[(\mu - 1)t + y\theta]}{\lambda}$$

(θ is the angle of the diffracted ray with respect to the incident ray.)

Using the expression for the intensity of an N -slit grating here, we have

$$I = \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N \delta/2}{\sin^2 \delta/2}$$

where $\beta = \frac{\pi}{\lambda} \cdot y \sin \theta = \frac{\pi}{\lambda} y \theta$

The factor $\frac{\sin^2 \beta}{\beta^2}$ is due to the single-slit diffraction while the factor $\sin^2 N(\delta/2)/\sin^2 (\delta/2)$ represents the interference terms for N slits.

When $\delta/2 = 0, \pi, 2\pi$, we have constructive interferences, i.e.

$$\pi \left[\frac{(\mu - 1)t + y\theta}{\lambda} \right] = m\pi$$

$m = 0, 1, 2, \dots$ This is the required condition.

(b) Now if we substitute the given values in the condition obtained above, we have

$$\pi \left[\frac{(1.5 - 1)0.5 + 0}{5000 \times 10^{-8}} \right] = m\pi$$

$$\text{or } 5000 \times 10^{-8} m = 0.25 \Rightarrow m = \frac{0.25}{5000 \times 10^{-8}} = \frac{0.25}{5 \times 10^{-5}} = 0.05 \times 10^5 = 5 \times 10^3$$

$$(c) \pi \left[\frac{(\mu - 1)t + y\theta}{\lambda} \right] = m\pi$$

$$\text{or } \theta = \frac{1}{y} [m\lambda - (\mu - 1)t]$$

$$\frac{d\theta}{d\lambda} = \frac{1}{y} \left[m - t \frac{d\mu}{d\lambda} \right]$$

(Since it is given that there is very slight variation of refractive index, it can be assumed that $d\mu/d\lambda = 0$)

$$\therefore \frac{d\theta}{d\lambda} = \frac{m}{y}$$

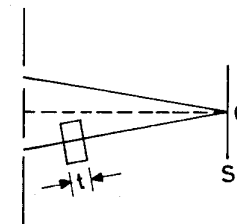
This is the angular dispersion.

$$\text{Resolving power} = \frac{\lambda_1}{\lambda_1 - \lambda_2} = \frac{\lambda}{d\lambda} = mN$$

Here m is the order and N is the total number of slits and the number of slits here means number of plates as each plate provides a slit.

$$\therefore \frac{\lambda}{d\lambda} = 5 \times 10^3 \times 50 = 25 \times 10^4$$

7. The figure shows Young's double slit experimental set-up. A light of wavelength λ from a distant source is incident upon two slits each of width W ($W \ll \lambda$) and the interference pattern is viewed on a distant screen S . Now a thin piece of glass of thickness t and refractive index μ is placed between one of the slits and the screen



and the intensity of the central point C is measured as a function of thickness t . If the intensity for $t = 0$ is given by I_0 , find the following:

- (a) intensity at C as a function of thickness t .
 (b) the value of t for which the intensity at C is a minimum.
 (c) suppose the width of one of the slits is now increased to $2W$, the other width remaining unchanged. What is the intensity at point C as a function of t ? Assume that the glass is 100% transparent.

Solution

- (a) The phase difference between the rays from the two slits at point C is

$$\phi = \frac{2\pi\mu t}{\lambda} - \frac{2\pi t}{\lambda} = (\mu - 1) \frac{2\pi t}{\lambda}$$

The intensity at point C is proportional to $\cos^2 \phi/2$ or,

$$I_C = I_0 \cos^2 \left[(\mu - 1) \frac{\pi t}{\lambda} \right]$$

- (b) For I_C to be minimum,

$$\frac{dI_C}{dt} = 0 \text{ or } \left\{ \sin 2(\mu - 1) \frac{\pi t}{\lambda} \right\} (\mu - 1) \frac{\pi}{\lambda} = 0$$

$$\text{or } 2(\mu - 1) \frac{\pi t}{\lambda} = \pi \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$$

- (c) The amplitude at point C is $A_C = 2Ae^{i\omega t} + Ae^{i(\omega t + \phi)}$
 $= Ae^{i\omega t} (2 + e^{i\phi})$

$$I_C \propto |A_C|^2 \propto 4 + 1 + 4R_e [e^{i\phi}] \quad (R_e \text{ here means real part})$$

$$\propto 5 + 4 \cos \phi$$

$$\propto 5 + 4 \cos \left[(\mu - 1) \frac{2\pi t}{\lambda} \right]$$

8. In a Newton's rings experiment, the diameters of the 4th and 12th dark rings are 0.400 and 0.700 cm respectively. Find the diameter of the 24th dark ring.

Solution

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

Here $(n + p) = 12$ and $n = 4$

$$\therefore p = 12 - 4 = 8$$

$$D_{n+p} = 0.700 \text{ cm} \quad \text{and} \quad D_n = 0.400 \text{ cm}$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda R \quad (1)$$

For the 24th and 4th rings,

$$D_{24}^2 - D_4^2 = 4 \times 20 \times \lambda R \quad (2)$$

(here $p = 24 - 4 = 20$)

Dividing (2) by (1), we get,

$$\frac{D_{24}^2 - D_4^2}{D_{12}^2 - D_4^2} = \frac{4 \times 20 \times \lambda R}{4 \times 8 \times \lambda R} = \frac{5}{2}$$

$$\frac{D_{24}^2 - (0.400)^2}{(0.700)^2 - (0.400)^2} = \frac{5}{2} \Rightarrow D_{24} = 0.992 \text{ cm}$$

9. A wedge of angle 0.5° is illuminated with sodium light whose two lines correspond to the wavelengths 5890 and 5896 Å. Find the distance from the apex at which the maxima due to the two wavelengths first coincide when observed in the reflected light. (The wedge contains air.)

Solution

Let the thickness of the wedge at the point where the maximum of 5896 Å coincides with the maximum of 5890 Å be t . For constructive interference, the condition is

$$2t = (2n + 1) \frac{\lambda_1}{2} \quad (i) \quad (\mu = 1)$$

$$\text{and } 2t = (2n + 3) \frac{\lambda_2}{2} \quad (ii)$$

Thus

$$(2n + 1) \left(\frac{5896}{2} \right) = (2n + 3) \left(\frac{5890}{2} \right)$$

$$\therefore t = 0.029 \text{ cm}$$

Let x cm be the distance from the apex where the maxima of the two wavelengths coincide. If θ be the angle of the wedge, then

$$t = x \tan \theta = x\theta \quad (\text{where } \theta \text{ is small})$$

$$x = \frac{t}{\theta}$$

$$\text{Here } \theta = 0.5^\circ = \frac{\pi}{180} \times 0.5 \text{ radian}$$

$$\therefore x = \frac{0.029}{(\pi/180) \times 0.5} = \frac{0.029 \times 180}{3.14 \times 0.5} = 3.32 \text{ cm}$$

10. In a Young's double slit experiment, fringes are produced using light of wavelength 5890 Å. One slit is covered by a thin glass plate of refractive index 1.4 and the other slit by a glass plate of the same thickness but of refractive index 1.7. On doing so, the central bright fringe shifts to the position originally occupied by the fifth bright fringe from the centre. Find the thickness of the glass plate.

Solution

We know that the shift of a fringe is given by

$$y = \frac{W}{\lambda} (\mu - 1)t$$

where W is the fringe width, μ is the refractive index of the glass plate and t is its thickness. If another plate of the same thickness of refractive index μ' is placed in the path of the second interfering beam, then the net shift is given by

$$y' = \frac{W}{\lambda} (\mu - \mu')t \Rightarrow t = \frac{\lambda y'}{W(\mu - \mu')}$$

Here, $y' = 5W$, $\lambda = 5890 \text{ \AA}$, $\mu - \mu' = 0.3$

$$t = \frac{5W \times 5890 \times 10^{-8}}{W \times 0.3} = \frac{5 \times 5890}{0.3} \times 10^{-8} \text{ cm} \\ = 9.8 \times 10^{-4} \text{ cm}$$

11. In a biprism experiment, the micrometer reading for zero order and n th order fringes are x_0 and x_n respectively when the wavelength of light used is λ_1 . What will be the position of zero order and n th order when the light source is replaced by one which has wavelength λ_2 ?

Solution

Since the zero order fringe corresponds to zero path difference between the interfering waves, its position remains unchanged with change in wavelength. Therefore, the position of the zero order fringe will be x_0 even when the wavelength is changed to λ_2 .

But since the fringe width W is a function of λ , the position of other fringes must change with change in λ .

For λ_1 , the distance between zero order fringe and n th order fringe is $(x_n - x_0) = [W_n]_{\lambda_1}$

For n fringes,

$$\text{Fringe width} = n \times W_n$$

When λ_1 is changed to λ_2 , let the fringe width be W_2 and we know that

$$[W_n]_{\lambda_2} = \frac{D\lambda_2}{2d} \quad \text{and} \quad [W_n]_{\lambda_1} = \frac{D\lambda_1}{2d}$$

$$\frac{[W_n]_{\lambda_2}}{[W_n]_{\lambda_1}} = \frac{\lambda_2}{\lambda_1}$$

Therefore,

$$[W_n]_{\lambda_2} = [W_n]_{\lambda_1} \frac{\lambda_2}{\lambda_1}$$

and for n fringes, the fringe width is

$$n[W_n]_{\lambda_2} = n[W_n]_{\lambda_1} \times \frac{\lambda_2}{\lambda_1}$$

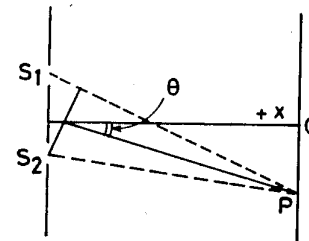
This is the distance between the zero order and the n th order fringes for λ_2 .

Since the micrometer reading for the zero order fringe remains the same (x_0), the micrometer reading for the n th order fringe becomes

$$\left[x_0 + n[W_n]_{\lambda_1} \times \frac{\lambda_2}{\lambda_1} \right] = \left[x_0 + n(x_n - x_0) \times \frac{\lambda_2}{\lambda_1} \right]$$

EXERCISES

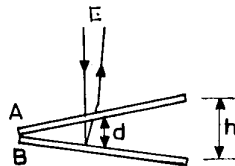
- In an experiment with a biprism, the width of 25 bands was 5 mm. The distance between the eye-piece and the slit was 1 m. When a convex lens was mounted on the axis at a distance of 20 cm from the slit, well-defined images of the virtual sources having a separation of 1.20 cm between them were observed in the eye-piece. The wavelength of light used in the experiment was
(a) $3 \times 10^{-5} \text{ cm}$ (b) $8 \times 10^{-5} \text{ cm}$
(c) $6 \times 10^{-5} \text{ cm}$ (d) $1 \times 10^{-5} \text{ cm}$
- Interference fringes are produced having a spacing of 2.2 mm between them. The two parallel slits have a separation of 0.2 mm and the screen is 75 cm away from the slits. The wavelength of light employed is
(a) 4440 Å (b) 7500 Å
(c) 6000 Å (d) 5869 Å
- A thin piece of glass of refractive index 1.50 is introduced in one of the interfering beams of the double slit experiment and it is observed that the central bright fringe is shifted to the position previously occupied by the 7th bright fringe. If the wavelength of light employed is 5890 Å, the thickness of the glass piece is
(a) $8.254 \times 10^{-4} \text{ cm}$ (b) $6.320 \times 10^{-4} \text{ cm}$
(c) $4.134 \times 10^{-4} \text{ cm}$ (d) $2.416 \times 10^{-4} \text{ cm}$
- With two slits spaced 0.2 mm apart and a screen at a distance of 1 m, the third bright fringe is found to be displaced 7.5 mm from the central fringe. The wavelength of light used is
(a) 5000 Å (b) 4000 Å
(c) 5500 Å (d) 4890 Å
- In a double slit set-up for the study of interference fringes, the two slits have two sources of identical radio antennas instead of light sources placed 10 m apart and these radiate waves of frequency of 30 MHz in all directions. If the intensity in the $+x$ direction (corresponding to $\theta = 0$), $I_0 = 0.02 \text{ W/m}^2$, then the intensity in the direction $\theta = 45^\circ$ is



- (a) 0.073 W/m^2
 (c) 0.0037 W/m^2

- (b) 0.0073 W/m^2
 (d) 0.0137 W/m^2

6. Two glass slides A and B each of length L are placed with one end in contact and the other separated by a spacer of thickness h . Monochromatic light of wavelength λ falls normally on the plate A and interference fringes are observed by the eye E through a microscope. The spacing between the successive resulting dark fringes is



(a) $\frac{2h}{L\lambda}$

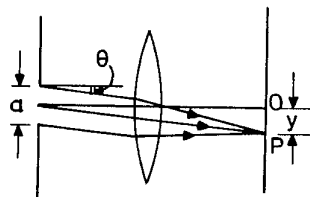
(b) $\frac{L\lambda}{2h}$

(c) $\frac{\lambda L}{h}$

(d) $\frac{2L\lambda}{h}$

7. The length of the plate in the above problem is 10 cm and they are separated by a spacer of thickness 0.02 mm. The wavelength of the monochromatic light employed is $5000 \text{ \AA}$. If the space between the two plates occupied by air is replaced by water, the spacing between two successive dark fringes will be
- (a) 0.06 mm (b) 0.36 mm
 (c) 0.44 mm (d) 0.94 mm

8. In a single slit diffraction as shown in the figure here, I_0 is the intensity at 0 corresponding to $\theta = 0$. Then the intensity at P is



(a) $I = I_0 \left[\frac{\sin \pi a (\sin \theta) / \lambda}{(\pi a \sin \theta) / \lambda} \right]$

(b) $I = I_0 \left[\frac{\sin \{ \pi a (\sin \theta) / \lambda \}}{(\pi a \sin \theta) / \lambda} \right]^2$

(c) $I = I_0 \left[\frac{\sin a \sin \theta / \lambda}{\pi a \sin \theta / \lambda} \right]$

(d) $I = I_0 \left[\frac{\{ \sin (a \sin \theta) / \lambda \}^2}{a \sin \theta / \lambda} \right]$

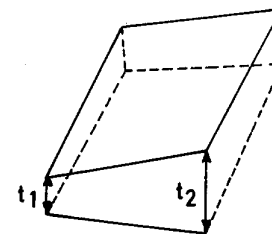
9. The lowest and highest wavelengths in a visible spectrum are approximately $4000 \text{ \AA}$ and $7000 \text{ \AA}$ respectively. A plane diffraction grating having 5000 lines per centimetre is placed in such a way that light falls on it at normal incidence. The angular width of the first order visible spectrum produced is
- (a) 4.26° (b) 13.60°
 (c) 10.25° (d) 8.99°
10. In a set-up to observe Newton's rings, the rings are first observed in the reflected light from a convex lens in contact with a flat glass plate, and are then viewed in the transmitted light from below. Choose the correct statement among the following.
- (a) The patterns will be exactly identical in every respect—the diameter of the rings, their contrast and their intensities.
 (b) The diameter of the n th ring in reflected light will be larger than that of the same ring in transmitted light.
 (c) Every aspect will be identical except that the intensity in reflected light will be more than in transmitted light.

- (d) The intensity of the transmitted fringe system will be more than in the reflected fringe system, and the rest will be identical.
11. The angle subtended by the first diffraction minimum for a point source viewed in the hydrogen line at 1420 MHz with a radio telescope having an aperture of 25 m is
- (a) 0.8° (b) 0.64°
 (c) 1.2° (d) 2.2°
12. The headlights of an automobile are 1.2 m apart. The diameter of the iris of the human eye is 7 mm at full dilation. Assume a wavelength of $5600 \text{ \AA}$ for the light from an automobile headlamp. The distance at which these headlights could be resolved is
- (a) 8.6 km (b) 12.3 km
 (c) 4.2 km (d) 10.5 km
13. Newton's rings are formed by reflection in the air film between a plane surface and a spherical surface of radius 50 cm . If the radius of the third bright ring is 0.09 cm and that of the twenty-third 0.25 cm , the wavelength of light used is
- (a) $5440 \text{ \AA}$ (b) $6500 \text{ \AA}$
 (c) $7200 \text{ \AA}$ (d) $4850 \text{ \AA}$
14. A glass plate 0.40 micron thick is illuminated by a beam of white light normal to the plate. The refractive index of glass is 1.50 and the limits of the visible spectrum are $\lambda_v = 4000 \text{ \AA}$ and $\lambda_r = 7000 \text{ \AA}$. The wavelengths that get intensified in the reflected beam are
- (a) $4800 \text{ \AA}$ and $5200 \text{ \AA}$ (b) $4800 \text{ \AA}$ and $6700 \text{ \AA}$
 (c) $4800 \text{ \AA}$ only (d) $5200 \text{ \AA}$ only
15. A beam of monochromatic light is incident normally on a thin wedge-shaped film of transparent plastic ($\mu = 1.40$) and the angle of the wedge is 10^{-4} radian. Interference fringes are observed, with a separation of 0.25 cm between adjacent bright fringes. The wavelength of incident light (in air) is
- (a) $4500 \text{ \AA}$ (b) $5000 \text{ \AA}$
 (c) $6500 \text{ \AA}$ (d) $7000 \text{ \AA}$
16. A planoconvex lens is placed on a plane glass plate and the system is illuminated from above by a monochromatic light to observe Newton's rings. The radius of the first bright ring is 1 mm and that of the convex surface is 4 m . The wavelength of the light used is
- (a) $4000 \text{ \AA}$ (b) $5000 \text{ \AA}$
 (c) $5600 \text{ \AA}$ (d) $6500 \text{ \AA}$
17. In the above problem, everything remains the same except that the air film between the plate and the lens is replaced with water ($\mu_w = 1.33$), the radius of the first bright ring is
- (a) 1.33 mm (b) 0.64 mm
 (c) 0.87 mm (d) 0.435 mm
18. The half-angular breadth of the central bright band in the Fraunhofer diffraction pattern of a slit $14 \times 10^{-5} \text{ cm}$ wide, when the slit is illuminated by a parallel beam of light of wavelength $7000 \text{ \AA}$, is
- (a) 12.4° (b) 25.6°
 (c) 18.0° (d) 9.4°
19. A slit of width 0.25 mm is placed in front of a convex lens and illuminated by plane waves of wavelength $5000 \text{ \AA}$. In the diffraction pattern formed in the focal plane of the lens, the distance from the third minimum on the left to the third minimum on the right is found to be 3 mm . The focal length of the lens is

- (a) 25 cm (b) 20 cm
(c) 40 cm (d) 30 cm
20. In an experiment, sodium light ($\lambda = 5890 \text{ \AA}$) is employed and interference fringes are obtained in which 20 fringes equally spaced occupy 2.30 cm on the screen. When sodium light is replaced by blue light, the set-up remaining the same otherwise, 30 fringes occupy 2.80 cm. The wavelength of blue light is
(a) 4780 $\text{\AA}$ (b) 5220 $\text{\AA}$
(c) 4250 $\text{\AA}$ (d) 4000 $\text{\AA}$
21. In Young's double slit experiment, sodium vapour lamp is made to illuminate the two slits which are 0.1 cm apart and the interference fringes are obtained on a screen placed at a distance of 1 m from the slits. The separation between the second dark and fifth bright fringe is
(a) 0.2945 cm (b) 0.3140 cm
(c) 0.2061 cm (d) 0.08835 cm
22. A wedged shaped air film having an angle of 40 seconds is illuminated by a monochromatic light and the fringes are observed vertically down through a microscope. The fringe separation between two consecutive bright fringes is 0.12 cm. The wavelength of light is
(a) 5545 $\text{\AA}$ (b) 6025 $\text{\AA}$
(c) 4925 $\text{\AA}$ (d) 4655 $\text{\AA}$
23. Two rectangular glass plates are in contact at one edge while the other edges are separated by a spacer of some suitable thickness so as to form a low angle wedge. The spacer is placed parallel to the line of contact and is at a distance of 10 cm from it. When viewed normally in light of wavelength 5500 $\text{\AA}$, a series of evenly spaced dark bands 0.5 mm apart are seen. The thickness of the spacer is
(a) 0.0425 cm (b) 0.0036 cm
(c) 0.0055 cm (d) 0.0254 cm
24. White light is incident on two parallel glass plates separated by a liquid film of refractive index 1.40 and thickness 0.001 cm and on examining the reflected light through a spectroscopic, a number of dark bands are seen between the wavelengths $4 \times 10^{-5} \text{ cm}$ and $7 \times 10^{-5} \text{ cm}$. The light is incident at an angle of 30° to the normal to the surfaces. The number of dark bands observed is
(a) 34 (b) 27
(c) 44 (d) 19
25. A parallel beam of sodium light ($\lambda = 5890 \text{ \AA}$) is incident on a thin glass plate of refractive index 1.50 such that the angle of refraction into the plate is 60° . The smallest thickness of the plate which will make it appear dark in the reflected light is
(a) $3.046 \times 10^{-5} \text{ cm}$ (b) $3.926 \times 10^{-5} \text{ cm}$
(c) $1.408 \times 10^{-5} \text{ cm}$ (d) $2.006 \times 10^{-5} \text{ cm}$
26. Mark the incorrect statement among the following:
(a) The radius of the n th bright ring in a Newton's rings experiment is $\sqrt{(2n-1) \frac{\lambda R}{2\mu}}$
(b) The radius of the dark ring of the second order in a Newton's rings experiment is $\sqrt{\frac{2\lambda R}{\mu}}$
(c) In a Newton's rings experiment, the diameter of the n th ring when there is an air film is D_n . When air is replaced by a liquid of refractive index

μ , the diameter of the n th ring is D'_n . The refractive index of the liquid is (D_n/D'_n)

- (d) The condition for constructive interference in case of a parallel-sided oil film of thickness t and refractive index μ is $2\mu \cos r = (2n \pm 1) \lambda/2$.
27. In a Newton's rings experiment, employing sodium light of wavelength 5890 $\text{\AA}$, the diameters of the 4th and 12th dark rings are measured as 0.400 and 0.700 cms respectively. The diameter of the 20th dark ring will be
(a) 0.892 cm (b) 0.874 cm
(c) 0.984 cm (d) 0.906 cm
28. An object is placed 0.50 cm in front of the objective of a microscope. The diameter of the objective is 0.50 cm. The smallest size of the object that the microscope can see is (take wavelength of light employed to be 5000 $\text{\AA}$)
(a) $4.8 \times 10^{-5} \text{ cm}$ (b) $5.4 \times 10^{-5} \text{ cm}$
(c) $6.1 \times 10^{-5} \text{ cm}$ (d) $4.0 \times 10^{-5} \text{ cm}$
29. A beam of light consists of two wavelengths λ_1 and λ_2 which fall normally on a convex lens of radius of curvature R resting on a glass plate. If the n th dark ring due to λ_1 coincides with the $(n+1)$ th dark ring due to λ_2 , then the radius of the n th dark ring of λ_1 is
(a) $\sqrt{\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2 R}}$ (b) $\sqrt{\frac{\lambda_1 \lambda_2 R}{(\lambda_1 - \lambda_2)}}$
(c) $\sqrt{\frac{(\lambda_1 - \lambda_2) R}{\lambda_1 \lambda_2}}$ (d) $\sqrt{\frac{\lambda_1 - \lambda_2}{R}}$
30. The deviation of the second order diffracted images formed by an optical grating having 5000 lines per centimetre is 35° . The wavelength of light used is
(a) 5730 $\text{\AA}$ (b) 5500 $\text{\AA}$
(c) 4920 $\text{\AA}$ (d) 6000 $\text{\AA}$
31. In a Newton's rings experiment, the rings are being observed normally in sodium light. The order of the dark ring whose diameter will be twice that of the 20th dark ring will be
(a) 40 (b) 160
(c) 80 (d) 120
32. The limits of the visible spectrum are approximately 4000 $\text{\AA}$ and 7000 $\text{\AA}$. The angular breadth of the first order visible spectrum produced by a plane transmission grating having 6000 lines per centimetre is (presume that the light is falling normally on the grating)
(a) $13^\circ 45'$ (b) $24^\circ 35'$
(c) $38^\circ 20'$ (d) $10^\circ 50'$
33. Two square surfaces of a transparent plastic block having a shape as shown in the figure have thickness t_1 at one edge and t_2 at the other. The refractive index of the material is 1.6. When viewed at normal incidence, using a light of wavelength 5000 $\text{\AA}$, 20 fringes are observed. The difference $(t_2 - t_1)$ is

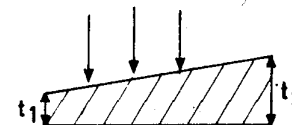


- (a) 0.3×10^{-4} cm (b) 3.1×10^{-4} cm
(c) 1.3×10^{-4} cm (d) 4.1×10^{-5} cm
34. Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.50. The fringe spacing is 1 mm and the wavelength of light is 5890 Å. The angle of the wedge in seconds of an arc is
(a) 48.42 (b) 22.46
(c) 40.48 (d) 36.24
35. White light falls normally on a film of soapy water whose thickness is 5×10^{-5} cm and refractive index is 1.40. The wavelengths in the visible region that are reflected the most strongly are
(a) 5600 Å and 4000 Å (b) 5400 Å and 4000 Å
(c) 6000 Å and 5000 Å (d) 4500 Å only
36. A liquid film of refractive index 1.4 and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 60° . The reflected light has a dark band corresponding to a wavelength 5×10^{-5} cm. The order of this interference band is
(a) 12 (b) 15
(c) 7 (d) 4
37. Light of wavelength 5000 Å is incident perpendicularly from air on a film of thickness 10^{-4} cm and refractive index 1.375. Part of the light is reflected from the first surface of the film and part enters the film and is reflected back from the second face. The number of waves contained along the path of this light in the film is
(a) 5.5 (b) 11
(c) 7 (d) 8.5
38. In a Fresnel biprism set-up, the separation between the two virtual sources is d , a is the distance of the biprism from the source (slit). A is the angle of the prism and μ is the refractive index of the material of the biprism. Then the separation between the virtual sources will be equal to
(a) $\frac{1}{2} aA(\mu - 1)$ (b) $aA(\mu - 1)$
(c) $\frac{1}{4} aA(\mu - 1)$ (d) $2aA(\mu - 1)$
39. In the above problem, green light of wavelength 5000 Å is employed to observe fringes on a screen placed at a distance of 2 m from the biprism. If $a = 20$ cm, $A = 0.005$ rad and $\mu = 1.5$, the spacing between the fringes is
(a) 0.11 mm (b) 0.5 mm
(c) 1.1 mm (d) 0.25 mm
40. Fringes are formed due to interference from an air wedge employing monochromatic light of wavelength 5800 Å. If the number of bands observed per centimetre is 18, the angle of the wedge is
(a) 107.6 s (b) 92.4 s
(c) 99.4 s (d) 112
41. A parallel beam of sodium light ($\lambda = 5890$ Å) is incident normally on a diffraction grating. The angle between the two first-order spectra on either side of the normal is $24^\circ 30'$. The number of rulings per centimetre on the grating is
(a) 3000 (b) 3601
(c) 1500 (d) 4500
42. Plane monochromatic waves of wavelength 6000 Å are incident normally on a plane transmission grating having 500 lines per millimetre. The angles of deviation in the first, second and third order respectively are

- (a) 21.5° , 38.2° and 66.2° (b) 27.5° , 41.6° and 72.4°
(c) 17.5° , 36.9° and 64.1° (d) 13.4° , 23.5° and 41.2°
43. Interference bands are produced by a Fresnel's biprism in the focal plane of a micrometer eye-piece. The focal plane is 1 m away from the slit. A lens is placed between the biprism and the eye-piece, and produces two images of the slit in two positions of the lens. In one position of the lens, the separation between the two images is measured as 4.05 mm while in the other position, it is 2.90 mm. If the light source is a sodium lamp ($\lambda = 5893$ Å), the distance between the two consecutive interference bands is
(a) 0.0241 cm (b) 0.0426 cm
(c) 0.1046 cm (d) 0.0172 cm
44. A parallel beam of white light is allowed to fall axially on the slit of the collimator of a spectrometer and a thin air film of uniform thickness enclosed between two glass plates placed in front of the slit with the surface of air film normal to the path of light. On viewing through the telescope, 250 fringes are observed in the spectral region between wavelengths $\lambda_1 = 4000$ Å and $\lambda_2 = 6500$ Å as measured in air. The thickness of the air film is
(a) 0.086 cm (b) 0.013 cm
(c) 0.046 cm (d) 0.142 cm
45. White light is incident on a soap film at an angle of incidence $\sin^{-1}(4/5)$ and the reflected light on examination by a spectroscope shows dark bands. Two consecutive dark bands correspond to wavelengths 6.1×10^{-5} and 6.0×10^{-5} cm. If the refractive index of the soap film is $4/3$, the thickness of the film is
(a) 0.00172 cm (b) 0.01722 cm
(c) 0.00244 cm (d) 0.00412 cm
46. A Fresnel's biprism, having an angle of one degree and refractive index 1.5, forms interference fringes on a screen placed 1 m from the prism. If the distance between the source and the biprism is 25 cm, and the wavelength of light employed is 6900 Å, the fringe separation is
(a) 0.0132 cm (b) 0.0198 cm
(c) 0.1324 cm (d) 0.0246 cm
47. In a biprism experiment, the micrometer readings for zero order and tenth order fringes are 1.25 and 2.37 mm respectively when light of wavelength 6000 Å is used. If the wavelength is changed to 7000 Å, the positions of the zero order and tenth order fringes will be
(a) 1.25 mm and 2.55 mm (b) 1.25 mm and 2.65 mm
(c) 2.50 mm and 3.10 mm (d) 0.50 mm and 1.89 mm
48. In a certain Young's double slit experiment, the slit separation is 0.050 cm. The distance between the slit and the micrometer eye-piece is 1 m. When green light is used, the distance from the central fringe to the 4th order fringe is 0.44 cm. The wavelength of light is
(a) 5000 Å (b) 5500 Å
(c) 5750 Å (d) 5900 Å
49. A plane wave of monochromatic light falls normally on a uniform thin film of oil formed on a glass plate. The wavelength of the source can be varied continuously. Complete destructive interference is observed in the reflected light for wavelengths 7000 Å and 5000 Å and for no other wavelength in between. If the index of refraction of oil is 1.40 and that of glass is 1.50, the thickness of the oil film is
(a) 3500 Å (b) 4250 Å
(c) 6730 Å (d) 6250 Å

50. In a Young's double slit experiment, if the incident light consists of two wavelengths λ_1 and λ_2 , the slit separation is d , and the distance between the slit and the screen is D , the maxima due to the two wavelengths will coincide at a distance from the central maxima, given by
- (a) $\frac{\lambda_1 + \lambda_2}{2Dd}$ (b) $(\lambda_1 - \lambda_2) \cdot \frac{2d}{D}$
- (c) LCM of $\lambda_1 \cdot \frac{D}{d}$ and $\lambda_2 \cdot \frac{D}{d}$ (d) HCF of $\frac{\lambda_1 D}{d}$ and $\frac{\lambda_2 D}{d}$
51. Light from a source emitting two wavelengths λ_1 and λ_2 is allowed to fall on a Young's double slit apparatus after one of the wavelengths is filtered. The position of interference maxima is noted. When the filter is removed, i.e. both the wavelengths are now incident on the slits, it is found that the maximum intensity is now produced where the fourth maxima occurred previously. If the other wavelength is filtered, the third maxima is found at the same location. The ratio of the two wavelengths is
- (a) 3/4 (b) 4/5
- (c) 3/5 (d) 4/7
52. To prevent the reflection of a camera lens, a thin film of magnesium fluoride is provided. In order that light of wavelength 5000 Å not be reflected, the thickness of the film should be (μ for magnesium fluoride is 1.25 and $\mu_g = 1.5$)
- (a) 2×10^{-7} m (b) 1×10^{-7} m
- (c) 2×10^{-5} m (d) 2×10^{-8} m
53. A double slit is illuminated with light from a helium-neon laser ($\lambda = 6330$ Å). Two successive maxima are found to be 1.50 cm apart when projected on a screen 275 cm behind the slits. Therefore,
- (a) the slits must be 0.0116 cm apart
- (b) the slits must be 0.116 cm apart
- (c) each slit must be 0.0116 cm wide
- (d) one cannot determine the separation of the slits unless one knows which two maxima are measured.
54. When a certain double slit is illuminated with light having a wavelength of 6200 Å, the interference fringes are spaced 2.0 cm apart on a screen 6.4 m from the slits. The light source is now replaced with one having a wavelength of 4500 Å. We can say that
- (a) the spacing of the fringes remains the same
- (b) the fringes are now 1.45 cm apart
- (c) the fringes are now 2.75 cm apart
- (d) the slit separation is 0.273 cm
55. Two slits are illuminated with light from an argon ion laser ($\lambda = 4880$ Å). The slits are separated by a distance of 39000 nm ($= 39000 \times 10^{-9}$ m). In this case
- (a) if the observation screen is 3 m behind the slits, the interference maxima will be 18.75 cm apart.
- (b) if the maxima are separated by 1 cm, the screen is 80 cm behind the slits
- (c) if the 514 nm line that the same laser can also emit were used, the maxima would be closer together
- (d) if the 514 nm line were used, the maxima would be in the same place.
56. A glass plate is set upon an optical flat and the interference is observed using a green light from mercury at 5460 Å. A dark spot is noted indicating the point at which the plates are in contact. The thickness of the film at the adjacent dark fringe is

- (a) 5460 Å
- (b) some unknown integral multiple of 5460 Å
- (c) 273 nm ($1 \text{ nm} = 10^{-9} \text{ m}$)
- (d) zero
57. When the diffraction pattern from a certain slit illuminated with laser light ($\lambda = 6330$ Å) is projected on a screen 150 cm from the slit, the second minima on each side are separated by 8 cm. This tells us that
- (a) the slit is approximately 0.005 cm wide
- (b) the slit is approximately 0.05 cm wide
- (c) a/λ is approximately 7.5 (a is the slit width)
- (d) a/λ is approximately 750
58. A Fresnel's biprism having a refracting angle of 2° and index of refraction 1.5 receives light from a narrow slit illuminated by the green line of mercury ($\lambda = 5461$ Å). A soap film is introduced in the path of one of the interfering beams and the fringes are observed to shift by 2.5 times the fringe width. If refractive index of soap film is 1.33, the thickness of the film is
- (a) 6.241×10^{-4} cm (b) 2.603×10^{-4} cm
- (c) 1.481×10^{-4} cm (d) 4.137×10^{-4} cm
59. A square piece of cellophane film with index of refraction of 1.5 has a wedge-shaped section so that its thicknesses at two opposite sides are t_1 and t_2 . Monochromatic light of wavelength 6000 Å is incident normally upon the film. The number of interference fringes appearing at the film is 10. Difference ($t_2 - t_1$) is
- (a) 4×10^{-4} cm (b) 5×10^{-5} cm
- (c) 2×10^{-4} cm (d) 1×10^{-4} cm
60. A wedge shaped vertical soap film ($\mu = 4/3$) is illuminated with monochromatic light ($\lambda = 6000$ Å). The upper edge of the film is observed to be black when viewed by the reflected light and six bright fringes are observed, the centre of the sixth bright band coinciding with the bottom of the film. If the film is a square of side 2.75 cm, the angle of wedge is
- (a) 4.5×10^{-5} rad. (b) 4.5×10^{-4} rad.
- (c) 5.4×10^{-4} rad. (d) 4.5×10^{-3} rad.



Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (d) | 3. (a) | 4. (a) | 5. (b) |
| 6. (b) | 7. (d) | 8. (b) | 9. (d) | 10. (d) |
| 11. (c) | 12. (b) | 13. (a) | 14. (c) | 15. (d) |
| 16. (b) | 17. (c) | 18. (b) | 19. (a) | 20. (a) |
| 21. (c) | 22. (d) | 23. (c) | 24. (b) | 25. (b) |
| 26. (c) | 27. (d) | 28. (c) | 29. (b) | 30. (a) |
| 31. (c) | 32. (d) | 33. (b) | 34. (c) | 35. (a) |
| 36. (c) | 37. (a) | 38. (d) | 39. (c) | 40. (a) |
| 41. (b) | 42. (c) | 43. (d) | 44. (b) | 45. (a) |
| 46. (b) | 47. (a) | 48. (b) | 49. (d) | 50. (c) |
| 51. (a) | 52. (b) | 53. (a) | 54. (b) | 55. (b) |
| 56. (c) | 57. (a) | 58. (d) | 59. (c) | 60. (a) |

Note: Each problem should take 4 to 5 minutes where numerical calculations are involved. The rest should be solved in about 3 minutes each.

18

Photometry

Total Radiant Flux

This is the total energy of radiation emitted by a source per unit time. Its SI unit is the watt.

Luminosity of Radiant Flux (I)

This is the measure of the capacity to produce the sensation of brightness in the eye. A source of any colour composition is said to have a luminous intensity I of one candela (1 cd) if it is perceived by the eye to have the same intensity as a certain standard source. Most sources have different luminous intensities when viewed from different directions and so the value of I for a source may vary with the angle at which it is viewed. An older unit for I is the *Candle* or *Candle power*.

$$1 \text{ Candle} = 1 \text{ Candle power} = 0.981 \text{ cd}$$

Luminous Flux (F)

Luminous flux is a quantity directly representing the total brightness-producing capacity of the source. Its unit is the *lumen*. The luminous flux of a source of 1/685 W emitting monochromatic light of wavelength 5550 Å (555 nm) is called one lumen. In other words, a 1-W source emitting monochromatic light of wavelength 555 nm emits 685 lumen.

Relative Luminosity

$$L = \frac{\text{luminous flux of a source of given wavelength}}{\text{luminous flux of a 555 nm source of same power}}$$

The flux leaving a point source through a solid angle $\Delta\omega$ steradian is given by

$$\Delta F = I \Delta\omega, \text{ where } I \text{ is the luminous intensity}$$

Because the solid angle subtended by an entire sphere is 4π steradians, this yields $F = 4\pi I$ for the total flux emanating from an isotropic point source. Since I is in candelas and $\Delta\omega$ is in steradians (sr), we have

$$1 \text{ lm} = (1 \text{ cd}) (1 \text{ sr}) \text{ or } 1 \text{ cd} = 1 \text{ lm/sr}$$

Illuminance

The illuminance (E) of a surface (also called illumination) is a measure of how much light falls on a unit area of the surface. If a flux ΔF strikes a small area ΔA , then the illuminance at the point is given by

$$E = \frac{\Delta F}{\Delta A}$$

and, for a finite A ,

$$E = \frac{F}{A}$$

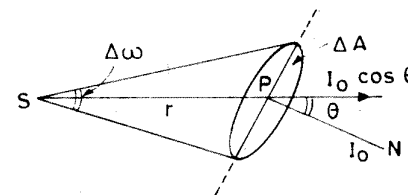
The units of E are lm/m^2 or lux or lx. The units are also expressed in foot-candle and 1 ft candle = 1 lm/ft^2 .

The photometric terms, their symbols, and units, are tabulated here.

Quantity	Symbol	Unit	Definition
✓ Luminous Flux	F <i>Visible</i>	Lumen	1 lumen = 0.0146 W of radiant flux of wavelength 555 mμ
Luminous efficiency	(F/P)	Lumen/watt	Ratio of luminous flux to radiant flux
Luminous intensity	I	Lumen or candle steradian	Luminous flux emitted per steradian
Bole Luminous emittance	L	Lumen/m ²	Luminous flux emitted per unit area
E = 0.1 Illuminance	E	Lumen/m ²	Luminous flux incident per unit area
✓ Luminance	B	Candle/m ²	Luminous intensity per unit of projected area

Inverse Square Law

Let a point source S be situated at a distance r from a point P on an area ΔA . The normal to the surface ΔA , PN makes an angle θ with SP . If the luminous intensity of the source in the direction SP is I , then



$$\Delta F = I \frac{\Delta A \cos \theta}{r^2}$$

and illumination at ΔA is

$$E = \frac{\Delta F}{\Delta A} = \frac{I \cos \theta}{r^2}$$

Thus, the illuminance due to a point source decreases inversely as the square of the distance from the source.

Lambert's Cosine Law

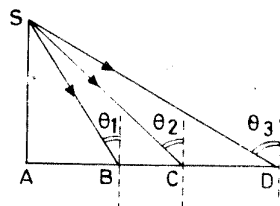
If the luminous intensity along the normal is I_0 , then $I = I_0 \cos \theta$, in a direction making an angle θ with the normal.

Principle of Photometry

Consider two sources having intensities I_1 and I_2 , which are at distances r_1 and r_2 from a screen and produce fluxes perpendicular to the screen. If r_1 and r_2 are so adjusted that the illumination at the screen is the same for the two sources, then $E_1 = E_2$ and

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$$

Illumination falls off as cube of the cosine of angle of incidence



$$E_B = E_A \cos^3 \theta_1,$$

$$E_C = E_A \cos^3 \theta_2,$$

$$E_D = E_A \cos^3 \theta_3, \text{ and so on.}$$

Reflection and Transmission Coefficients of a Surface

$$\text{Reflection coefficient } r = \frac{\text{energy reflected}}{\text{energy incident}}$$

$$\text{and Transmission coefficient } t = \frac{\text{energy transmitted}}{\text{energy incident}}$$

ILLUSTRATIONS

1. A projector lens has a focal length of 10 cm. The image of a 3 cm $\times$ 3 cm slide is formed on a screen placed at a distance of 4 m from the lens.

(i) Find the size of the picture on the screen.

(ii) Find the ratio of illumination of the slide and of the picture on the screen.

Solution

$$(i) \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{400} - \frac{1}{10} = \frac{-39}{400} \Rightarrow u = -\frac{400}{39} \text{ cm}$$

$$\frac{I}{O} = \frac{v}{u} = \frac{400 \times 39}{400} \Rightarrow O = 3 \times 39 = 117 \text{ cm}$$

$$(ii) \quad \text{Illumination of the slide} = E_1 = \frac{I}{9}$$

$$\text{and Illumination of the picture} = E_2 = \frac{I}{(117)^2}$$

$$\therefore \frac{E_1}{E_2} = \frac{117 \times 117}{9}$$

$$= 117 \times 13 = 1521$$

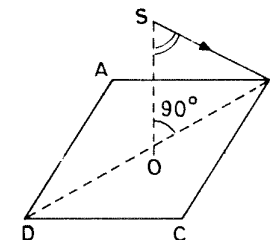
2. A bulb of luminous intensity of 60 cd (candela) is at a vertical distance of 2 m over the centre of a square table of side 2 m. Find the illumination at the centre and also at one corner of the table.

Solution

The illumination at the centre O of the square table is

$$I_0 = \frac{60}{(2)^2} = 15 \text{ cd/m}^2$$

The diagonal $BD = \sqrt{8} \text{ m}$.



$$SB^2 = SO^2 + BO^2 = (2)^2 + \left(\frac{\sqrt{8}}{2}\right)^2 = 4 + \frac{8}{4} = 6$$

$$\cos(\angle BSO) = \frac{2}{\sqrt{6}}$$

and, hence, the illumination at a corner, say, B is

$$I_0 \cos \theta = \frac{60}{(\sqrt{6})^2} \times \frac{2}{\sqrt{6}} = \frac{120}{6\sqrt{6}} = \frac{20}{2.45} = 8.16 \text{ cd/m}^2$$

3. Two lamps A and B produce equal illumination on the screen of a photometer when A is 50 cm and B is 60 cm from the screen. A plane mirror is then placed 5 cm behind A, the plane of the mirror being at right angles to the line from A to the screen. It is now found that, to restore equality of

illumination on the screen, the source B has to be moved 10 cm nearer to the screen. What is the reflecting power of the mirror?

Solution

If P_1 and P_2 be the powers of A and B,

$$\frac{P_1}{50^2} = \frac{P_2}{60^2} \text{ or } \frac{P_1}{P_2} = \frac{25}{36} \quad (i)$$

If k be the reflecting power of the mirror, the effect of the mirror is to place another source of power kP_1 5 cm behind the mirror, i.e. $50 + 5 + 5$ or 60 cm, from the screen. The equality of illumination on the screen is restored when B is placed $60 - 10$ or 50 cm from the screen. Hence,

$$\frac{P_1}{50^2} + \frac{kP_1}{60^2} = \frac{P_2}{50^2} \text{ or } P_1 \left(\frac{1}{25} + \frac{k}{36} \right) = \frac{P_2}{25}$$

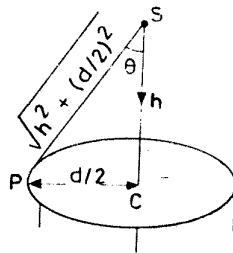
$$\text{or } \frac{P_1}{P_2} = \frac{36}{36 + 25k} \quad (ii)$$

From Eqs. (i) and (ii), we get $k = 0.63$, i.e. the mirror reflects 63% of the light that falls on it.

4. If an electric lamp is suspended at a height h from a circular table of diameter d , prove that the illumination at the centre of the table is $[1 + (d^2/4h^2)]^{3/2}$ times the illumination at the edge of the table.

Solution

Let E_c and E_p be the illuminations at the centre and the edge of the table respectively and let I be the illuminating power of the lamp. Then,



$$E_c = \frac{I}{(SC)^2} = \frac{I}{h^2} \quad \text{and} \quad E_p = \frac{I \cos \theta}{(SP)^2} = \frac{I \times h}{[h^2 + (d/2)^2]^{3/2}}$$

$$\begin{aligned} \frac{E_c}{E_p} &= \frac{I}{h^2} \times \frac{[h^2 + d^2/4]^{3/2}}{I \times h} = \frac{(h^2 + d^2/4)^{3/2}}{h^3} = \frac{h^3 (1 + d^2/4h^2)^{3/2}}{h^3} \\ &= \left(1 + \frac{d^2}{4h^2} \right)^{3/2} \end{aligned}$$

5. In a photometer, light from a bulb B_1 is exactly balanced by that from another bulb B_2 placed 30 cm from the screen. On placing a transparent glass sheet between the bulb B_1 and the screen, B_2 had to be shifted by 10 cm to restore the balance. Find the fractions of the light transmitted and absorbed by the glass sheet.

Solution

Let I_1 and I_2 be the illuminating powers of bulbs B_1 and B_2 respectively. Let B_1 be at a distance d from the screen in the first photometric balance. Then,

$$\frac{I_1}{d^2} = \frac{I_2}{(30)^2} \quad (i)$$

When the glass sheet is introduced between bulb B_1 and the screen, let x be the fraction of light transmitted through it. The effective illuminating power of B_1 will now be xI_1 and we have for the balance,

$$\frac{xI_1}{d^2} = \frac{I_2}{(40)^2} \quad (ii)$$

On solving (i) and (ii) we get $x = 9/16$. This means that 56% of the light is transmitted by the sheet. The fraction of light absorbed is 44%.

6. A good photographic print is obtained by an exposure of 2 s at a distance of 1 m from a 75 cd (candela) bulb. What should be the time of exposure if one is to obtain an equally satisfactory result at a distance of 2 m from a 50 cd bulb.

Solution

Intensity of illumination produced by a 75 cd bulb,

$$I_1 = \frac{75}{(1)^2} = 75 \text{ cd/m}^2$$

Light energy falling on unit area of the print in 2 s = $75 \text{ cd/m}^2 \times 2 \text{ s} = 150 \text{ cd} \cdot \text{s/m}^2$

Intensity of illumination produced by a 50 cd bulb at a distance of 2 m is

$$I_2 = \frac{50}{(2)^2} = 12.5 \text{ cd/m}^2$$

Let t be the time for which light is allowed to fall on unit area of the print. Then light energy on the print is

$$(12.5 \times t) \text{ cd/m}^2 \cdot \text{s}$$

Since the light energy falling on the print in the two cases has to be the same, we have

$$150 = 12.5 \times t \Rightarrow t = \frac{150}{12.5} = 12 \text{ s}$$

7. Two small lamps A and B are placed at distances of 60 and 100 cm respectively from a Bunsen photometer, one lamp on each side of the photometers, so as to produce equal illumination at the photometer. A large perfectly reflecting mirror is then placed 20 cm to the left of A with its reflecting surface normal to the axis of the bench so that the light from A is reflected on the photometer. The illumination on the photometer from the two sides is thus made unequal. Through what distance must the lamp B be moved in order to restore equality of illumination of the photometer?

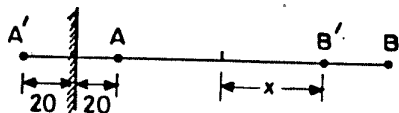
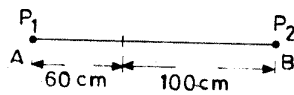
Solution

Let P_1 and P_2 be the illuminating powers of lamps A and B respectively.

$$\frac{P_1}{(60)^2} = \frac{P_2}{(100)^2} \Rightarrow P_2 = \frac{25}{9} P_1 \quad (i)$$

Let P_3 be the illuminating power of the image of A in the mirror. Intensity of illumination on the left side of the screen

$$\begin{aligned} &= \frac{P_1}{(60)^2} + \frac{P_3}{(60 + 20 + 20)^2} \\ &= \frac{P_1}{(60)^2} + \frac{P_3}{(100)^2} \end{aligned}$$



Let the distance between the photometer screen and the new position of B (shown as B') be x , such that the intensities of illumination on the screen due to lamp B, lamp A, and its image A' in the mirror are equal.

$\therefore$ Intensity of illumination on the right side of the screen

$$= \frac{P_2}{x^2}$$

Therefore,

$$\frac{P_1}{(60)^2} + \frac{P_3}{(100)^2} = \frac{P_2}{x^2} \quad (ii)$$

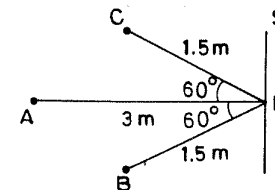
Since the mirror is perfectly reflecting, it means that no light is absorbed and, therefore, $P_1 \approx P_3$.

$$\therefore \frac{P_1}{(60)^2} + \frac{P_3}{(100)^2} = \frac{25}{9} \frac{P_1}{x^2}$$

$$\frac{1}{3600} + \frac{1}{10000} = \frac{25}{9x^2} \Rightarrow x = 85.74 \text{ cm}$$

The distance between the lamp B and the photometer should be reduced to 85.74 cm from 100 cm, i.e. it should be moved a distance of 14.26 cm towards the screen of the photometer.

8. Screen S is illuminated by two point sources A and B. Another source C sends a parallel beam of light towards the point P on the screen (refer to the figure). Line AP is normal to the screen and the lines AP, BP and CP are in one plane. The distances AP, BP and CP are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of sources A and B are 90 and 180 W respectively and the beam from C is of the intensity 20 W/m^2 . Calculate the intensity at P on the screen.



Solution

$$I_A = \frac{P_A}{4\pi r^2} = \frac{90}{4\pi(3)^2} = \frac{10}{4\pi} \text{ W/m}^2$$

$$I_B = \frac{180 \cos 60}{4\pi(1.5)^2} = \frac{10}{\pi} \text{ W/m}^2$$

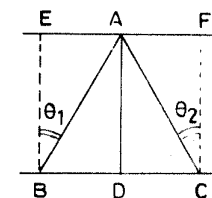
$$I_C = 20 \cos 60 = 10 \text{ W/m}^2$$

$$\text{Total intensity } I = I_A + I_B + I_C$$

$$= \frac{10}{4\pi} + \frac{10}{\pi} + 10$$

$$= 10 \left[\frac{1}{4\pi} + \frac{1}{\pi} + 1 \right] = 10 \left[\frac{4\pi + 5}{4\pi} \right]$$

9. At the vertex A of an equilateral triangle ABC, a screen is placed parallel to the base BC. Three lamps placed at B, C, D give equal illumination at A on the screen. Also, $BD = DC$. Find the ratio of these Candle powers.



Solution

Let P_1 , P_2 and P_3 be the illuminating powers of the three lamps B, C and D respectively. For oblique incident light, we have

$$I = \frac{P \cos \theta}{r^2}$$

Let I_1 , I_2 and I_3 be the intensities of illumination of P_1 , P_2 and P_3 at A, B and C.

$$I_1 = \frac{P_1 \cos \theta_1}{(BA)^2}, I_2 = \frac{P_2 \cos \theta_2}{(CA)^2} \text{ and } I_3 = \frac{P_3}{(AD)^2}$$

Since they give equal illumination at A, it means that $I_1 = I_2 = I_3$ and θ_1 and θ_2 are both equal to 30° , $AB = BC = AC = a$ and $AD = \frac{\sqrt{3}a}{2}$

$$\frac{P_1 \sqrt{3}}{2a^2} = \frac{4P_3}{3a^2} \Rightarrow \frac{P_2}{P_3} = \frac{8}{3\sqrt{3}} \text{ and } \frac{P_1}{P_3} = \frac{8}{3\sqrt{3}}$$

$$\therefore P_1 : P_2 : P_3 :: 8 : 8 : 3\sqrt{3}$$

10. A room measuring 30 m by 20 m is lighted by 20 bulbs. Each bulb is rated 500 W and has an efficiency of 15 lm/W. If the amount of flux actually utilized is 60% of the total flux, calculate the illumination on the working plane.

Solution

$$\text{Output of each lamp} = 500 \text{ W} \times 15 \text{ lm/W} = 7500 \text{ lm}$$

$$\text{Amount of flux actually utilized} = 0.6 \times 7500 = 4500 \text{ lm}$$

$$\text{Total flux received by the working plane} = 20 \times 4500 = 90000 \text{ lm}$$

$$\text{Area illuminated} = 600 \text{ m}^2$$

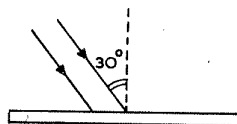
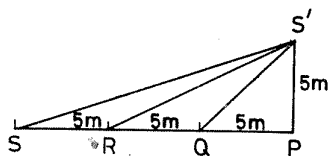
$$\therefore \text{Illumination} = \frac{90000}{600} = 150 \text{ lm/m}^2$$

EXERCISES

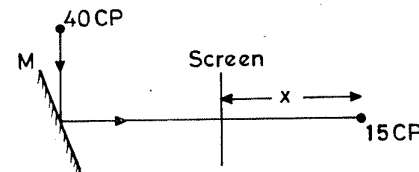
- A lamp is hanging along the axis of a circular table of radius r . It is desired that the intensity of illumination at the edge of the table be $1/8$ of that at its centre. To achieve this, the height at which the lamp should be placed is
 - $r/\sqrt{2}$
 - $r/\sqrt{3}$
 - $r/3$
 - $\sqrt{3}r$
- An electric bulb illuminates a plane surface and the intensity of illumination on the surface at a point 2 m away from the bulb is 5×10^{-4} phot (lumen/cm²). The line joining the bulb to the point makes an angle of 60° with the normal to the surface. The intensity of the bulb in Candela (Candle power) is
 - $40\sqrt{3}$
 - 20
 - 40×10^{-4}
 - 40
- A drawing hall has a number of boards having total effective area of 60 m². It is lit by a number of 60 W bulbs giving 15 lm/W. An illumination of 100 lm/m² is required on the drawing boards. Assume that 60% of the total light emitted by the bulbs is available for illuminating the drawing boards. To achieve this, the number of bulbs required is
 - 11
 - 15
 - 20
 - 25
- Two lamps are 40 m apart and 5 m high. Each lamp is of power 100 W, and has an efficiency of 2 candles per watt. The illumination at a point on the ground midway between the lamps is
 - 0.023 lm/m²
 - 0.32 lm/m²
 - 0.23 lm/m²
 - 0.032 lm/m²

- A surface receives light normally from a source which is at a distance of 3 m from it. If the source is moved closer until the distance is only 2 m, the illumination of the surface increases. In order to restore the original illumination on the surface, the angle through which the surface should be turned is
 - $34^\circ 04'$
 - $43^\circ 40'$
 - $36^\circ 63'$
 - $63^\circ 36'$
- Two lamps of 50 CP and 20 CP are placed at distances of 1 m and 30 cm respectively from the screen of a photometer on the same side of it. In order to equalize the illumination on the screen from this side, a third lamp of 40 CP is placed in line with the other two, normal to the screen on the other side of it. The distance of the 40 CP lamp from the screen is
 - 22.6 cm
 - 38.3 cm
 - 30.6 cm
 - 46.2 cm
- Two lamps A and B produce equal illumination on a screen when they are placed on opposite sides of it at distances 20 cm and 30 cm respectively. A glass plate is then introduced in front of lamp B. To restore the equality of illumination, lamp A has to be moved to a position 25 cm from the screen. The percentage of light transmitted by the glass plate is
 - 64
 - 46
 - 75
 - 55
- The standard photometer bar is 3 m long. A lamp whose intensity in the direction of the photometer is known to be 30 candles is set up at one end of the bar and a lamp of unknown intensity at the other end. The field of view of a Lummer-Brodhun photometer appears to be of uniform brightness when it is 80 cm from the 30-candle lamp. The intensity of the other lamp in the direction of the photometer is
 - 127 Candles
 - 227 Candles
 - 187 Candles
 - 280 Candles
- The full moon is capable of producing an illuminance of 0.2 lumen/m² on the surface of the earth. Assuming the full moon to be optically equivalent to a uniform circular disc 2200 miles in diameter and at a distance of 250,000 miles from the surface of the earth, the luminance of the moon is (neglect absorption of light by the atmosphere)
 - 6.6×10^3 Candles/m²
 - 3.3×10^3 Candles/m²
 - 2.2×10^3 Candles/m²
 - 5.5×10^3 Candles/m²
- A uniform point source of intensity 100 Candles is mounted 50 cm above a horizontal screen containing a hole of radius 1 cm located directly under the source. Light passing through this hole falls on a second horizontal screen 50 cm below the first. The total number of lumens from the source that fall on the second screen is
 - 12.6
 - 1.26
 - 0.126
 - 126
- Two lamps placed 20 cm and 30 cm respectively from a screen, illuminate it equally. When a transparent glass plate is introduced between the brighter lamp and the screen, the weaker lamp is moved 10 cm to restore the same illumination on both sides of the screen. The percentage of light absorbed by the glass is
 - 44.54
 - 36.46
 - 63.54
 - 55.56

12. Two lamps A and B equally illuminate the screen of a photometer when they are 30 and 60 cm respectively from the screen. A plane mirror with its reflecting face at right angles to the photometer bench is placed 7.5 cm from A on the side farther away from the screen. To get equal illumination again, B has to be moved 9 cm. The ratio of the illuminating powers of the image of A in the plane mirror and that of A is
 (a) 0.864 (b) 0.684
 (c) 0.468 (d) 0.543
13. The minimum luminous flux required to operate a certain photoelectric circuit is 3 lumens. The diameter of the sensitive surface of the photocell is 8 cm. There is a uniform point source of light located 1 m above the cell. The minimum luminous intensity of the source required for operating the circuit is
 (a) 490 Candelas (b) 597 Candelas
 (c) 795 Candelas (d) 975 Candelas
14. A plane mirror is placed 8 cm behind A, the plane of the mirror being normal to the line from A to the screen. It is found that, for a photometric balance, a source B must be moved 10 cm nearer the screen. In the beginning the two lamps A and B produced equal illumination on the screen when A was 60 cm and B 70 cm away from the screen. The reflecting power of the mirror is
 (a) 73% (b) 58%
 (c) 54% (d) 80%
15. A source S' of light gives 100 Candela uniformly in all directions. The illuminations at P, Q, R and S lying in the vertical plane passing through P respectively are
 (a) 4, 2, 1, 0.5 (lumen/m^2)
 (b) 4, $1/4$, $1/16$, $1/64$ (lumen/m^2)
 (c) 4, 1, $1/4$, $1/16$ (lumen/m^2)
 (d) 4, 1.41, 0.36, 0.13 (lumen/m^2)
16. A light meter aimed directly into a beam of light reads 1000 lm/m^2 . If the beam is oriented as shown in the figure, the illumination of a horizontal tabletop at the same position is
 (a) 745 lm/m^2 (b) 866 lm/m^2
 (c) 686 lm/m^2 (d) 668 lm/m^2
17. Two light sources are placed 140 cm apart and a Bunsen's photometer screen is adjusted to such a position on the straight line joining them that the screen is equally illuminated from both sides. The two sources are now interchanged and it is found that the screen has to be shifted through 20 cm to get equal illumination on it. The ratio of the illuminating powers of two sources is
 (a) $4/3$ (b) $5/6$
 (c) $16/9$ (d) $5/2$
18. A lamp produces a certain intensity of illumination on a screen when situated at a distance of 85 cm from it. On placing a sheet of transparent glass between the lamp and the screen, the lamp must be moved 5 cm nearer to the screen to produce the same illumination as before. The percentage of light absorbed by the glass is
 (a) 17.54 (b) 22.80
 (c) 6.54 (d) 11.42



19. A satisfactory photographic print is obtained when the exposure was for 20 s at a distance of 0.5 m from a 20 CP lamp. In order to get the same print again, a lamp of 25 CP is used and the exposure time is changed to 36 s. The distance from the lamp in this case should be
 (a) 0.75 m (b) 0.650 m
 (c) 1.0 m (d) 1.25 m
20. Light from a 40 CP lamp falls on a silvered mirror and is reflected from there to a grease spot photometer. The distance from the lamp to the screen via the mirror is 150 cm. The mirror reflects 80% of the light falling on it. A 15 CP lamp is placed so that the grease spot vanishes. The distance (x) of this lamp from the screen is



- (a) 102.7 cm (b) 72.5 cm
 (c) 110.4 cm (d) 88.2 cm
21. An intense flood light has an efficiency of 100 lumen/watt and is operated by means of a generator having a pulley of 50 cm diameter and run by a belt having a pressure difference of 100 N in its two arms. The efficiency of the generator is 75%. It is desired to have an illumination of 50 lumen/m^2 on the ground covering a circular area of 30 m diameter. To achieve this, the angular speed of the generator should be
 (a) 150 rev/s (b) 15 rev/s
 (c) 30 rev/s (d) 3 rev/s
22. An illuminating source is in the shape of a plane horizontal disc of radius 25 cm and is suspended over a table at a height of 75 cm. The illuminance at the table below the centre of the disc source is equal to 70 lm/m^2 . The luminosity of the source is
 (a) 70 lm/m^2 (b) 700 lm/m^2
 (c) 350 lm/m^2 (d) 1050 lm/m^2

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (a) | 4. (c) | 5. (d) |
| 6. (b) | 7. (a) | 8. (b) | 9. (b) | 10. (c) |
| 11. (d) | 12. (a) | 13. (b) | 14. (b) | 15. (d) |
| 16. (b) | 17. (c) | 18. (d) | 19. (a) | 20. (a) |
| 21. (d) | 22. (b) | | | |

Note: All problems in this chapter should take three to five minutes each.

19

Electrostatics— Coulomb's Law, Electric Fields and Potentials

Coulomb's Law

The electrical interaction between two charged particles is described in terms of the forces they exert on each other. Suppose two point charges q_1 and q_2 are separated by a distance r_{12} and are placed in vacuum. If the two charges are of the same sign, then the force of repulsion between them is given by Coulomb's law.

$$F = k \frac{q_1 q_2}{r_{12}^2} \quad (\text{in vacuum})$$

The constant k has the value $k = 8.988 \times 10^9 \text{ N.m}^2/\text{C}^2 \approx 9 \times 10^9 \text{ N.m}^2/\text{C}^2$. This constant is equal to $(1/4\pi\epsilon_0)$ where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2$ and is called the permittivity of free space. The law can be rewritten as

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \hat{u}_r$$

where $\hat{u}_r$ is a unit vector directed from q_2 to q_1 and $\vec{F}_{12}$ is the force exerted on q_1 by q_2 . If the charges are of opposite sign, the force between q_1 and q_2 will be attractive. The force exerted on q_2 by q_1 is directed opposite to $\vec{F}_{12}$ and we have $\vec{F}_{21} = -\vec{F}_{12}$. When more than two point charges are present, the net force on any one charge is the linear vector sum of the electrostatic forces individually exerted by each of the other charges. If three charges are present, the force on q_1 is

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}^2} \hat{u}_2 + \frac{q_1 q_3}{r_{13}^2} \hat{u}_3 \right]$$

where $\hat{u}_3 = \frac{\vec{r}_{31}}{|\vec{r}_{31}|}$ and $\hat{u}_2 = \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$

Charge is Quantized

The magnitude of the smallest charge ever found is denoted by e , called the charge quantum and its value is $1.6 \times 10^{-19} \text{ C}$. All other charges are integer multiples of e . The proton has a charge $+e$ and the electron $-e$.

Conservation of Charge

The total charge in a system remains constant, i.e. there may be a transfer of charge from one object to another but charges cannot be created or destroyed. For example, when an electron ($-e$) and a positron ($+e$) are brought close to each other, the two particles may simply disappear, converting all their rest mass into energy ($E = mc^2$) and the process is known as annihilation [$e^- + e^+ \rightarrow \gamma + \gamma$]. The net charge is zero both before and after the event so that charge is conserved.

The Test Charge

In order to find the field at a given point, we assume an imaginary charge of negligible magnitude at that point. This is supposed to be non-disturbing to the set-up of other charges in the vicinity.

An Electric Field

To define the electric field operationally, we place a small test charge (q_0) assumed to be positive for convenience at the point in space that is to be examined, and measure the electric force that acts on this test charge. The electric field $\vec{E}$ is defined as $\vec{E} = \vec{F}/q_0$. Here, $\vec{E}$ is a vector because $\vec{F}$ is a vector and q_0 is a scalar. The direction of $\vec{E}$ is the direction of $\vec{F}$ and its units are N/C . $\vec{E}$ is also known as electric field intensity or electric field strength.

Electric Intensity due to a Point Charge

The electric field intensity at a distance r from a point charge q is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

where $\hat{r}$ is a unit vector.

If q is positive, E is positive and $\vec{E}$ is directed radially outward from q ; for negative q , E is negative and $\vec{E}$ is directed radially inward.

To Find $\vec{E}$ for a Group of Point Charges

(a) Calculate $\vec{E}$ due to each charge at the given point as if it were the only charge present.

(b) Add these separately calculated fields vectorially to find the resultant field $\vec{E}$ at that point.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots = \sum \vec{E}_n \quad n = 1, 2, 3$$

This is known as the principle of superposition.

If this charge distribution is continuous, the field it sets up at any point can be computed by dividing the charge into infinitesimal elements dq . The field $d\vec{E}$ due to each element at the point in question is then calculated, treating the elements as point charges. The magnitude of $d\vec{E}$ is given by

$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

and the resultant field at that point is found by integrating dE over the total charge distribution:

$$E = \int dE$$

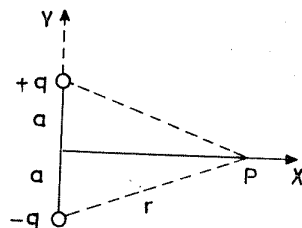
Electric Field due to a Dipole

(a) At a point on the perpendicular bisector at a distance r

$$E = \frac{1}{4\pi\epsilon_0} \frac{(2a) \cdot q}{r^3}$$

The direction of the field is from the positive charge to the negative charge of the dipole. If the axis of the dipole is the Y-axis and the perpendicular bisector is the X-axis, we can write $\vec{E}$ at P in the vector form

$$\vec{E} = -\frac{1}{4\pi\epsilon_0} \frac{(2a) \cdot q}{r^3} \hat{j} \quad [\text{Take } r \gg a]$$



(b) At a point on the axis of the dipole at a distance r from the centre of the dipole (take $r \gg a$)

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{r^3}$$

where p is equal to $(2aq)$, called the electric dipole moment. In vector notation,

$$\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\vec{p}}{r^3}$$

This means that the direction of the electric dipole moment vector is same as that of the electric field vector.

Electric Field on the Axis of a Ring of Charge

If a is the radius of a ring of charge q uniformly distributed over it, then the

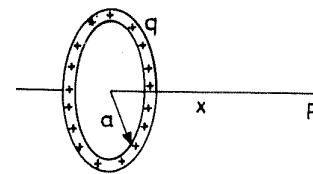
electric field at a point P, distant x from the centre of the ring is

$$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{(a^2 + x^2)^{3/2}}$$

If $x \gg a$, we have

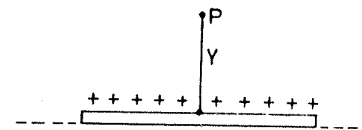
$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

(at a large distance compared to radius of the ring, the ring behaves as a point charge)



Electric Field at a Point due to an Infinite Line of Charge

Uniform linear charge density = λ C/m. y is the distance of point P from the line of charge. Then, the electric field intensity at point P is



$$E = \frac{\lambda}{2\pi\epsilon_0 y}$$

A point charge in an electric field experiences a force given by $\vec{F} = \vec{E}q$ and produces an acceleration

$$\vec{a} = \frac{\vec{E}q}{m}$$

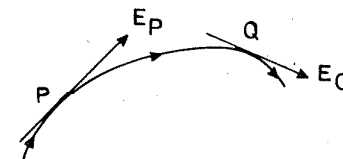
A Dipole in an Electric Field

An electric dipole placed in an external electric field $\vec{E}$ experiences a torque tending to align it with the field.

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{where } \vec{p} \text{ is the dipole moment}$$

Field Lines

These are imaginary lines drawn in such a way that its direction at any point, i.e. the direction of its tangent, is the same as the direction of the field at that point.



Electric Potential

The electric field around a charged particle can be described not only by the vector $\vec{E}$ but also by a scalar quantity called the electric potential, V . These two quantities are intimately related. To find the electric potential difference between two points A and B in an electric field, we move a test charge q_0 from A to B, always keeping it in equilibrium, and we measure the work W_{AB} that must be done by the agent in moving the charge. The electric potential difference is defined from

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

The work W_{AB} may be (i) positive (ii) negative or (iii) zero depending upon whether the electric potential at B is (i) higher, (ii) lower, or (iii) the same as the electric potential at A. If point A is chosen to be at a large distance (ideally infinite) from all charges, the potential at A is arbitrarily taken as zero. Putting $V_A = 0$, we have

$$V = \frac{W}{q_0}$$

where W is the work that an external agent must do to move the test charge q_0 from infinity to that point.

The SI unit of potential difference is joule/coulomb and

$$1 \text{ V} = \frac{1 \text{ J}}{\text{C}}$$

Both W_{AB} and $V_B - V_A$ are independent of path as shown by I and II in the figure above.

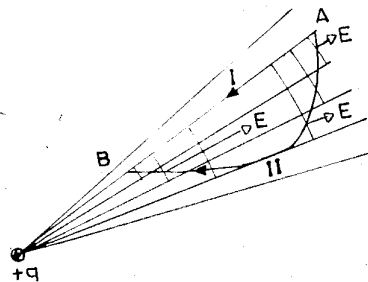
$$W_{AB} = F \times d = q_0 E \cdot d \quad (\text{d is the displacement of the charge } q_0 \text{ in a field } E)$$

$$\text{or} \quad V_B - V_A = \frac{W_{AB}}{q_0} = E \cdot d \quad (\text{This is a relation for a special case})$$

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{l} = -q_0 \int_A^B \vec{E} \cdot d\vec{l} \quad (\vec{F} = -q\vec{E})$$

$$V = - \int_{\infty}^B \vec{E} \cdot d\vec{l}$$

$$\text{Potential at a distance } r \text{ from a point charge } Q = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



Potential due to a group of point charges is given by

$$V = \sum_n V_n = \frac{1}{4\pi\epsilon_0} \sum_n \frac{q_n}{r_n}$$

where q_n is the value of the n th charge and r_n is the distance of this charge from the point in question.

For a continuous distribution of charge,

$$V = \int dV = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

Electric Potential Energy

The potential energy of a system ($q_1 + q_2$) is given as (q_1 and q_2 are separated by a distance r_{12})

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}}$$

Calculation of $\vec{E}$ from V

$E = - \left(\frac{dV}{dr} \right)$ and the three components of $\vec{E}$ can be written as

$$E_x = - \frac{\partial V}{\partial x}; E_y = - \frac{\partial V}{\partial y}; E_z = - \frac{\partial V}{\partial z}$$

Thus, if V is known for all points in space, that is, if the function $V(x, y, z)$ is known, the components of $\vec{E}$ and thus $\vec{E}$ itself can be found by taking its derivative.

ILLUSTRATIONS

1. The original experiments on the scattering of alpha particles from a gold foil were done with particles of speed 1.78×10^7 m/s. Some of the alpha particles were scattered directly backward. It is assumed that the gold nucleus and the alpha particles have spherically symmetric charge distributions of radius R . Calculate the maximum value of R on the assumption that the nuclei do not "touch" even in backward scattering. Neglect the motion of the gold nucleus which is 50 times as heavy as the helium nucleus. Find also the force acting on and the acceleration of the alpha particle at the point of closest approach. (Charge of gold nucleus = $+79e$, mass of alpha particle = 6.64×10^{-27} kg)

Solution

Treat the two spherical charge distributions as point charges concentrated at the centres of the distributions. Thus, we can calculate the distance of closest approach.

Potential in the electric field of the gold nucleus is

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{79e}{r} \right)$$

The potential energy of the helium nucleus (charge $+2e$) at a distance r from the gold nucleus is obtained by multiplying the potential (the potential energy per unit charge) by the charge:

$$\begin{aligned} E_p &= V(2e) = \frac{1}{4\pi\epsilon_0} \left(\frac{79e}{r} \right) \times 2e \\ &= 8.99 \times 10^9 \times \frac{158}{r} \times (1.6 \times 10^{-19})^2 \\ &= \frac{3.64 \times 10^{-26}}{r} \text{ J.m} \end{aligned}$$

It starts its approach to the gold nucleus from a very large distance r with negligible potential energy and kinetic energy equal to

$$\begin{aligned} E_k &= \frac{1}{2} mV^2 = \frac{1}{2} \times (6.64 \times 10^{-27} \text{ kg}) (1.78 \times 10^7 \text{ m/s})^2 \\ &= 1.05 \times 10^{-12} \text{ J} \end{aligned}$$

As it approaches the gold nucleus, the repulsive electric force slows it down; it loses kinetic energy and gains potential energy. It comes to rest at such distance r that all the kinetic energy has been converted to potential energy. Hence,

$$\frac{3.64 \times 10^{-26}}{r} \text{ J.m} = 1.05 \times 10^{-12} \text{ J} \Rightarrow r = 3.48 \times 10^{-14} \text{ m}$$

The radius R assumed for the nuclei must be less than half of this distance between the centres if the nuclei are not to "touch".

$$\therefore R < 1.74 \times 10^{-14} \text{ m}$$

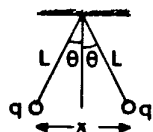
At the distance of closest approach, the force acting between the charges is

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{(79e)(2e)}{r^2} = \frac{3.64 \times 10^{-26} \text{ N.m}^2}{(3.48 \times 10^{-14} \text{ m})^2} = 30.1 \text{ N}$$

The acceleration of the alpha particle when acted by this relatively enormous force is

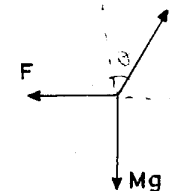
$$a = \frac{F}{m} = \frac{30.1 \text{ N}}{6.64 \times 10^{-27} \text{ kg}} = 4.53 \times 10^{27} \text{ m/s}^2$$

2. Two similar balls of mass m are hung from silk threads of length L and carry similar charges q as shown in the figure. (Assume that θ is small.) The balls start losing charge at the rate of dq/dt . Find an expression for the instantaneous relative velocity with which the balls approach initially.



Solution

Consider one of the balls which is in equilibrium under the action of forces due to gravity, mg , tension in the string T , and Coulomb's repulsive force F . The free-body diagram of the ball is shown in the figure. The balancing force equations are



$$\left. \begin{aligned} T \sin \theta &= F & (i) \\ T \cos \theta &= mg & (ii) \end{aligned} \right\} \Rightarrow \tan \theta = \frac{F}{mg}$$

As θ is taken to be small,

$$F = mg\theta = mg \cdot \frac{x}{2L}$$

$$\text{Coulomb's force of repulsion} = \frac{q^2}{4\pi\epsilon_0 x^2}$$

Therefore,

$$\frac{q^2}{4\pi\epsilon_0 x^2} = mg \cdot \frac{x}{2L} \Rightarrow x^3 = \frac{2Lq^2}{4\pi\epsilon_0 mg} = \frac{Lq^2}{2\pi\epsilon_0 mg}$$

$$\text{or } x = \left(\frac{Lq^2}{2\pi\epsilon_0 mg} \right)^{1/3} \quad (iii)$$

The instantaneous velocity V , at the instant when ball starts losing charge at the rate dq/dt , is

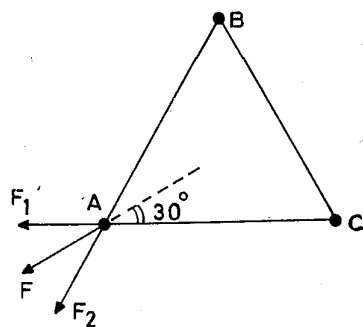
$$V = \frac{dx}{dt} = \frac{dx}{dq} \cdot \frac{dq}{dt}$$

Now differentiating (iii) with respect to q , we have

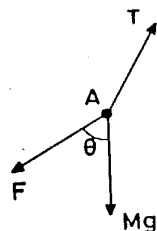
$$\begin{aligned} \frac{dx}{dt} &= \frac{2}{3} q^{-1/3} \left(\frac{L}{2\pi\epsilon_0 mg} \right)^{1/3} \frac{dq}{dt} \\ &= \frac{2}{3q^{1/3}} \left(\frac{L}{2\pi\epsilon_0 mg} \right)^{1/3} \frac{dq}{dt} \\ &= \frac{2}{3q} \left(\frac{Lq^2}{2\pi\epsilon_0 mg} \right)^{1/3} \frac{dq}{dt} = \frac{2x}{3q} \cdot \frac{dq}{dt} \end{aligned}$$

$$\therefore V = \frac{dx}{dt} = \frac{2x}{3q} \cdot \frac{dq}{dt}$$

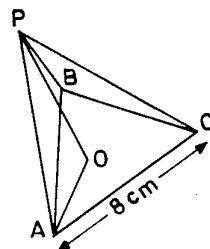
3. Three small balls, each of mass 20 g, are suspended separately from a common point P by silk threads, each 1.0 m long. The balls are identically charged and hang at the corners of an equilateral triangle of side 8 cm. What is the charge on each ball?



(a)

Free body diagram
of a ball (A)

(b)



(c)

Solution

The repulsive forces on the ball A are shown in Fig. (a). F_1 is the force on A due to C and F_2 is the force on A due to B. The resultant force on A is shown by F .

$$F = F_1 \cos 30 + F_2 \cos 30 \quad (1)$$

$$F_1 = F_2 = \frac{q^2}{4\pi\epsilon_0 x^2} \quad (2)$$

where x is the side of equilateral triangle = 8 cm.

Figure (b) shows the free-body diagram of ball A which is in equilibrium under the joint action of three forces: (i) the Coulomb repulsive force F , the tension in the string T and the weight Mg . These three forces are in the same plane.

The force equations are

$$T \sin \theta = F \quad (3)$$

$$\text{and } T \cos \theta = Mg \quad (4)$$

$$\therefore \tan \theta = \frac{F}{Mg} \text{ and so } F = Mg \tan \theta$$

From the geometry of Fig. (c),

$$\begin{aligned} OA &= \frac{x}{2} \sec 30 = \frac{x}{2} \cdot \frac{2}{\sqrt{3}} \\ &= \frac{x}{\sqrt{3}} \text{ and } \sin \theta = \frac{x}{\sqrt{3}} \end{aligned}$$

Since θ is small, $\tan \theta = \sin \theta$ and, therefore,

$$\begin{aligned} F &= Mg \tan \theta = Mg \sin \theta = F_1 \cos 30 + F_2 \cos 30 \\ &= \frac{q^2}{4\pi\epsilon_0 x^2} \times \frac{\sqrt{3}}{2} \times 2 = \frac{\sqrt{3} q^2}{4\pi\epsilon_0 x^2} \end{aligned}$$

$$\therefore q^2 = \frac{4\pi\epsilon_0 x^2 \cdot Mg \sin \theta}{\sqrt{3}} = \frac{4\pi\epsilon_0 x^2 \cdot Mg}{\sqrt{3}} \times \frac{x}{\sqrt{3}} = \frac{4\pi\epsilon_0 x^3 \cdot Mg}{3}$$

Substituting the values $x = 0.08 \text{ m}$, $M = 20 \times 10^{-3} \text{ kg}$ and $g = 9.8 \text{ m/s}^2$, we get

$$\begin{aligned} q^2 &= \frac{(0.08)^3 \times 20 \times 10^{-3} \times 9.8}{9 \times 10^9 \times 3} = \frac{512 \times 10^{-6} \times 20 \times 10^{-3} \times 9.8}{27 \times 10^9} \\ &= \frac{512 \times 2 \times 98}{27} \times 10^{-18} \text{ C}^2 \\ &= 3716.7 \times 10^{-18} \text{ C}^2 \end{aligned}$$

$$\therefore q = 61 \times 10^{-9} \text{ C} = 6.1 \times 10^{-8} \text{ C}$$

Q4 Two free point charges $+q$ and $+4q$ are placed a distance x apart. A third charge is so placed that all the three charges are in equilibrium. Find the location, magnitude and sign of the third charge.

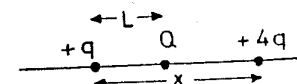
Solution

For equilibrium, it is necessary that the third charge Q be placed on the line joining the other two charges.

Let Q be located at a distance L from $+q$.

$$\text{Force on } Q \text{ due to } +q = \frac{1}{4\pi\epsilon_0} \frac{qQ}{L^2}$$

Similarly,



$$\text{Force on } Q \text{ due to } +4q = \frac{1}{4\pi\epsilon_0} \cdot \frac{4qQ}{(x-L)^2}$$

For the equilibrium of Q , the net force on it should be zero. Hence

$$\frac{1}{4\pi\epsilon_0} \frac{qQ}{L^2} = \frac{1}{4\pi\epsilon_0} \frac{4qQ}{(x-L)^2} \Rightarrow L = \frac{x}{3}$$

Let us consider the equilibrium of $+q$:

$$\text{Force on } +q \text{ due to } Q = \frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L^2}$$

$$\text{Force on } +q \text{ due to } +4q = \frac{1}{4\pi\epsilon_0} \frac{4q^2}{x^2}$$

(This force must be repulsive.)

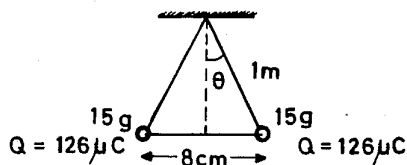
This means that the force on $+q$ due to Q must be attractive and therefore the nature of the charge Q should be negative. Also,

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{qQ}{L^2} = -\frac{1}{4\pi\epsilon_0} \cdot \frac{4q^2}{x^2} \quad \text{or} \quad \frac{qQ}{L^2} = -\frac{4q^2}{x^2}$$

Putting $L = x/3$, we have

$$\frac{qQ}{(x/3)^2} = -\frac{4q^2}{x^2} \quad \text{or} \quad Q = -\frac{4q}{9}$$

5. Two conducting balls, each of mass 15 g and having equal charges on them of $126 \mu\text{C}$, are suspended by two 1-m-long silk threads as shown. The separation between them is 8 cm when they are in equilibrium. When one of the balls is discharged to have half the original charge, find the new equilibrium separation.



Solution

When one of the balls gets discharged, they come in contact and share the charge of the other ball so that each of them now carries a charge of $63 \mu\text{C}$, and the balls mutually repel each other. We know that the separation is given by

$$x = \left(\frac{Q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$$

and when Q is reduced to $Q/2$, we have

$$x' = \left[\frac{(Q/2)^2 L}{2\pi\epsilon_0 mg} \right]^{1/3}$$

and from these two relations, we have,

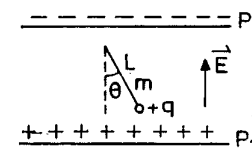
$$\frac{x'}{x} = \frac{1}{(4)^{1/3}} \quad \text{or} \quad x' = \frac{0.08}{(4)^{1/3}} = 0.08 \times 0.63$$

$$= 0.05 \text{ m}$$

$$= 5 \text{ cm}$$

Therefore, the new equilibrium separation is 5 cm.

6. A uniform vertical field $\vec{E}$ is set up between two large parallel plates P_1 and P_2 as shown in the figure. A simple pendulum of length L having a small conducting sphere of mass m and charge $+q$ is oscillating between the plates. Find the period of oscillation of this pendulum.



Solution

A force qE acts on the sphere and thus the net force on the sphere is $(mg - qE)$.

The restoring force is $-(mg - qE) \sin \theta$

Since q is small, $\sin \theta = \theta = x/L$ where x is the displacement.

$\therefore F = -(mg - qE) \cdot x/L$ and acceleration a is given by

$$-\left(g - \frac{qE}{m}\right) \frac{x}{L} = -\omega^2 x$$

where $\omega^2 = \frac{1}{L} \left(g - \frac{qE}{m}\right)$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{\left(g - \frac{qE}{m}\right)}}$$

Consequently, the time period increases.

Note: If the polarity of the charges on the plates is reversed,

$$T = 2\pi \sqrt{\frac{L}{g + qE/m}}$$

Consequently, the time period decreases.

7. Two balls of the same radius and mass are suspended on threads of length L so that their surfaces are in contact. A charge Q_0 applied to the balls repels them so that they make an angle 2α with each other. Now the balls are immersed in kerosene, and the angle of divergence becomes 2θ . Find the density of the material of the balls.

Solution

Each ball is acted upon by (i) gravitational force W , and (ii) Coulombic force F_1 . Their resultant is F as shown in the figure.

$$F_1 = \frac{1}{4\pi\epsilon_0\epsilon_r} \frac{Q^2}{r^2} \quad (i)$$

and $\frac{r}{2} = L \sin \alpha$ or $r = 2L \sin \alpha$ (ii)

(ϵ_r is the permittivity of the medium.)

$$W = \frac{F_1}{\tan \alpha} = \frac{Q^2}{4\pi\epsilon_0\epsilon_a 4L^2 \sin^2 \alpha \tan \alpha} \quad (iii)$$

As the total charge given is Q_0 and it is shared by both the balls,

$$Q = \frac{Q_0}{2}$$

$$W = \frac{Q_0^2}{4\pi\epsilon_0\epsilon_a 4L^2 \sin^2 \alpha \tan \alpha \times 4} \quad (iv)$$

(ϵ_0 is for vacuum and ϵ_a is for air.)

When the balls are immersed in kerosene, each ball is acted upon by the buoyant force, say W_1 .

For a ball in kerosene,

$$W - W_1 = \frac{Q^2}{4\pi\epsilon_0\epsilon_k 4L^2 \sin^2 \theta \tan \theta} \quad (v)$$

(ϵ_k is the permittivity of kerosene.)

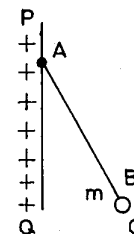
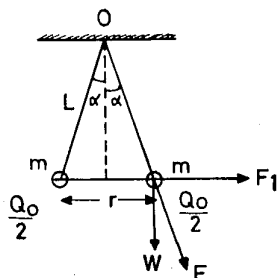
$$W - W_1 = (\rho_1 - \rho_2) Vg \quad (vi)$$

where ρ_1 is the density of the material of the ball and ρ_2 is the density of kerosene and V is the volume of the ball.

$$\frac{W - W_1}{W} = \frac{\rho_1 - \rho_2}{\rho_1} = \frac{\sin^2 \alpha \tan \alpha \epsilon_a}{\sin^2 \theta \tan \theta \epsilon_k} \quad (vii)$$

or $\rho_1 = \rho_2 \frac{\sin^2 \theta \tan \theta \epsilon_k}{\sin^2 \alpha \tan \alpha \epsilon_a} \quad (viii)$

8. An infinite plane of positive charge has a surface charge density of σ C/m². A metallic ball B of mass m and charge $+Q$ is attached to a thread and tied to a point A on the sheet PQ. Find the angle θ which AB makes with the plane PQ.



Solution

Electric field intensity due to an infinite plane of charge having a surface charge density σ is given by

$$E = \frac{\sigma}{2\epsilon_0\epsilon_a}$$

where ϵ_0 is permittivity in vacuum and ϵ_a is permittivity in air. The electrostatic force on B whose charge is Q and which is placed in an electric field E is QE .

$$QE = \frac{Q\sigma}{2\epsilon_0\epsilon_a} \quad \text{and} \quad \tan \theta = \frac{QE}{mg}$$

$$\tan \theta = \frac{Q\sigma}{2\epsilon_0\epsilon_a mg}$$

or $\theta = \tan^{-1} \left(\frac{Q\sigma}{2\epsilon_0\epsilon_a mg} \right)$

9. A copper ball of diameter d is immersed in oil of density ρ_o . What is the charge on the ball if, in a homogeneous electric field E directed vertically upward, it is suspended in the oil? The density of copper is ρ_c .

Solution

Three forces are acting on the copper ball: (i) the force of the electric field, F_e , directed upwards, (ii) the force of gravity mg , directed downwards, and (iii) the force of buoyancy, F_B , acting upward. In equilibrium,

$$mg = F_e + F_B \quad (i)$$

$$mg = \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 \rho_c \cdot g \quad (ii)$$

where ρ_c is the density of copper.

$$F_B = \frac{4}{3} \pi \left(\frac{d}{2} \right)^3 \rho_o \cdot g \quad (iii)$$

where ρ_o is the density of oil.

$$F_e = Q.E$$

where Q is the charge on the ball.

$$\therefore \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 \rho_c \cdot g = Q.E + \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 \rho_o \cdot g$$

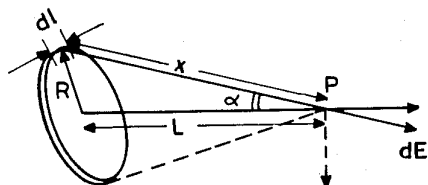
$$\text{or } \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 g (\rho_c - \rho_o) = QE$$

$$\text{or } Q = \frac{\pi d^3 (\rho_c - \rho_o) g}{6E}$$

10. A ring of radius R carries a charge Q , uniformly distributed along its circumference. Find the electric field intensity at a point P distant L from the centre of the ring. Find also the point on the axis where the field will be maximum.

Solution

Consider an element dI of the ring which carries a charge dQ . Let this element be at a distance x from the point P as shown in the figure.



The electric field intensity at P due to this element,

$$dE = \frac{dQ}{4\pi\epsilon_0 x^2}$$

Components of dE along the axis of the ring add up while the normal components cancel due to symmetry of the charge distribution. Hence, the total field

$$E = \int dE \cos \alpha = \int dE \cdot \frac{L}{x} = \int \frac{dQ \cdot L}{4\pi\epsilon_0 x^3}$$

$$\text{as } R^2 + L^2 = x^2$$

we have

$$E = \frac{QL}{4\pi\epsilon_0 (R^2 + L^2)^{3/2}}$$

and in a special case, when $L \gg R$,

$$E = \frac{QL}{4\pi\epsilon_0 L^3} = \frac{Q}{4\pi\epsilon_0 L^2} \quad (\text{the ring behaves like a point charge})$$

Maximum Intensity of Electric Field

$$R = x \sin \alpha, \quad L = x \cos \alpha \Rightarrow x^2 = \frac{R^2}{\sin^2 \alpha}$$

$$E = \frac{Q \cos \alpha \sin^2 \alpha}{4\pi\epsilon_0 R^2}$$

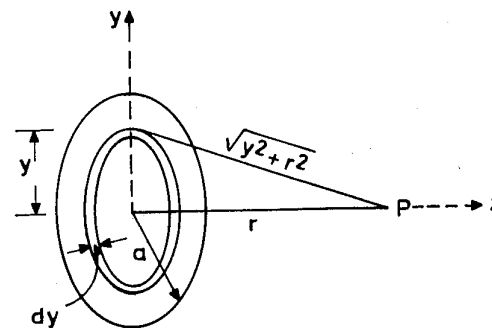
Differentiating E with respect to α , we get

$$\frac{dE}{d\alpha} = \frac{Q}{4\pi\epsilon_0 R^2} (\cos^2 \alpha \cdot 2 \sin \alpha - \sin^3 \alpha)$$

and for a maximum, $dE/d\alpha = 0$ or $\tan^2 \alpha = 2$. Therefore, the maximum intensity is at a point distant L from the centre of the ring, given by

$$L = \frac{R}{\tan \alpha} = \frac{R}{\sqrt{2}}$$

* 11. Find the electric potential for points on the axis of a uniformly charged circular disc whose surface charge density is σ .



Solution

Consider a charge element dq consisting of a flat circular strip of radius y and width dy . We have

$$dq = \sigma(2\pi y)dy$$

where $(2\pi y)dy$ is the area of the strip. All parts of this element are at the same distance $\sqrt{y^2 + x^2}$ from the axial point P . The potential at P due to this element is

$$dV = \frac{1}{4\pi\epsilon_0} \cdot \frac{\alpha \cdot 2\pi y \cdot dy}{\sqrt{y^2 + r^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0} \int_0^a (y^2 + r^2)^{-1/2} y \cdot dy = \frac{\sigma}{2\epsilon_0} (\sqrt{a^2 + r^2} - r)$$

12. A charged particle of mass m and charge Q falls between two vertical plates at an equal distance from them at a constant speed v . A potential difference V is applied between the plates. Find the time required for the particle to reach one of the plates. Find the vertical distance L that will be covered during this time interval. The separation between the plates is d .

Solution

In the absence of an electric field, Stokes' law gives

$$F = mg = 6\pi\eta rv_1$$

where η is the coefficient of viscosity, r is the radius of the particle falling down with a velocity v_1 .

As the particle is moving with a constant speed, it means that the net force acting on the particle is zero.

When the electric field E exists, the force on the charged particle is QE . This particle is accelerated due to this force but the drag force of the medium balances it. Hence,

$$QE = mg + 6\pi\eta rv_2 \quad (2)$$

The resultant of these velocities v_1 and v_2 is directed at an angle α , and

$$\tan \alpha = \frac{v_2}{v_1} = \frac{QE}{mg} = \frac{Q}{mg} \cdot \frac{V}{d}$$

Also,

$$\tan \alpha = \frac{d/2}{L} = \frac{d}{2L}$$

and therefore

$$\frac{v_2}{v_1} = \frac{d}{2L}$$

$$\therefore L = \frac{v_1 d}{2v_2} = \frac{1}{2} \frac{mgd}{QE} = \frac{1}{2} \frac{mgd^2}{QV} \quad (3)$$

Further,

$$v_2 = \frac{v_1 d}{2L}$$

and the required time t can be obtained from the formula

$$t = \frac{d}{2v_2} \quad \text{or} \quad t = \frac{L}{v_1} \quad (4)$$

13. Two fixed charges $-2Q$ and Q are located at points $(-3a, 0)$ and $(+3a, 0)$ respectively in the X-Y plane.

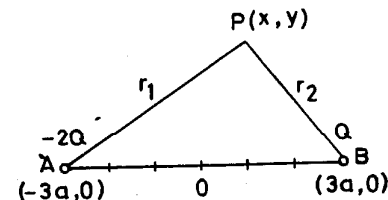
(a) Show that all the points in X-Y plane, where the electric potential due to the two charges is zero, lie on a circle. Find the radius and location of its centre.

(b) Give the expression for the potential $V(x)$ at a general point on the X-axis and sketch the function $V(x)$ on the entire X-axis.

(c) If a particle of charge $+q$ starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find the speed when it does so.

Solution

Let A and B be the two points $(-3a, 0)$ and $(3a, 0)$ where charges $-2Q$ and Q are located. If $P(x, y)$ be any point in the plane where the potential is zero, we have,



$$V_P = \frac{1}{4\pi\epsilon_0} \cdot \frac{(-2Q)}{r_1} + \frac{1}{4\pi\epsilon_0} \cdot \frac{(Q)}{r_2} = 0 \Rightarrow r_1 = 2r_2$$

Now,

$$r_1 = [(x - (-3a))^2 + (y - 0)^2]^{1/2}$$

$$\text{and } r_2 = [(x - (3a))^2 + (y - 0)^2]^{1/2}$$

The locus of all points (x, y) where the potential is zero is given by

$$[(x + 3a)^2 + y^2]^{1/2} = 2[(x - 3a)^2 + y^2]^{1/2}$$

$$\text{or } x^2 + 6ax + 9a^2 + y^2 = 4(x^2 - 6ax + 9a^2 + y^2)$$

$$\text{or } 3x^2 + 3y^2 + 27a^2 - 30ax = 0$$

$$\text{or } x^2 + y^2 - 10ax + 9a^2 = 0$$

$$\text{or } (x - 5a)^2 + (y - 0)^2 = (4a)^2 \quad (A)$$

This is the equation of a circle of radius $4a$ and its centre is at $(5a, 0)$. Hence, all points in the X-Y plane, where the electric potential due to the two charges is zero, lie on a circle of radius $4a$ and the coordinates of its centre are $(5a, 0)$.

(b) Let $(X, 0)$ be the general point on the X-axis. Its distance from the charges $-2Q$ and Q are then $(X + 3a)$ and $(X - 3a)$ respectively. Hence, the potential at this point is

$$V(X) = \frac{1}{4\pi\epsilon_0} \left(\frac{-2Q}{X+3a} + \frac{Q}{X-3a} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{X-3a} - \frac{2}{X+3a} \right) \quad (B)$$

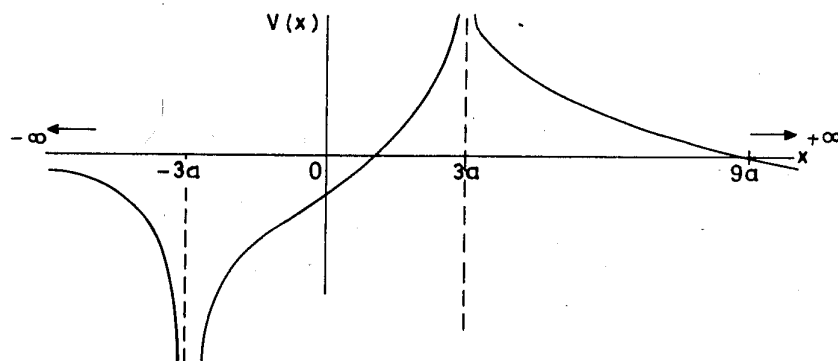
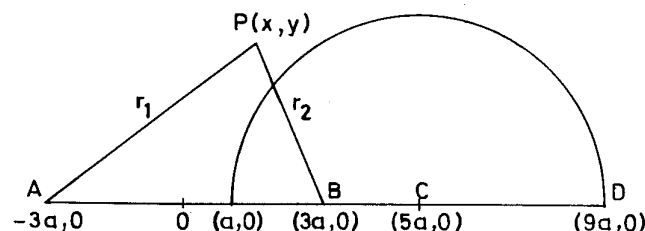
Potential function $V(x)$ vs x :

(i) The circle of zero potential cuts the X -axis at $(a, 0)$ and $(9a, 0)$ (obtained from part (a)). Hence $V(x) = 0$ for $x = a$ and $x = 9a$.

(ii) From (B), we get $V(x) \rightarrow \infty$ as $x \rightarrow 3a$ and $V(x) \rightarrow -\infty$ at $x = -3a$.

(iii) It is also clear that $V(x) \rightarrow 0$ as $x \rightarrow \pm \infty$

(iv) In general $V(x)$ varies as $1/x$.



(c) At the centre of the circle, the charge $+q$ experiences a net force directed to the right. It, therefore, moves away from the centre towards the boundary of the circle. The potential energy of this charge when at the centre is obtained by calculating the potential at $(5a, 0)$,

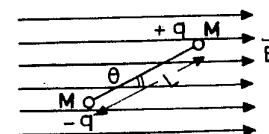
$$V_c = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{2a} + \frac{-2Q}{8a} \right] = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{4a}$$

or
$$U_c = \frac{1}{4\pi\epsilon_0} \frac{Qq}{4a}$$

At the point $(9a, 0)$, the potential being zero, the potential energy is also zero and it has only kinetic energy at that moment. Therefore,

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{4a} \Rightarrow v = \sqrt{\frac{Qq}{8\pi\epsilon_0 ma}}$$

14. A point particle of mass M is attached to one end of a massless rigid rod made of nonconducting material. Another particle of the same mass is attached to the other end of the rod.



The two particles carry charges of $+q$ and $-q$ respectively. The arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ with the field direction. Find the time needed for the rod to become parallel to the field.

Solution

The situation is similar to an electric dipole in an electric field. Each charge experiences a force qE . Restoring torque $\tau = qE \cdot L \cdot \sin \theta = qEL\theta$ (as θ is small). This shows that torque is proportional to angular displacement, and hence the motion is simple harmonic.

$$\therefore \text{Time period } T = 2\pi \sqrt{\frac{I}{qEL}}$$

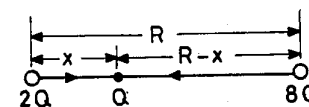
$$\text{where } I = 2M \left(\frac{L}{2} \right)^2 = \frac{ML^2}{2}$$

$$\begin{aligned} \text{Minimum time required} &= \frac{T}{4} = \frac{2\pi}{4} \sqrt{\frac{ML^2/2}{qEL}} \\ &= \frac{\pi}{2} \sqrt{\frac{ML}{2qE}} \end{aligned}$$

15. Three charges Q , $2Q$ and $8Q$ are to be placed on a line whose length is R metres. Find the positions where these charges should be placed such that the potential energy of the system is minimum. In this situation, find the electric field at the position of the charge Q due to the other two.

Solution

For minimum potential energy, the largest charge should be farthest and therefore, $2Q$ and $8Q$ should occupy the end positions. Let Q be placed at a distance x from $2Q$. Then, the potential energy of configuration of these charges is



$$4\pi\epsilon_0 U = \frac{2Q^2}{x} + \frac{8Q^2}{R-x} + \frac{16Q^2}{R}$$

For a minimum, $dU/dx = 0$ and so,

$$4\pi\epsilon_0 \frac{dU}{dx} = \frac{-2Q^2}{x^2} + \frac{8Q^2}{(R-x)^2} + 0 = 0$$

$$\text{or } \frac{2Q^2}{x^2} = \frac{8Q^2}{(R-x)^2} \Rightarrow 4x^2 = (R-x)^2$$

$$\text{or } 2x = R - x$$

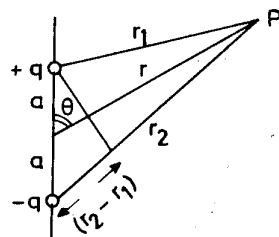
$$\text{or } x = \frac{R}{3}$$

Therefore, the lowest charge should be placed in the middle at a distance $R/3$ from charge $2Q$ or $2R/3$ from charge $8Q$.

Electric Field at Q

$$E_Q = \frac{1}{4\pi\epsilon_0} \left[\frac{2Q^2}{x^2} - \frac{8Q^2}{(R-x)^2} \right] = \frac{2Q^2}{4\pi\epsilon_0} \left(\frac{9}{R^2} - \frac{9}{R^2} \right) = 0$$

16. A dipole consists of two charges of opposite sign $\pm q$, separated by a distance $2a$. Find the potential at a point P , located at a distance r from the centre of the dipole. From the expression of potential at P , calculate the electric field at that point.



Solution

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_1} - \frac{q}{r_2} \right)$$

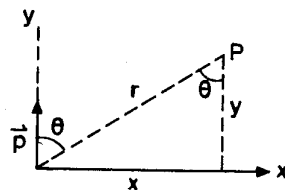
$$\text{or } V = \frac{q}{4\pi\epsilon_0} \frac{r_2 - r_1}{r_1 r_2}$$

If $r \gg 2a$, then from the figure, we can have the approximate relations: $r_2 - r_1 = 2a \cos \theta$ and $r_1 r_2 = r^2$. Therefore the potential becomes

$$V = \frac{q}{4\pi\epsilon_0} \frac{2a \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2}$$

where $2aq = p$, the electric dipole moment.

Calculation of Electric Field: From considerations of symmetry, $\vec{E}$, for points in the plane of the figure here, lies in this plane. Thus, it can be expressed in terms of its components E_x and E_y ; $E_z = 0$.



$$r = (x^2 + y^2)^{1/2} \text{ and } \cos \theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

$$\therefore V = \frac{p}{4\pi\epsilon_0} \cdot \frac{y}{(x^2 + y^2)^{3/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{p}{4\pi\epsilon_0} \frac{(x^2 + y^2)^{3/2} - y^{3/2}(x^2 + y^2)^{1/2}(2y)}{(x^2 + y^2)^3}$$

$$= -\frac{p}{4\pi\epsilon_0} \cdot \frac{x^2 - 2y^2}{(x^2 + y^2)^{5/2}}$$

If the point P lies on the Y -axis, $x = 0$, and

$$E_y = \frac{2p}{4\pi\epsilon_0} \cdot \frac{1}{y^3}$$

Putting $y = 0$,

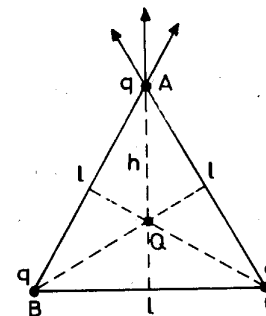
$$E_y = -\frac{p}{4\pi\epsilon_0} \frac{1}{x^3}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{py}{4\pi\epsilon_0} (-3/2)(x^2 + y^2)^{-5/2}(2x)$$

$$= \frac{3p}{4\pi\epsilon_0} \frac{xy}{(x^2 + y^2)^{5/2}}$$

E_x vanishes both on the dipole axis ($x = 0$) and in the median plane ($y = 0$).

17. Three charges, each of value q , are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle. If $Q = -q$, will the charges at the corners move towards the centre or fly away from it? For what value of Q will the charge remain stationary? In this situation how much work is done in removing the charges to infinity?



Solution

Let l be the length of each side.

$$\text{Height } h = \frac{l\sqrt{3}}{2}$$

$$\text{or } 2h = \frac{l\sqrt{3} \times \sqrt{3}}{\sqrt{3}} \Rightarrow \frac{2h}{3} = \frac{l}{\sqrt{3}}$$

Total force on the charge at A in the upward direction

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{qQ}{l^2/3} + \frac{2q^2(\sqrt{3}/2)}{l^2} \right]$$

(i) If $Q = -q$,

$$F = \frac{1}{4\pi\epsilon_0} \left(\frac{-3q^2}{l^2} + \frac{\sqrt{3}q^2}{l^2} \right)$$

This force is in the downward direction; the particles converge to the centre.

(ii) For the charges to remain stationary, the resultant force on them should be zero. Therefore,

$$\frac{1}{4\pi\epsilon_0} \times \frac{-Qq \times 3}{l^2} = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{3}q^2}{l^2} \text{ or } Q = -\frac{q}{\sqrt{3}}$$

This is the required value of Q to keep all charges in equilibrium.

(iii) To calculate the required work in moving these charges to infinity, let us calculate the potential energy of the configuration

$$\frac{1}{4\pi\epsilon_0} \left(\frac{3q^2}{l} + \frac{3qQ}{l/\sqrt{3}} \right)$$

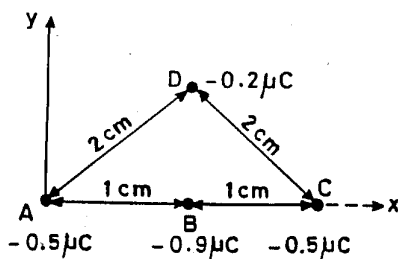
Substituting $Q = -q/\sqrt{3}$, we get

$$\frac{1}{4\pi\epsilon_0} \left(\frac{3q^2}{l} + \frac{3q \times (-q)}{\sqrt{3}} \times \frac{\sqrt{3}}{l} \right) = \frac{1}{4\pi\epsilon_0} \left(\frac{3q^2}{l} - \frac{3q^2}{l} \right) = 0$$

As the potential energy of configuration is zero, work done in removing all the charges to infinity is zero.

EXERCISES

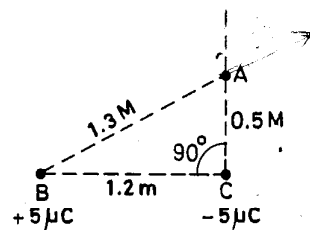
1. Three charges A, B and C are placed in a line with a separation of 1 cm between them and a fourth charge D is placed at a distance of 2 cm from A and C as shown in the figure. The value of charges at A, B, C and D are -0.5 , -0.9 , -0.5 and $-0.2 \mu\text{C}$ respectively. The net force experienced by D is



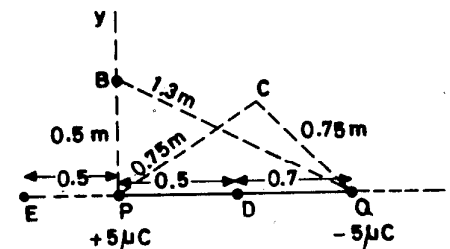
- (a) $9.29 \hat{i} \text{ N}$
(b) $9.29 \hat{j} \text{ N}$
(c) $5.39 \hat{j} \text{ N}$

2. Charges of $+5 \mu\text{C}$ and $-5 \mu\text{C}$ are placed at B and C respectively. $BC = 1.2 \text{ m}$. The electric intensity at a point A due to these charges is
- (a) $172 \times 10^3 \text{ N/C}$, making an angle of 15° with the line AC
(b) $17.2 \times 10^3 \text{ N/C}$, making an angle of 7.5° with the line AC
(c) $172 \times 10^3 \text{ N/C}$, making an angle of 8.1° with the line AC
(d) $17.2 \times 10^3 \text{ N/C}$, making an angle of 8.1° with the line AC

(d) $3.90 \hat{j} \text{ N}$

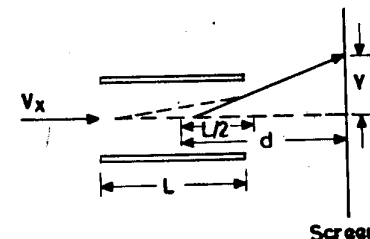


3. In the field of two charges at P and Q of values $+5 \mu\text{C}$ and $-5 \mu\text{C}$ respectively, the electric intensity at points B, C, D and E (shown in the figure) is calculated. The correct answer is



- (a) $E_B = 172 \times 10^3 \text{ N/C}$ at angle of 8.1° with the $+y$ direction, $E_C = 128 \times 10^3 \hat{i} \text{ N/C}$, $E_D = 272 \times 10^3 \hat{i} \text{ N/C}$, $E_E = -164 \hat{i} \text{ N/C}$
(b) $E_B = 172 \times 10^3 \hat{i} \text{ N/C}$; $E_C = 164 \times 10^3 \hat{i} \text{ N/C}$, $E_D = 180 \hat{i} \text{ N/C}$, $E_E = 164 \hat{i} \text{ N/C}$
(c) $E_B = 172 \times 10^3 \hat{j} \text{ N/C}$; $E_C = -128 \hat{i} \text{ N/C}$, $E_D = 180 \hat{i} \text{ N/C}$, $E_E = -164 \hat{i} \text{ N/C}$
(d) $E_B = 172 \times 10^3 \text{ N/C}$ at an angle of 8.1° with the $+y$ direction, $E_C = -128 \times 10^3 \hat{i} \text{ N/C}$, $E_D = -272 \times 10^3 \hat{i} \text{ N/C}$, $E_E = -180 \hat{i} \text{ N/C}$

4. In an electron-ray tube, the speed of the electron is 10^7 m/s , the length of the plates 2 cm and the distance between the centre of the plates and the screen is 30 cm. When a uniform field of intensity 15000 N/C is set up between the plates, the deflection of the electron beam on the screen is (neglect effect of gravity)

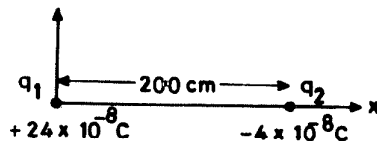


- (a) 5.2 cm
(b) 10.4 cm
(c) 7.8 cm
(d) 15.6 cm
5. In an electron-ray tube, an electron enters an electric field E between the two plates with a velocity V_x (shown in the figure alongside Problem 4) and emerges from the field with a velocity V so as to strike the screen. The separation of the screen from the centre of the plates is d and the length of the plates is L . If the charge of the electron is q , then the deflection Y on the screen is (m is the mass of the electron)

- (a) $\frac{qLD}{mV^2} E$
(b) $\frac{qLD}{mV_x^2} E$
(c) $\frac{qL^2}{mV_x^2} E$
(d) $\frac{2qLD}{mV_x^2}$

6. In Problem 4, with the same electron tube, the effect of gravity is also taken into account. The deflection on account of gravity alone is
- (a) $4.4 \times 10^{-16} \text{ cm}$
(b) $4.4 \times 10^{-10} \text{ cm}$
(c) $4.4 \times 10^{-13} \text{ cm}$
(d) $4.4 \times 10^{-8} \text{ cm}$

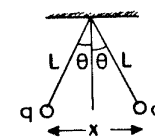
7. A charged particle of mass 0.003 g is held stationary in space by placing it in a downward directed electric field of 48×10^4 N/C. The charge on the particle in μC is
 (a) -6.13×10^{-5} (b) -3.65×10^{-5}
 (c) -12.26×10^{-4} (d) -6.13×10^{-4}
8. The magnitude of the force on a charge of $12 \mu\text{C}$ placed at a point where the potential gradient has a magnitude of 6×10^5 V/m is
 (a) 14.40 N (b) 21.60 N
 (c) 7.20 N (d) 3.60 N
9. In an oil droplet experiment, the horizontal plates are 0.8 cm apart, the radius of the oil droplet is 2.8×10^{-4} cm and the density of oil is 0.92 g/cm^3 . If a single excess electron is attached to the droplet, and the droplet is held stationary in the field, the potential difference between the plates is
 (a) 4.1×10^3 V (b) 410 V
 (c) 41×10^4 V (d) 4.15×10^4 V
- 10.* The experimentally determined value of the electric dipole moment of an HCl molecule is 3.46×10^{-30} C.m. The separation between two fictitious charges $+1e$ and $-1e$ in this molecule is
 (a) 0.0216 m μ (b) 0.2160 m μ
 (c) 2.0000 m μ (d) 0.2600 m μ
11. In Problem 10, the electric field intensity along the axis of the dipole at a distance of 1 m μ from the centre of the dipole is
 (a) 3.11×10^7 V/m (b) 6.23×10^7 V/m
 (c) 1.40×10^7 V/m (d) 24.82×10^7 V/m
12. Two copper spheres, each of mass 1 kg, are separated by 1 m. The number of electrons that each sphere has is
 (a) 2.61×10^{24} (b) 2.61×10^{20}
 (c) 2.61×10^{26} (d) 2.61×10^{30}
13. In the above problem, it is desired that the two spheres attract each other with a force of 10^4 N. The number of electrons that should be removed from one sphere and added to the other is
 (a) 6.6×10^{30} (b) 6.6×10^{10}
 (c) 6.6×10^{12} (d) 6.6×10^{15}
14. In the Bohr model of the hydrogen atom, an electron revolves round a proton in an orbit of radius 5.3×10^{-11} m. The speed of the electron is ($m_e = 9.1 \times 10^{-31}$ kg and $1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N.m}^2/\text{C}^2$)
 (a) 4.4×10^6 m/s (b) 2.2×10^6 m/s
 (c) 2.2×10^5 m/s (d) 4.4×10^5 m/s
- 15.* Two charges are placed on the X-axis such that a charge of $+4\mu\text{C}$ is at $x = 0$ and $-6 \mu\text{C}$ is at $x = 40$ cm. If a third charge is so placed that the net force experienced by it is zero, then its distance as measured from the origin is
 (a) 1.78 m on the negative X-axis (b) 1.78 m on the positive X-axis
 (c) 0.10 m on the negative X-axis (d) 0.18 m on the positive X-axis
- 16.* Two charges $q_1 = 24 \times 10^{-8}$ C and $q_2 = -4 \times 10^{-8}$ C are placed at a separation of 20 cm. The point in the region where the electric field would be zero lies from the charge q_2 at a distance of
 (a) 0.25 m (b) -0.33 m
 (c) -0.25 m (d) 0.33 m



- 17.* A negative point charge of magnitude q is located on the Y-axis at a point $y = +a$ and a positive charge of the same magnitude is located at $y = -a$. A third positive charge of the same magnitude is located at a distance x from the origin. The force on the third charge is

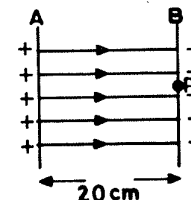
(a) $\frac{q^2 a}{2\pi\epsilon_0 (a^2 + x^2)^{3/2}}$ (b) $\frac{q^2 a}{4\pi\epsilon_0 (a^2 + x^2)^{1/2}}$
 (c) $\frac{q^2 a^2}{2\pi\epsilon_0 (a^2 + x^2)^{3/2}}$ (d) $\frac{q^2}{4\pi\epsilon_0 (a^2 + x^2)^{1/2}}$

18. Two similar conducting balls of mass m are hung from silk threads of length L and carry similar charges q as shown in the figure. Assuming θ to be small, the distance x between the balls is



(a) $\left(\frac{q^2 L}{4\pi\epsilon_0 mg} \right)$ (b) $\left(\frac{q^2 L}{2\pi\epsilon_0 mg} \right)^{1/3}$
 (c) $\left(\frac{q^2 L}{4\pi\epsilon_0 mg} \right)^{3/2}$ (d) $\left(\frac{q^2 L}{4\pi\epsilon_0 mg} \right)^{1/2}$

19. In the Problem 18, if $L = 200$ cm, $m = 10$ g, and $x = 10$ cm, the value of q is
 (a) 27×10^{-16} C (b) 27×10^{-8} C
 (c) 5.2×10^{-6} C (d) 5.2×10^{-8} C
20. In Problem 19, each ball loses charge at the rate of 5×10^{-9} C/s, the initial instantaneous relative speed of the balls is
 (a) 6.4 cm/s (b) 3.2 mm/s
 (c) 6.4 mm/s (d) 3.2 cm/s
21. Two identical conducting spheres, having charges of opposite sign, attract each other with a force of 0.108 N when separated by 0.5 m. The spheres are connected by a conducting wire, which is then removed, and thereafter, they repel each other with a force of 0.036 N. The initial charges on the spheres are
 (a) $\pm 5 \times 10^{-6}$ C and $\mp 15 \times 10^{-6}$ C
 (b) $\pm 1.0 \times 10^{-6}$ C and $\mp 3.0 \times 10^{-6}$ C
 (c) $\pm 2.0 \times 10^{-6}$ C and $\mp 6.0 \times 10^{-6}$ C
 (d) $\pm 0.5 \times 10^{-6}$ C and $\mp 1.5 \times 10^{-6}$ C
22. A uniform electric field of 5000 N/C exists between two charged metal plates in vacuum. An electron is released from rest at point P on the plate B. The time it takes to reach the other plate A is
 (a) 2×10^{-6} s
 (b) 2×10^{-8} s
 (c) 4×10^{-8} s
 (d) 4×10^{-6} s
- 23.* A cubical block of mass m containing a net positive charge Q is placed on a smooth horizontal surface which terminates in a vertical wall. The distance between the wall and the block is d . A horizontal electric field E directed towards the wall is suddenly switched on. Assuming elastic collision (if any), the time period of the resulting oscillation is



(a) $\sqrt{\frac{4md}{QE}}$

(b) $\sqrt{\frac{8md}{QE}}$

(c) $\sqrt{\frac{md}{8QE}}$

(d) $\sqrt{\frac{md}{4QE}}$

24. A particle of mass m and charge q is placed at rest in a uniform electric field $\vec{E}$ as shown and released. The kinetic energy it attains after moving a distance y is

(a) $\frac{1}{2} qEy$

(b) qE^2y

(c) qEy

(d) $\frac{1}{2} m(qEy)$

25. An electric field of 1.5×10^4 N/C exists between two parallel plates of length 2 cm. An electron enters the region between the plates at right angles to the field with a kinetic energy of 2000 eV. The deflection that the electron experiences at the deflecting plates is

(a) 0.34 mm

(b) 0.57 mm

(c) 7.5 mm

(d) 0.75 mm

26. A particle of charge $-q$ and mass m moves in a circular orbit of radius r about a fixed charge $+Q$. The relation between the radius of the orbit r and the time period T is

(a) $r = \frac{Qq}{16\pi^2 \epsilon_0 m} T^2$

(b) $r^3 = \frac{Qq}{16\pi^3 \epsilon_0 m} T^2$

(c) $r^2 = \frac{Qq}{16\pi^3 \epsilon_0 m} T^3$

(d) $r^2 = \frac{Qq}{4\pi \epsilon_0 m} T^3$

27. An electron is projected with an initial speed of 5×10^5 m/s directly toward a proton which is at rest. Initially, the electron is supposed to be at a fairly large distance from the proton. The distance of the electron from the proton when its instantaneous speed becomes twice the initial speed is

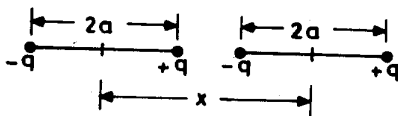
(a) 6.7×10^{-10} m

(b) 6.7×10^{-8} m

(c) 6.7×10^{-9} m

(d) 6.7×10^{-6} m

28. Two dipoles are oriented as shown in the figure, and their centres are separated by x . The force exerted on the left dipole is



(a) $\frac{q^2}{4\pi\epsilon_0} \left[\frac{x^2 + a^2}{(x^2 - a^2)^2} - \frac{1}{x^2} \right]$

(b) $\frac{q^2}{4\pi\epsilon_0} \left[\frac{x^2 + 4a^2}{(x^2 - 4a^2)^2} - \frac{1}{x^2} \right]$

(c) $\frac{q^2}{2\pi\epsilon_0} \left[\frac{x^2 + 4a^2}{(x^2 - 4a^2)^2} - \frac{1}{x^2} \right]$

(d) $\frac{q^2}{2\pi\epsilon_0} \left[\frac{x^2 - 4a^2}{(x^2 + 4a^2)^2} - \frac{1}{x^2} \right]$

29. A point charge q is at a distance d from a conducting plane and is to be moved infinitely away from the plane. The energy required to achieve this is

(a) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{d}$

(b) $\frac{1}{4\pi\epsilon_0} \frac{q}{d}$

(c) $\frac{1}{4\pi\epsilon_0} \frac{q}{2d}$

(d) $\frac{1}{4\pi\epsilon_0} \frac{-q^2}{4d}$

30. An electric dipole of moment $\vec{P}$ is located in a region of constant electric field $\vec{E}$ at an angle θ to the field. The dipole is to be rotated by 180° about an axis perpendicular to $\vec{P}$. The work required to achieve this is

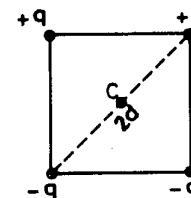
(a) $pE \cos \theta$

(b) $2pE \sin \theta$

(c) $2pE \cos \theta$

(d) $-pE \sin \theta$

- 31.* Four point charges are placed at the vertices of a square whose diagonal is of length $2d$ as shown. A point P is symmetrically located with respect to its vertices at a distance x from its centre C. The electric field at P is



(a) $\frac{qd}{\sqrt{2} \pi \epsilon_0 (d^2 + x^2)^{3/2}}$

(b) $\frac{2qd}{\pi \epsilon_0 (d^2 + x^2)^2}$

(c) $\frac{qd}{4\pi \epsilon_0 (x^2 + d^2)^2}$

(d) $\frac{4qd}{\sqrt{2} \pi \epsilon_0 (x^2 + d^2)^{1/2}}$

32. A semicircular ring of radius 0.5 m is uniformly charged with a total charge of 1.4×10^{-9} C. The electric field intensity at the centre of curvature of this ring is

(a) 100 V/m

(b) 320 V/m

(c) 64 V/m

(d) 32 V/m

33. A point charge Q is located at the centre of a thin ring of radius R with uniformly distributed charge $-Q$. The magnitude of the electric field strength vector at the point lying on the axis of the ring at a distance x from its centre is (assume that $x \gg R$)

(a) $\frac{3QR^2}{4\pi\epsilon_0 x^3} \hat{i}$

(b) $\frac{2QR}{4\pi\epsilon_0 x^2} \hat{i}$

(c) $\frac{3QR^2}{4\pi\epsilon_0 x^4} \hat{i}$

(d) $\frac{3QR^2}{2\pi\epsilon_0 x^3} \hat{i}$

- 34.* A thin uniform rod of length $2a$ carrying a uniformly distributed charge Q is located in vacuum. The magnitude of the electric field as a function of the distance r from the centre of the rod along the rod is

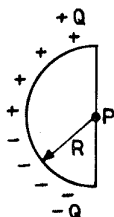
(a) $\frac{Q}{4\pi\epsilon_0 (r^2 - a^2)^2}$

(b) $\frac{Q}{2\pi\epsilon_0 (r^2 - a^2)^2}$

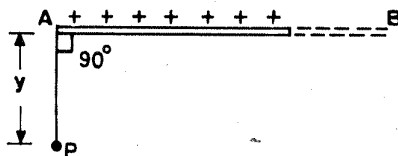
(c) $\frac{Q}{4\pi\epsilon_0 (r^2 - a^2)}$

(d) $\frac{2Q}{\pi\epsilon_0 (r^2 - a^2)}$

- 35.* A thin glass rod is bent into a semicircular shape of radius R . A charge $+Q$ is uniformly distributed along the upper half and a charge $-Q$ is uniformly distributed along the lower half as shown in the figure. The electric field at the centre P , is

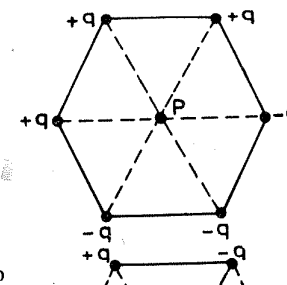


- (a) $\frac{Q}{4\pi\epsilon_0 R^2}$
 (b) $\frac{Q}{2\pi\epsilon_0 R^2}$
 (c) $\frac{Q}{(4\pi\epsilon_0 R)^2}$
 (d) $\frac{Q}{\pi^2 \epsilon_0 R^2}$
- 36.* A "semi-infinite" insulating rod has a linear charge density λ . The electric field at P is
- (a) $\frac{2\lambda^2}{(4\pi\epsilon_0 y)^2}$ along AP
 (b) $\frac{2\lambda^2}{(4\pi\epsilon_0 y)^2}$ at an angle of 45° with AP or 135° with AB
 (c) $\frac{\lambda^2}{(4\pi\epsilon_0 y)^2}$ along AP
 (d) $\frac{2\lambda^2}{(4\pi\epsilon_0 y)^2}$ at an angle of 45° with AP or 45° with AB .

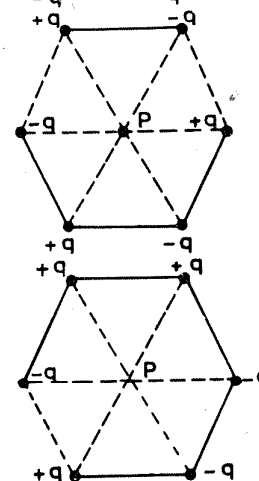


- 37.* Two point charges q_1 and q_2 are placed at a distance of 50 cm from each other in air, and interact with a certain force. Now the same charges are put in an oil whose relative permittivity is 5. If the interacting force between them is still the same, their separation now is
- (a) 16.6 cm (b) 22.4 cm
 (c) 28.4 cm (d) 30.4 cm
38. Two protons have Newtonian attraction equal to F_N and the Coulomb repulsion between them is F_C . The ratio of F_C and F_N is
- (a) 1.25×10^{36} (b) 1.25×10^{30}
 (c) 2.5×10^{40} (d) 2.5×10^{36}
39. Two identically charged metal balls, each of mass 0.5 kg are placed at a certain distance from each other. If their electrostatic energy at this distance is one million times greater than their mutual gravitational energy, the charge on each ball is
- (a) 3.4×10^{-8} C (b) 4.3×10^{-6} C
 (c) 4.3×10^{-8} C (d) 4.3×10^{-9} C
- 40.* A negative charge is placed at the centre of a square, each vertex of which contains a charge of $100 \mu\text{C}$. The resulting force acting on each charge is zero. The magnitude of the negative charge is
- (a) $-9.6 \mu\text{C}$ (b) $-6.9 \mu\text{C}$
 (c) $-0.69 \mu\text{C}$ (d) $-0.96 \mu\text{C}$

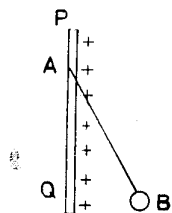
41. Three vertices of a regular polygon of side 3 cm have equal charges of $+q = 1.5 \times 10^{-9}$ C and the remaining three vertices have $-q = 1.5 \times 10^{-9}$ C as shown in the figure. The electric field intensity at the centre of the polygon P is
- (a) 1.5×10^4 V/m
 (b) 6×10^4 V/m
 (c) 3×10^4 V/m



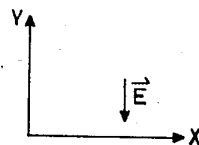
- (d) zero
42. In the above problem, if the charges at the vertices are placed as shown in the figure here, the field at the centre P is
- (a) zero
 (b) 3×10^4 V/m
 (c) 1.5×10^4 V/m
 (d) 6×10^4 V/m
43. In Problem 41, if the charges are now placed as shown in the figure here, the field at P is
- (a) 3×10^4 V/m
 (b) zero
 (c) 6×10^4 V/m
 (d) 1.5×10^4 V/m



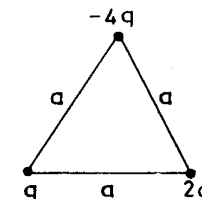
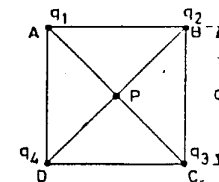
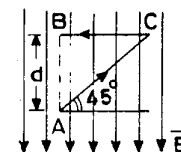
44. Two balls of the same radius and weight are suspended on threads so that their surfaces are in contact. A charge of 4×10^{-7} C is applied to the balls to make them repel each other to an angle of 60° . If the length of the thread from this point of suspension to the centre of the ball is 20 cm, the mass of the ball is
- (a) 1.6×10^{-3} kg (b) 1.6×10^{-4} kg
 (c) 0.016 kg (d) 0.16 kg
- 45.* Two metallic balls of the same radius and weight are suspended on two threads so that their surfaces are in contact. The length of the thread from the point of suspension to the centre of the ball is 10 cm and each ball has a mass of 10 g. The tension in the threads is 0.196 N when the charge is applied to the balls. The charge applied to the balls is
- (a) 0.11×10^{-6} C (b) 4.6×10^{-6} C
 (c) 1.5×10^{-6} C (d) 2.3×10^{-6} C
46. A simple pendulum of length 1 m has a metallic bob of mass 10 g and is suspended between two large metallic horizontal parallel plates. The time period of this pendulum is recorded. Now the upper plate is given a negative charge and an equal positive charge to the lower plate so that a uniform electric field of 50 V/m exists between the plates. The bob of the pendulum is given a positive charge of $500 \mu\text{C}$. The time period of the pendulum is again recorded under the present set-up. The time period in the second case
- (a) increases by 0.32 s (b) decreases by 0.32 s
 (c) increases by 0.64 s (d) decreases by 0.64 s
47. PQ is an infinitely positively charged plane with surface charge density 5×10^{-5} C/m² and B is a ball with a positive charge 5×10^{-9} C and mass 10 g attached to a point A with a thread AB. The angle θ is

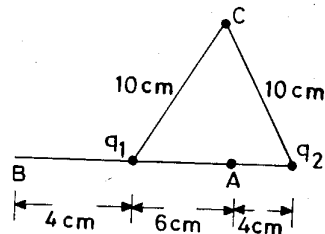


- (a) 13.0° (b) 18.2°
(c) 22.5° (d) 8.2°
48. A copper ball 2 cm in diameter is immersed in a liquid (relative permittivity 5) of density 800 kg/m^3 . The electric field of intensity 400 V/m directed vertically upward is applied and a charge Q is given to the ball so that the ball remains suspended in the field. The value of the charge Q is ($\rho_{\text{Cu}} = 8600 \text{ kg/m}^3$)
(a) $8 \times 10^{-6} \text{ C}$ (b) $4 \times 10^{-6} \text{ C}$
(c) $4 \times 10^{-3} \text{ C}$ (d) $4 \times 10^{-4} \text{ C}$
- 49.* A charged spherical drop of mercury is in equilibrium in a plane horizontal air capacitor and the intensity of the electric field is $6 \times 10^4 \text{ V/m}$. The charge on the drop is $8 \times 10^{-18} \text{ C}$. The radius of the drop is [$\rho_{\text{Air}} = 1.29 \text{ kg/m}^3$ and $\rho_{\text{Hg}} = 13600 \text{ kg/m}^3$]
(a) $0.95 \times 10^{-8} \text{ m}$ (b) $2.7 \times 10^{-10} \text{ m}$
(c) $2.7 \times 10^{-8} \text{ m}$ (d) $0.95 \times 10^{-6} \text{ m}$
50. A ring made of wire with a radius of 10 cm is charged negatively and carries a charge of $-5 \times 10^{-9} \text{ C}$. The distance from the centre to the point on the axis of the ring where the intensity of the electric field is maximum is
(a) 0.71 m (b) $7.1 \times 10^{-2} \text{ m}$
(c) $7.1 \times 10^{-3} \text{ m}$ (d) $1.7 \times 10^{-2} \text{ m}$
51. A particle of mass m and charge $-q$ is projected with a horizontal speed V into an electric field of intensity E directed downward. The following statements are given. Mark the one which is incorrect.
(a) the equation of the trajectory is given by $y = \frac{1}{2} \left(\frac{E \cdot q}{mV^2} \right) x^2$
(b) the horizontal and vertical components of acceleration are $a_x = 0$, $a_y = Eq/m$
(c) the horizontal and vertical displacements, x and y , after a time interval t are $x = Vt$ and $y = \frac{1}{2} a_y \cdot t^2$
(d) the kinetic energy attained after moving a distance y is qEy .
52. An electric dipole consists of two opposite charges of magnitude $5 \mu\text{C}$ separated by a distance of 4.0 cm. The dipole is placed in an external field of $4.0 \times 10^5 \text{ N/C}$. The maximum torque experienced by the dipole is
(a) $8 \times 10^{-2} \text{ N.m}$ (b) $8 \times 10^{-3} \text{ N.m}$
(c) 80 N.m (d) 0.8 N.m
53. In Problem 52, the dipole is aligned along the external field ($\theta = 0$). In order to turn the dipole end for end, the work that an external agent must do is



- (a) 16 J (b) 160 J
(c) 0.16 J (d) 1.6 J
54. A charge Q is fixed at each of two opposite corners of a square. A charge q is placed at each of the other two corners. If the resultant electrical force on Q is zero, then Q and q are related as
(a) $Q = \sqrt{2} q$ (b) $Q = -2\sqrt{2} q$
(c) $Q = \sqrt{2} q^2$ (d) $Q = -2\sqrt{2} q^2$
55. Two equally charged particles are held $5 \times 10^{-3} \text{ m}$ apart and are released from rest. The acceleration of the first particle is observed to be 10 m/s^2 and that of the second to be 15 m/s^2 . The mass of the first particle is $1 \times 10^{-5} \text{ kg}$. The common charge that the particles have and the mass of the second particle respectively are
(a) $5.3 \times 10^{-11} \text{ C}$ and $0.66 \times 10^{-6} \text{ kg}$
(b) $0.66 \times 10^{-10} \text{ C}$ and $5.3 \times 10^{-6} \text{ kg}$
(c) $0.33 \times 10^{-10} \text{ C}$ and $3.5 \times 10^{-5} \text{ kg}$
(d) $5.3 \times 10^{-10} \text{ C}$ and $0.66 \times 10^{-5} \text{ kg}$
56. A test charge q_0 is moved without acceleration from point A to B through a path $A \rightarrow C \rightarrow B$. The potential difference between A and B is
(a) $E(AC \cos 45^\circ + BC)$
(b) $-E \times \vec{BC}$
(c) $E \cdot d \cos 45^\circ$ (d) $E \cdot d$
57. The radius of the gold nucleus is $6.6 \times 10^{-15} \text{ m}$ and the atomic number is 79. The electric potential at the surface of the gold nucleus is
(a) $1.7 \times 10^7 \text{ V}$ (b) $7.1 \times 10^7 \text{ V}$
(c) $1.7 \times 10^9 \text{ V}$ (d) $7.1 \times 10^9 \text{ V}$
58. $q_1 = +1.0 \times 10^{-8} \text{ C}$, $q_2 = -2.0 \times 10^{-8} \text{ C}$, $q_3 = +3.0 \times 10^{-8} \text{ C}$, $q_4 = +2.0 \times 10^{-8} \text{ C}$, and $a = 1.0 \text{ m}$. The potential at the centre of the square P, is
(a) 300 V
(b) 200 V
(c) 500 V
(d) 800 V
59. Three charges, $-4q$, $+q$ and $+2q$ are held fixed on the vertices of an equilateral triangle of side a . $q = 5 \mu\text{C}$ and $a = 15 \text{ cm}$. The total energy of configuration is
(a) 1.5 J
(b) -15 J
(c) -1.5 J
(d) +15 J
60. $q_1 = +12 \times 10^{-9} \text{ C}$ and $q_2 = -12 \times 10^{-9} \text{ C}$ are placed at the vertices of an equilateral triangle of side 10 cm along the base line as shown in the figure. The potentials in volts at A, B and C respectively are





- (a) 900, -1930, 0
(c) -900, 1930, 0

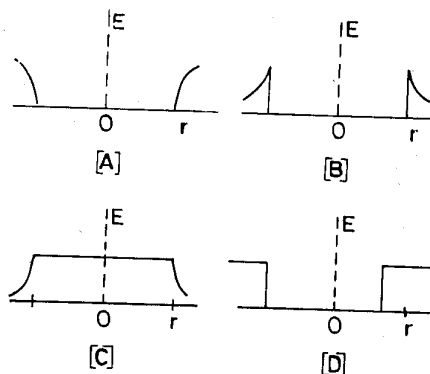
- (b) 1930, -900, 0
(d) -1930, -900, 0

61. A particle of mass 10 g and charge 5×10^{-8} C starts from rest at point A and moves in a straight line to point B. The speed of the particle when it reaches point B is

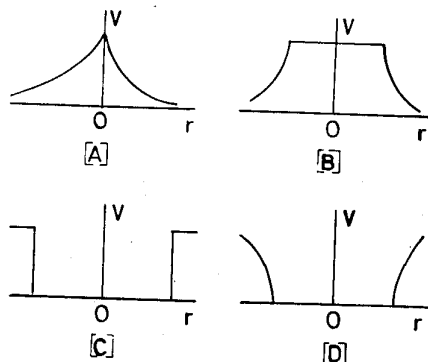
- (a) 5.9 cm/s
(b) 12.5 cm/s
(c) 15.5 cm/s

- (d) 9.5 cm/s

62. A charged spherical conductor of radius R is in vacuum. The electric field is plotted against the distance r from the centre of the conductor, starting from $r = 0$ to $r = R$ and then $r > R$. The plot is as shown in:



63. For the above mentioned conductor, the plot of potential V against r is



64. Three concentric spherical metallic shells A, B and C of radii a , b and c ($a < b < c$) have charge densities σ , $-\sigma$ and σ respectively. The potentials of A, B and C are

(a) $V_A = \frac{1}{\epsilon_0} (a + b + c) \sigma$

(b) $V_B = \frac{1}{\epsilon_0} \left(\frac{a^2}{b} - b + c \right) \sigma$

(c) $V_C = \frac{1}{\epsilon_0} \left(\frac{a^2}{c} + \frac{b^2}{c} + c \right) \sigma$

(d) $V_A = V_B = V_C = \frac{1}{\epsilon_0} (a + b + c) \sigma$

65. In the above problem, if the shells A and C are at the same potential, then the relation between a , b and c is

- (a) $a + b + c = 0$ (b) $a + c = b$
(c) $a + b = c$ (d) $a = b + c$

66. Point charges of 3×10^{-9} C are situated at each of three corners of a square whose side is 15 cm. The magnitude and direction of the electric field at the vacant corner of the square is

- (a) 2296 V/m along the diagonal
(b) 9622 V/m along the diagonal
(c) 22.0 V/m along the diagonal
(d) zero

67. Two point charges $-q$ and $+q/2$ are situated at the origin and at the point $(a, 0, 0)$ respectively. The point along the X-axis where the electric field vanishes is

- (a) $x = \frac{a}{\sqrt{2}}$ (b) $x = \sqrt{2} a$
(c) $x = \frac{\sqrt{2} a}{\sqrt{2} - 1}$ (d) $x = \frac{\sqrt{2} a}{\sqrt{2} + 1}$

68. The electric field in the atmosphere at the earth's surface is approximately 200 V/m, directed downward. 1400 m above the earth's surface, the electric field in the atmosphere is only 20 V/m, again directed downward. The average charge density in the atmosphere below 1400 m is

- (a) 2.2×10^{-10} C/m³ (b) 1.1×10^{-10} C/m³
(c) 1.1×10^{-6} C/m³ (d) 1.1×10^{-12} C/m³

69. Two balls with charges $5 \mu\text{C}$ and $10 \mu\text{C}$ are at a distance of 1 m from each other. In order to reduce the distance between them to 0.5 m the amount of work to be performed is

- (a) 45 J (b) 0.45×10^{-6} J
(c) 1.2×10^{-4} J (d) 0.45 J

70. A point charge of 5×10^{-9} C is transferred from infinity to a point 5 cm from the surface of a ball of radius 5 cm and surface charge density 5×10^{-4} C/m². The amount of work done in the process is

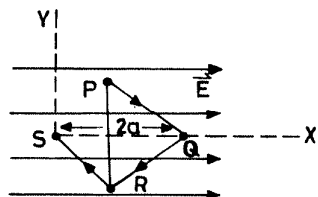
- (a) 7.07×10^{-3} J (b) 7.07×10^{-6} J
(c) 7.07×10^{-4} J (d) 70 J

71. A nonconducting massless rod of length 10 cm carries two small metallic balls of mass 5 g each. There is a charge of $1 \mu\text{C}$ on each ball, fixed at each end of

the rod. The rod is held in an electrical field of 50 V/m, making a small angle θ with the field. When the rod is released, it executes simple harmonic motion. Its time period is

- (a) 2.8 s (b) 0.14 s
(c) 14 s (d) 1.4 s

72. A point charge q moves from point P to a point S along a path PQRS in a uniform electric field E pointing parallel to the X-axis. The coordinates of P, Q, R and S are $(a, b, 0)$, $(2a, 0, 0)$, $(a, -b, 0)$ and $(0, 0, 0)$. The work done by the field in the above process is



- (a) zero
(b) $qEab$
(c) qEa
(d) $-qEa$
- 73.* A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow spherical shell be V . If the shell is now given a charge $-3Q$ the new potential difference between the same surfaces is

- (a) V (b) $2V$
(c) $4V$ (d) $-2V$

74. Three identical metallic uncharged spheres A, B and C of radius a are kept on the corners of an equilateral triangle of side d ($d \gg a$). A fourth sphere (radius a) which has a charge Q touches A and is then removed to a position far away. B is earthed and then the earth connection is removed. C is then earthed. The charge on C is

- (a) $\frac{Qa}{2d} \left(\frac{2d-a}{2d} \right)$ (b) $\frac{Qa}{2d} \left(\frac{2d-a}{d} \right)$
(c) $\frac{Qa}{2d} \left(\frac{d-a}{d} \right)$ (d) $\frac{2Qa}{d} \left(\frac{d-a}{2d} \right)$

75. A charge q is placed at the centre of the line joining two equal charges Q . The system will be in equilibrium if q is equal to

- (a) $-Q/2$ (b) $-Q/4$
(c) $+Q/4$ (d) $+Q/2$

76. Three point charges q , $2q$ and $8q$ are to be placed on a straight line 9 cm long. In order that the potential energy of the system be minimum, their positions should be

- (a) $2q$ and q at the ends (9 cm apart) and $8q$ at a distance of 3 cm from the $2q$ charge
(b) $2q$ and $8q$ at the ends and q at a distance of 3 cm from the $8q$ charge
(c) $2q$ and $8q$ at the ends and q at a distance of 6 cm from the $8q$ charge
(d) q and $8q$ at the ends and $2q$ at a distance of 6 cm from the q charge

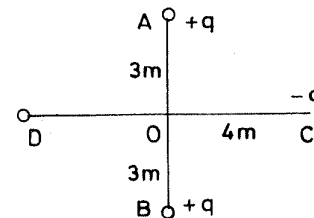
77. In the above problem, the electric field at the point where charge q is located is

- (a) $\frac{1}{4\pi\epsilon_0} \frac{8q}{9}$ (b) $\frac{1}{4\pi\epsilon_0} \frac{2q^2}{9}$
(c) zero (d) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{30}$

78. Two small balls, each having equal positive charge Q , C are suspended by two insulating strings of equal length L (metres) from a hook fixed to a stand. The whole set-up is taken into a satellite in space where there is no gravity. In this state, the angle between the strings and the tension in each string respectively are

- (a) 18° and $\frac{4Q^2}{\pi\epsilon_0 L^2} N$ (b) 36° and $\frac{Q^2}{4\pi\epsilon_0 L^2} N$
(c) 9° and $\frac{Q^2}{4\pi\epsilon_0 (2L)^2} N$ (d) 18° and $\frac{Q^2}{4\pi\epsilon_0 (2L)^2} N$

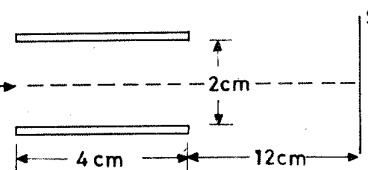
79. Two fixed, equal, positive charges, each of magnitude $5 \times 10^{-5} C$, are located at points A and B separated by a distance of 6 m. An equal and opposite charge moves towards them along the line COD, the perpendicular bisector of the line AB. The moving charge when it reaches the point C at a distance of 4 m from O has a kinetic energy of 4 J. The distance between the farthest point D where the negative charge will reach before returning towards C, and the point O is



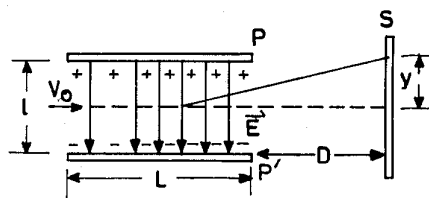
- (a) 4.84 m (b) 8.48 m
(c) 4.24 m (d) 2.14 m
80. A ball of mass 5 g and charge $5 \times 10^{-8} C$ moves from a point P whose potential is 800 V to a point Q whose potential is zero. If the velocity of the ball at the point Q is 20 cm/s, the velocity at the point P will be

- (a) $15.5 \times 10^{-2} m/s$ (b) 1.5 m/s
(c) $1.5 \times 10^{-2} m/s$ (d) $2.5 \times 10^{-2} m/s$

81. An electron is projected along the axis midway between the plates of a cathode ray tube with an initial velocity $V_0 = 2 \times 10^7 m/s$. The uniform electric field between the plates is $2 \times 10^4 N/C$, directed upward. Mark the statement which is incorrect.

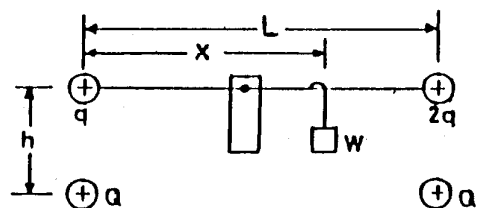


- (a) the downward velocity component of velocity just after the electron travels through the plates is $7 \times 10^6 m/s$.
(b) the electron moves a distance 0.704 cm below the axis when it reaches the end of the plates.
(c) the electron moves at an angle of 9.7° with the axis at the time when it leaves the plates
(d) it will strike the fluorescent screen S at a distance 4.92 cm below the axis.
82. An electron is accelerated horizontally at a constant voltage V_1 so that it enters and passes through two parallel plates P and P' between which an electric field E exists, as shown in the figure. The length of the plate is L and the distance from the plate to fluorescent screen S is D . The separation between the plates is l . If the voltage between the plates is V_2 , the deflection y on the screen is



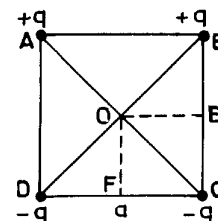
- (a) $\left[\frac{L}{l} (D + L) \right] \frac{V_2}{V_1}$ (b) $\left[\frac{L}{2l} \left(D + \frac{L}{2} \right) \right] \frac{V_2}{V_1}$
 (c) $\left[\frac{l}{2L} \left(D + \frac{L}{2} \right) \right] \frac{V_2}{V_1}$ (d) $\left[\frac{1}{2l} (2D + 1) \right] \frac{V_2}{V_1}$

83. An insulating long massless rod of length L , pivoted at its centre and balanced with a weight W at a distance x from the left end, is shown in the figure. Charges q and $2q$ are attached at the left and right ends of the rod. A distance h directly below each of these charges is a fixed positive charge Q . The distance x in terms of q , Q , L and W is



- (a) $\frac{qQL + \epsilon_0 h^2 WL}{\epsilon_0 h^2 W}$ (b) $\frac{4qQL + \epsilon_0 h^2 WL}{8\pi h^2 W}$
 (c) $\frac{qQL + 4\pi\epsilon_0 h^2 WL}{8\pi\epsilon_0 h^2 W}$ (d) $\frac{qQL + \epsilon_0 h^2 WL}{h^2 W}$

84. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. The potential at the centre of the sphere is
 (a) zero
 (b) 10 V
 (c) same as at a point 5 cm away from the surface
 (d) same as at a point 25 cm away from the surface
85. A thin fixed ring of radius 1 m has a positive charge of 1×10^{-5} C distributed over it. A particle of mass 0.9 g and having a negative charge of 1×10^{-6} C is placed on the axis at a distance 1 cm from the centre of the ring. The time period of oscillation of this charge is (take the approximation that the radius of the ring is much larger than the distance of the negative charge from the centre)
 (a) 6.3 s (b) 3.6 s
 (c) 0.36 s (d) 0.63 s
86. Four charges $+q$, $+q$, $-q$ and $-q$ are placed at A, B, C, D, the corners of a square of side a , and E is the mid-point of side BC. The work done in carrying a charge e from O to E is



- (a) $-e \frac{4q}{a\sqrt{5}}$ (b) $e \frac{4q}{a\sqrt{5}}$
 (c) zero (d) $e \frac{2q}{5a}$

87. In Problem 86, F is the midpoint of CD. A charge e is moved from O to F. The work done in the process is

- (a) $\frac{4qe}{a\sqrt{5}}$ (b) $\frac{4qe}{a\sqrt{5}} (\sqrt{5} - 1)$
 (c) zero (d) $\frac{2qe}{5a}$

88. Two identically charged spheres are suspended by strings of equal length. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g/cc, the angle remains the same. If the density of the material of the sphere is 1.6 g/cc, the dielectric constant of the liquid is

- (a) 2.0 (b) 3.5
 (c) 8.0 (d) 4.0

- 89.* Two equal charges A and B, each of $(1/3) \times 10^{-6}$ C, are placed 200 cm apart in air. A particle, carrying a charge of $(-1/3) \times 10^{-6}$ C is projected along the perpendicular bisector from the point O, midway between A and B with a kinetic energy of 10^{-3} J. Before the particle starts to return, it will cover a distance

- (a) 0.170 m (b) 0.137 m
 (c) 1.73 m (d) 3.71 m

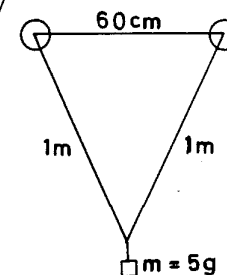
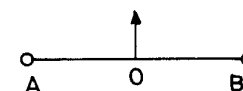
90. 27 identical drops of mercury are charged simultaneously to the same potential of 10 V. All these drops are combined to make one large drop. If the drops are assumed to be spherical, the potential of the large drop is

- (a) 90 V (b) 270 V
 (c) 135 V (d) 10 V

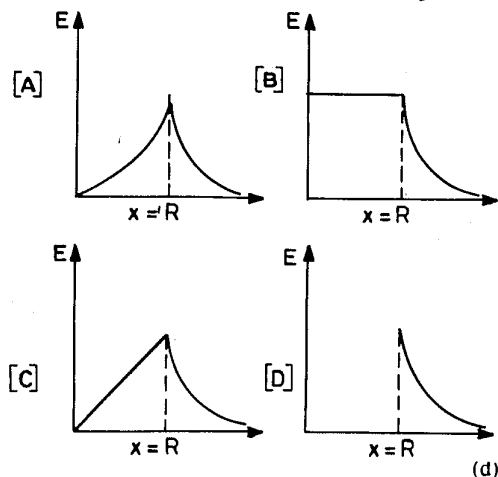
91. Two similar helium-filled spherical balloons tied to a 5 g weight with strings of length 1 m and each carrying an electric charge q float in equilibrium. If the charge on each balloon acts as if it were concentrated at its centre, the magnitude of q in coulombs is

- (a) 5.5×10^{-6}
 (b) 5.5×10^{-10}
 (c) 5.5×10^{-4}

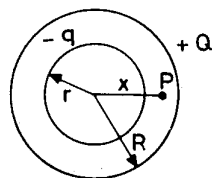
- (d) 5.5×10^{-7}



92. An infinite sheet of charge has a surface charge density of $1.0 \times 10^{-7} \text{ C/m}^2$. The separation between two equipotential surfaces whose potentials differ by 5.0 V is
 (a) 0.89 cm (b) 0.89 mm
 (c) 0.89 m (d) $5 \times 10^{-7} \text{ m}$
93. Two small spheres are positively charged, the total charge being $5 \times 10^{-5} \text{ C}$. If each sphere is repelled from the other with a force of 1 N when the spheres are 2.0 m apart, the charge distribution on the two spheres is
 (a) 4.0×10^{-5} and $1.0 \times 10^{-5} \text{ C}$
 (b) 3.0×10^{-5} and $2 \times 10^{-5} \text{ C}$
 (c) 1.2×10^{-5} and $3.8 \times 10^{-5} \text{ C}$
 (d) 2.5×10^{-5} and $2.5 \times 10^{-5} \text{ C}$
94. A 500 eV electron is fired directly toward a large metal plate that has a surface charge density of $-5 \times 10^{-6} \text{ C/m}^2$. The distance from where an electron may be fired so as to just fail to reach the plate is
 (a) 9.80 cm (b) 8.90 cm
 (c) 8.9 mm (d) 0.890 mm
95. A hollow charged sphere of radius R has a constant surface charge density σ . Of the following graphs, the one which shows the variation of the electric field strength E with distance x from the centre of the sphere is



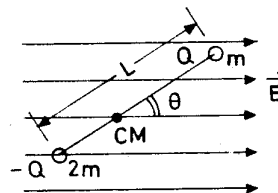
96. A hollow sphere of radius r is put inside another hollow sphere of radius R . The charges on the two are $+Q$ and $-q$ as shown in the figure. A point P is located at a distance x from the common centre such that $r < x < R$. The potential at the point P is



- (a) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q-q}{x} \right)$ (b) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} - \frac{q}{r} \right)$
 (c) $\frac{1}{4\pi\epsilon_0} \left(\frac{Q}{R} - \frac{q}{x} \right)$ (d) $\frac{1}{4\pi\epsilon_0} \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{x} \right)$

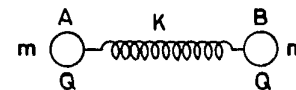
97. An electric dipole is placed in a homogeneous electric field increasing at the rate of λ per unit distance along the X-axis. The dipole undergoes

- (a) rotational motion only
 (b) rotational motion and translational motion along the Z-axis
 (c) rotational motion and translational motion along the X-axis
 (d) translational motion only along the X-axis
98. Two equal point charges separated by a distance $2l$ are placed symmetrically on opposite sides of a large uniformly charged nonconducting plane, as shown in the figure. The locations on the plane where the net electrostatic force on a charge Q due to the two charges is maximum, are
 (a) two points symmetrically located on the plane on either side of the q - q line at a distance $l\sqrt{2}$
 (b) at the centre of the line q - q only
 (c) all points lying on a circle of radius $l/\sqrt{2}$
 (d) two points on the circle of radius $l\sqrt{2}$ and on the diameter perpendicular to the q - q line
99. At a certain distance from a point charge the field intensity is 500 V/m and the potential is -3000 V . The distance to the charge and the magnitude of the charge respectively are
 (a) 6 m and $6 \mu\text{C}$ (b) 4 m and $2 \mu\text{C}$
 (c) 6 m and $4 \mu\text{C}$ (d) 6 m and $2 \mu\text{C}$
100. A dipole consists of two particles, one with charge Q and mass m , and the other with charge $-Q$ and mass $2m$, separated by a distance L . For small oscillations about its equilibrium position, the angular frequency when placed in a uniform electric field E , is



- (a) $\sqrt{\frac{3QE}{2mL}}$ (b) $\sqrt{\frac{2QE}{3mL}}$
 (c) $\sqrt{\frac{2QL}{2mE}}$ (d) $\sqrt{\frac{3Qm}{2LE}}$

101. Two bodies A and B of mass m and carrying a charge Q rotate with constant speed in a circle of radius $r/2$ on a horizontal table and are connected by a spring of unstretched length l_0 with a spring constant K . (Assume the PE of the two charges to be zero when they are separated by an infinite distance.) The total energy of the system is

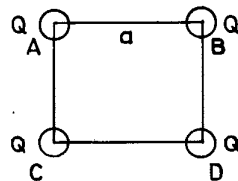


- (a) $mV^2 - \frac{Q^2}{4\pi\epsilon_0 r}$
 (b) $\frac{1}{2} K(r - l_0)^2$

$$(c) 2K(r-l_0)r + \frac{1}{2}K(r-l_0)^2 - \frac{Q^2}{4\pi\epsilon_0 r}$$

(d) infinite

102. Four identical spheres of radius r , A and B each carrying a charge Q and C and D each carrying a charge $-Q$, are located in the four corners of a square of side a . The energy required to take the spheres C and D to infinity (they are also infinitely separated from each other) is



$$(a) \frac{Q^2}{4\pi\epsilon_0 a}$$

$$(b) \frac{2Q^2}{4\pi\epsilon_0 a}$$

$$(c) \frac{Q^2}{4\pi\epsilon_0 a} (\sqrt{2} + 1)$$

(d) zero

Answers

- | | | | | |
|----------|----------|---------|---------|----------|
| 1. (b) | 2. (c) | 3. (a) | 4. (d) | 5. (b) |
| 6. (c) | 7. (a) | 8. (c) | 9. (d) | 10. (a) |
| 11. (b) | 12. (c) | 13. (d) | 14. (b) | 15. (a) |
| 16. (d) | 17. (a) | 18. (b) | 19. (d) | 20. (c) |
| 21. (b) | 22. (b) | 23. (b) | 24. (c) | 25. (d) |
| 26. (b) | 27. (a) | 28. (b) | 29. (d) | 30. (c) |
| 31. (a) | 32. (d) | 33. (c) | 34. (c) | 35. (d) |
| 36. (b) | 37. (b) | 38. (a) | 39. (c) | 40. (d) |
| 41. (b) | 42. (a) | 43. (a) | 44. (a) | 45. (c) |
| 46. (a) | 47. (d) | 48. (c) | 49. (d) | 50. (b) |
| 51. (d) | 52. (a) | 53. (c) | 54. (b) | 55. (d) |
| 56. (d) | 57. (a) | 58. (c) | 59. (b) | 60. (c) |
| 61. (d) | 62. (b) | 63. (b) | 64. (b) | 65. (c) |
| 66. (a) | 67. (c) | 68. (d) | 69. (d) | 70. (a) |
| 71. (c) | 72. (d) | 73. (a) | 74. (a) | 75. (b) |
| 76. (c) | 77. (c) | 78. (d) | 79. (b) | 80. (a) |
| 81. (c) | 82. (b) | 83. (c) | 84. (b) | 85. (d) |
| 86. (c) | 87. (b) | 88. (a) | 89. (c) | 90. (a) |
| 91. (d) | 92. (b) | 93. (c) | 94. (d) | 95. (d) |
| 96. (c) | 97. (c) | 98. (c) | 99. (d) | 100. (a) |
| 101. (c) | 102. (c) | | | |

Note: Problems marked with a star should take about four minutes each and the rest should take two to three minutes each.

20 Capacitors and Dielectrics

A *capacitor* or condenser consists of two conductors separated by an insulator or dielectric. The capacitance of a capacitor is defined as

$$\text{Capacitance } C = \frac{\text{magnitude of charge } q \text{ on either conductor}}{\text{magnitude of potential difference } V \text{ between the conductors}}$$

It is a device for storing electric charge. Its units are *coulombs per volt*, C/V, also called *farad* (F). 1 F = C/V. The farad is quite a large unit. The capacitance values are generally measured in microfarads (μF or 10^{-6} F) or picofarads (pF or 10^{-12} F).

Characteristics of a Capacitor

- The current through a capacitor is zero if the voltage across it does not change with time. A capacitor is therefore an open circuit to dc.
- A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero, such as when the voltage across it is constant.
- It is not possible to change the voltage across a capacitor by a finite amount in zero time, for this requires an infinite current through the capacitor.
- The capacitor never dissipates energy, but only stores it. Although this is true for the mathematical model, it is not true for a physical capacitor.

The Parallel Plate Capacitor

The capacitance of a parallel plate capacitor whose opposing plate faces, each of area A , are separated by a small distance d is given by

$$C = K\epsilon_0 \frac{A}{d}$$

where K is the dielectric constant of the nonconducting material between the plates and ϵ_0 is the permittivity constant ($= 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 = 8.85 \times 10^{-12} \text{ F/m}$). For vacuum, $K = 1$, so that a dielectric filled parallel plate capacitor has a capacitance K times larger than the same capacitor with vacuum between its plates. This result is true for a capacitor of any shape.

The Spherical Capacitor

Such a capacitor consists of two metallic concentric spheres A and B of radii a and b respectively, which are insulated from each other. The sphere A is charged with a positive charge of Q units and the other sphere B is earthed. In such a case, the capacitance of the capacitor is

$$C = \frac{4\pi\epsilon_0 ab}{(b-a)} \text{ F}$$

But when the inner sphere is earthed and the outer sphere is given a charge $+Q$, the capacitance of the spherical capacitor is

$$C = 4\pi\epsilon_0 \left(\frac{b^2}{b-a} \right)$$

Capacitance of an Isolated Sphere

Consider a metallic sphere of radius R which given a positive charge Q , and is placed away from any other metallic body. If the surrounding medium is of permittivity ϵ_r , then its capacitance $C = 4\pi\epsilon_0\epsilon_r R$. For air, $\epsilon_r = 1$.

$$\therefore C = 4\pi\epsilon_0 R$$

Capacitance of a Cylindrical Capacitor

Such a capacitor consists of two coaxial cylinders of radii a and b and length l . A charge $+Q$ is given to the outer cylinder while the inner cylinder has charge $-Q$. Its capacitance is given by

$$C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

Grouping of Capacitors

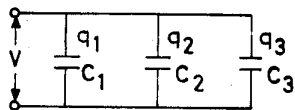
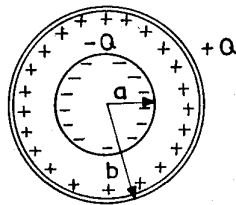
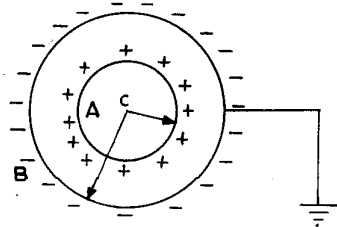
(a) Capacitors in Parallel

$$\text{Total charge } q = q_1 + q_2 + q_3$$

$$V = V_1 = V_2 = V_3$$

$$C_{eq} = C_1 + C_2 + C_3$$

(C_{eq} is the equivalent capacitance)

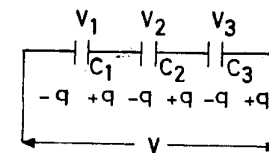


(b) Capacitors in Series

$$q = q_1 = q_2 = q_3$$

$$V = V_1 + V_2 + V_3$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Energy Stored in a Capacitor

The energy stored in a capacitor of capacitance C that has a charge Q and a potential difference V , is given by

$$W = \frac{1}{2} QV = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

Dielectric and its Relation with Capacitance, Electric Field, Potential difference and Charge distribution

$$(a) C = C_0 K$$

where C_0 is the capacitance without the dielectric and K is the dielectric constant

$$(b) E = \frac{1}{K} E_0$$

$$(c) V = \frac{V_0}{K}$$

(d) The charge on a capacitor with dielectric is greater than that without it.

$$\text{Electric field in the absence of dielectric, } E_0 = \frac{q}{\epsilon_0 A}$$

If dielectric is present,

$$E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

in which $-q'$ is the induced surface charge. (q is called the free charge.)

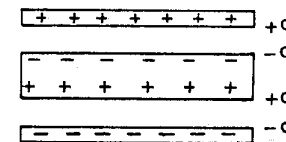
$$q' = q \left(1 - \frac{1}{K} \right)$$

is the relation between free charge, induced charge and dielectric constant.

Electric Field on Introduction of Dielectric between Plates of Parallel Plate Capacitor

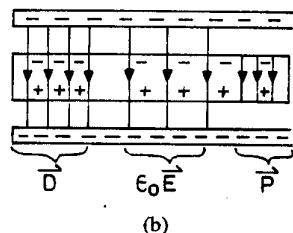
Figure (a) here shows that the charges are induced in the dielectric (q').

The induced surface charge per unit area is called electric polarization or $P = q'/A$; the direction of $\vec{P}$ is from



(a)

negative induced charge to positive induced charge, as for any dipole. In a region, where there is no dielectric, $\vec{P}$ is zero. Figure (b) here shows the electric lines of force associated with free charge $q(\vec{D})$, all charges ($\epsilon_0 \vec{E}$) and the induced charges ($\vec{P}$).



The relation between these three electric vectors is $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$. Other relations can also be written in the following forms

$$\vec{D} = K\epsilon_0 \vec{E}, \quad \vec{P} = \epsilon_0(K-1)\vec{E}$$

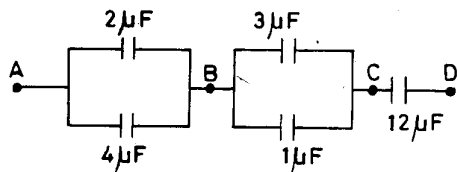
Energy Stored in an Electric Field

$$U = \frac{1}{2} K\epsilon_0 E^2$$

Here U is the energy density, called energy per unit volume.

ILLUSTRATIONS

1. A network of capacitors is shown in the figure here.



- (a) Find the capacitance of a single equivalent capacitor between points A and D.
 (b) What is the charge on $2 \mu\text{F}$ and $4 \mu\text{F}$ capacitors when the capacitors are fully charged by a 24 V battery connected to the terminals.
 (c) Determine the potential difference across each capacitor.

Solution

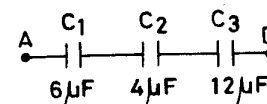
(a) First reduce the two parallel capacitors between A and B to a single equivalent capacitor C_1 .

$$C_1 = 2 \mu\text{F} + 4 \mu\text{F} = 6 \mu\text{F}$$

Similarly between B and C,

$$C_2 = 3 \mu\text{F} + 1 \mu\text{F} = 4 \mu\text{F}$$

Then the circuit reduces to



These three capacitors can be reduced to the single equivalent capacitor C where

$$\frac{1}{C} = \frac{1}{6} + \frac{1}{4} + \frac{1}{12} = \frac{1}{2} \Rightarrow C = 2 \mu\text{F}$$

(b) Total charge supplied by the battery to the capacitor network is $Q = CV = 2 \mu\text{F} \times 24 \text{ V} = 48 \mu\text{C}$
 $48 \mu\text{C}$ is shared between $2 \mu\text{F}$ and $4 \mu\text{F}$ capacitors in the proportion to their capacitances

$$Q_2 = \frac{2}{6} \times 48 = 16 \mu\text{C} \text{ and } Q_4 = \frac{4}{6} \times 48 = 32 \mu\text{C}$$

Similarly $48 \mu\text{C}$ is shared between $3 \mu\text{F}$ and $1 \mu\text{F}$ capacitors.

$$Q_3 = \frac{3}{4} \times 48 = 36 \mu\text{C} \text{ and } Q_1 = \frac{1}{4} \times 48 = 12 \mu\text{C}$$

$12 \mu\text{F}$ capacitor carries full $48 \mu\text{C}$ of charge.

(c) The potential difference between A and D is 24 V .

$$V_A - V_D = (V_A - V_B) + (V_B - V_C) + (V_C - V_D) \\ = V_1 + V_2 + V_3 = 24 \text{ V}$$

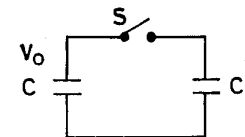
$$V_1 = \frac{Q_1}{C_1} = \frac{Q}{C_1} = \frac{48 \mu\text{C}}{6 \mu\text{F}} = 8 \text{ V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{Q}{C_2} = \frac{48 \mu\text{C}}{4 \mu\text{F}} = 12 \text{ V}$$

$$V_3 = \frac{Q_3}{C_3} = \frac{Q}{C_3} = \frac{48 \mu\text{C}}{12 \mu\text{F}} = 4 \text{ V}$$

It can be verified that the voltage distribution is equal to the total voltage applied, i.e. $(8 + 12 + 4) \text{ V} = 24 \text{ V}$.

2. A capacitor of capacitance C is charged to a potential difference V_0 . The charging battery is disconnected and the capacitor is connected to a capacitor of unknown capacitance C_x . The potential difference across the combination is V . What is the capacitance C_x in terms of C , V_0 and V ? Find the energy stored before and after the switch S is closed.



Solution

When the capacitor C is charged to a potential difference of V_0 , it has a charge

q_0 but when C and C_x are connected by closing the switch S , q_0 is shared by the two capacitors. Let q_1 and q_2 be the charge of C and C_x . Then

$$q_0 = q_1 + q_2 \text{ and } CV_0 = CV + C_x V$$

or $C_x V = CV_0 - CV = C(V_0 - V)$

or $C_x = \frac{C(V_0 - V)}{V}$

Initially stored energy in the capacitor C ,

$$U_0 = \frac{1}{2} CV_0^2$$

The final stored energy is

$$\begin{aligned} U &= \frac{1}{2} CV^2 + \frac{1}{2} C_x V^2 \\ &= \frac{1}{2} CV^2 + \frac{1}{2} \frac{C(V_0 - V)}{V} \times V^2 \\ &= \frac{1}{2} CV_0 V \end{aligned}$$

Note that the final stored energy is smaller than the initially stored energy. The "missing" energy appears as heat in the connecting wires as the charges move through them.

3. A parallel plate capacitor has plates with area A and separation d . A battery charges the plates to a potential difference of V_0 . The battery is then disconnected and a dielectric slab of constant K and thickness d is introduced. Calculate the positive work done by the (capacitor + slab) system on the man who introduces the slab.

Solution

The energy before the slab is introduced,

$$U_0 = \frac{1}{2} C_0 V_0^2.$$

The capacitance after the slab is introduced,

$$C = KC_0 \quad \text{and} \quad V = \frac{V_0}{K}$$

and thus, the energy becomes

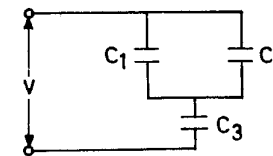
$$U = \frac{1}{2} CV^2 = \frac{1}{2} KC_0 \left(\frac{V_0}{K} \right)^2 = \frac{U_0}{K}$$

The difference $\Delta U = U_0 - U$ is the "missing" energy and would be apparent to the person who inserted the slab. If he does not want the slab to accelerate, he will have to apply a force to restrain it. This means that he would have to

do negative work on it. Or, in other words, the (capacitor + slab) system would do positive work on him. The positive work is

$$W = \Sigma U = U_0 - U = \frac{1}{2} C_0 V_0^2 \left(1 - \frac{1}{K} \right)$$

4. A combination of three capacitors is given a difference of potential V . Suppose capacitor C_3 breaks down electrically, becoming equivalent to a conducting path. What changes in (a) the charge, and (b) the potential difference, would occur for capacitance C_1 ?



(Given $C_1 = 10 \mu\text{F}$, $C_2 = 5 \mu\text{F}$, $C_3 = 4 \mu\text{F}$ and $V = 100 \text{ V}$)

Solution

The charge on C_3 is equal to that across the combination of C_1 and C_2 in parallel.

$$\therefore C_3 V_3 = (C_1 + C_2) V_1$$

where V_1 is the common potential of C_1 and C_2 and V_3 is the potential of C_3 .

$$V_3 = \left(\frac{C_1 + C_2}{C_3} \right) V_1 = \left(\frac{10 + 5}{4} \right) V_1 = \frac{15}{4} V_1$$

But $V_1 + V_3 = 100$

$$\therefore V_1 = 21 \text{ V}$$

When the capacitor C_3 breaks down, the potential difference across C_1 becomes 100 V. Therefore, the change in potential across C_1 is $(100 - 21) = 79 \text{ V}$.

$$\text{The initial charge on } C_1 \text{ was } V_1 C_1 = 21 \text{ V} \times 10 \mu\text{F} = 210 \mu\text{C}$$

$$\text{The charge after } C_3 \text{ breaks down} = 100 \text{ V} \times 10 \mu\text{F} = 1000 \mu\text{C}$$

$$\therefore \text{Change in charge on } C_1 = (1000 - 210) \mu\text{C} = 790 \times 10^{-6} \text{ C} = 7.9 \times 10^{-4} \text{ C}$$

5. A parallel plate capacitor has plates of area 0.5 m^2 and a separation of 1.5 cm . A battery charges the plates to a potential difference of 150 V after which the battery is removed. A dielectric slab of thickness 0.5 cm and dielectric constant 5.0 is then placed symmetrically between the plates.

(a) What is the free charge before and after the slab is inserted.

(b) What is the electric field strength in the space between the plates and the slab?

(c) What is the electric field strength in the dielectric?

(d) With the slab in place, what is the potential difference across the plates?

(e) How much external work is involved in the process of inserting the slab.

Solution

$$\begin{aligned} \text{(a) Capacitance without the slab } (C_0) &= \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 \cdot (0.5)}{(1.5 \times 10^{-2})} \\ &= \frac{8.8 \times 10^{-12} \times 0.5}{1.5 \times 10^{-2}} \text{ F} \\ &= 2.9 \times 10^{-10} \text{ F} \end{aligned}$$

$$\text{The capacitance with the dielectric } (C) = \frac{q}{V}$$

We will calculate this later.

$$\begin{aligned} \therefore \text{Free charge before the slab is inserted} &= q = C_0 V_0 \\ &= 2.9 \times 10^{-10} \times 150 \\ &= 435 \times 10^{-10} \text{ C} \end{aligned}$$

(for free charge after the slab is inserted, see part (d))

(b) Electric field between the plates and the slab

$$E_0 = \frac{V_0}{d} = \frac{150}{1.5 \times 10^{-2}} = 1 \times 10^4 \text{ V/m}$$

(c) Electric field in dielectric,

$$E = \frac{E_0}{K} = \frac{1 \times 10^4}{5} = 0.2 \times 10^4 \text{ V/m}$$

$$\text{(d) } V = E_0 \left[d - b \left(1 - \frac{1}{K} \right) \right]$$

$$= 10^4 \left[1.5 \times 10^{-2} - 0.5 \times 10^{-2} \left(1 - \frac{1}{5} \right) \right]$$

$$\begin{aligned} &= 10^4 [1.5 \times 10^{-2} - 0.4 \times 10^{-2}] = 1.1 \times 10^2 \\ &= 110 \text{ V} \end{aligned}$$

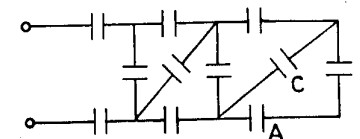
$$C = \frac{q}{V} = \frac{C_0 V_0}{V} = \frac{2.9 \times 10^{-10} \times 150}{110} = 3.95 \times 10^{-10} \text{ F}$$

$$\begin{aligned} \therefore \text{Free charge after inserting the slab} &= 3.95 \times 10^{-10} \times 110 \\ &= 435 \times 10^{-10} \\ &= 4.35 \times 10^{-8} \text{ C} \end{aligned}$$

(e) Work done in the process of inserting the slab

$$\begin{aligned} W &= \frac{1}{2} C_0 V_0^2 - \frac{1}{2} C V^2 \\ &= \frac{1}{2} \times (2.9 \times 10^{-10}) \times (150)^2 - \frac{1}{2} \times (3.95 \times 10^{-10}) \times (110)^2 \\ &= 326 \times 10^{-8} - 239 \times 10^{-8} = 87 \times 10^{-8} = 8.7 \times 10^{-7} \text{ J} \end{aligned}$$

6. Find the equivalent capacitance of the given network shown in the figure, if each capacitor is of capacitance 1 F.



Solution

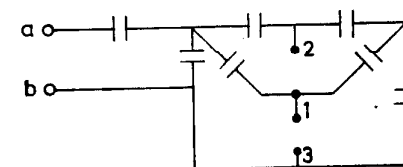
Let us start from the extreme right of the network where A and B are in series and C is in parallel to them. If we denote series by S, we can write the same like this

$$1 \text{ S } 1 = \frac{1}{2}; \frac{1}{2} + 1 = \frac{3}{2}; \frac{3}{2} \text{ S } 1 = \frac{3}{5}; \frac{3}{5} + 1 = \frac{8}{5};$$

$$\frac{8}{5} \text{ S } 1 = \frac{8}{13}; \frac{8}{13} + 1 = \frac{21}{13}; \frac{21}{13} \text{ S } 1 = \frac{21}{34}; \frac{21}{34} + 1 = \frac{55}{34};$$

$$\frac{55}{34} \text{ S } 1 = \frac{55}{89}; \frac{55}{89} \text{ S } 1 = 0.3819 \text{ F}$$

7. The given figure is a network of capacitors, each being of 1 μF . Find the equivalent capacitance at a-b if



- 1-2 and 1-3 are both open-circuited
- 1-2 and 1-3 are both short-circuited
- 1-2 is open-circuited and 1-3 is short-circuited.
- 1-2 is short-circuited and 1-3 is open-circuited.

Solution

(a) 1-2 and 1-3 in open circuit:

$$1 \text{ S } 1 = \frac{1}{2}; 1 \text{ S } 1 = \frac{1}{2}; \frac{1}{2} + \frac{1}{2} = 1; 1 \text{ S } 1 = \frac{1}{2}$$

$$\frac{1}{2} + 1 = \frac{3}{2}; \frac{3}{2} \text{ S } 1 = \frac{3}{5} = 0.6 \mu\text{F} = C_{eq}$$

(b) Both 1-2 and 1-3 short-circuited:

$$1 + 1 = 2; 2 + 1 = 3; 3 \text{ S } 1 = 0.75 \mu\text{F} = C_{eq}$$

(c) 1-2 open-circuited and 1-3 is short-circuited:

$$1 + 1 = 2; 1 \text{ S } 1 = \frac{1}{2}; \frac{1}{2} \text{ S } 2 = \frac{2}{5}; \frac{2}{5} + 1 + 1 = \frac{12}{5};$$

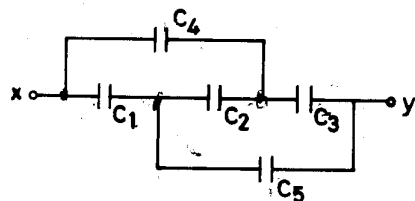
$$\frac{12}{5} \text{ S } 1 = 0.7059 \mu\text{F} = C_{eq}$$

(d) 1-2 short-circuited and 1-3 open-circuited:

$$1 + 1 = 2; 1 + 1 = 2; 2S2 = 1; 1S1 = \frac{1}{2};$$

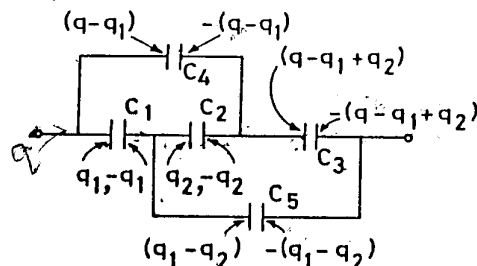
$$\frac{1}{2} + 1 = \frac{3}{2}; \frac{3}{2} S1 = 0.6 \mu\text{F} = C_{eq}$$

8. Find the effective capacitance between the points X and Y in the network of capacitors shown in the figure. Assume that $C_2 = 10 \mu\text{F}$ and all others have a capacitance of $4 \mu\text{F}$ each.



Solution

Let C be the effective capacitance between X and Y. Suppose this circuit is connected to a battery so that a potential difference V is applied between X and Y. Let the total charge be q . Let the charge across C_1 and C_2 be q_1 and q_2 respectively. The charges across the capacitors are shown in the figure here.



The potential drop across C_1 plus that across C_5 must be equal to the potential drop across C_4 plus that across C_3 .

$$V_1 + V_5 = V_3 + V_4$$

$$\therefore \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_5} = \frac{q - q_1}{C_4} + \frac{q - q_1 + q_2}{C_3} = V \quad (A)$$

But it is given that $C_1 = C_3 = C_4 = C_5 = 4 \mu\text{F}$

or from (A) we have

$$q_1 + q_1 - q_2 = C_1 V$$

$$\text{or } 2q_1 - q_2 = C_1 V \quad (i)$$

$$\text{Also } q - q_1 + q - q_1 + q_2 = C_1 V$$

$$\text{or } 2q - 2q_1 + q_2 = C_1 V \quad (ii)$$

Adding (i) and (ii) we get $2q = 2C_1 V$

$$C = \frac{q}{V} = C_1 = 4 \mu\text{F}$$

EXERCISES

- A circular parallel plate capacitor with spacing 3 mm is charged to produce an electric field strength of 3×10^6 V/m. If the stored charge is $1 \mu\text{C}$, the radius of the plate is
 - 5.45 cm
 - 2.72 cm
 - 7.50 cm
 - 10.90 cm
- Two spherical conductors of radii R_1 and R_2 are separated by a sufficiently large distance so that the induction effects are negligible. The spheres are connected by a thin conducting wire and brought to the same potential relative to $V(\infty) = 0$ and $C = (Q_1 + Q_2)/V$. Mark the one which is incorrect among the following conclusions drawn:
 - the common potential of the two spheres is

$$V = \frac{Q_1}{4\pi\epsilon_0 R_1} = \frac{Q_2}{4\pi\epsilon_0 R_2}$$

- the capacitance of the system, $C = 4\pi\epsilon_0 (R_1 + R_2)$
 - the ratio of charge density on sphere of radius R_1 and sphere of radius R_2 is R_1/R_2 .
 - the charge ratio is $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$
- The parallel plates of a capacitor of area A have charges $\pm Q$. The plates are maintained at a separation d . The force required to maintain this separation is
 - $\frac{Q^2}{2\epsilon_0 A}$
 - $\frac{Q^2}{4\pi\epsilon_0 A}$
 - $\frac{Q^2}{2\epsilon_0 dA}$
 - $\frac{Q^2 d}{2\epsilon_0 A}$
 - A circular plate capacitor has a radius of 50 cm and is charged to produce an electric field of intensity of 5×10^6 V/m. The force necessary to maintain the plates with a fixed separation is
 - 8.7 N
 - 28.9 N
 - 21.7 N
 - 86.9 N
 - In the above problem, if the spacing between the plates is 2 mm, the amount of energy stored in the capacitor field is
 - 0.348 J
 - 0.696 J
 - 0.174 J
 - 1.392 J
 - A parallel plate capacitor of area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is inserted between the plates of the capacitor so as to fill the space between the plates. If Q , E and W respectively denote the magnitude of charge on each

plate, the electric field between the plates (after the slab is inserted) and the work done on the system in the process of inserting the slab, then

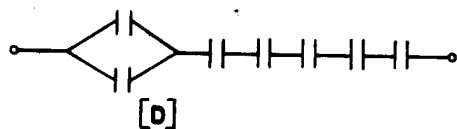
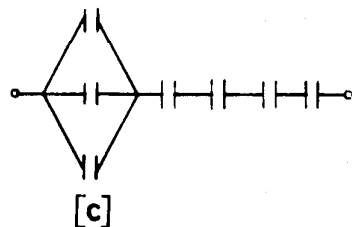
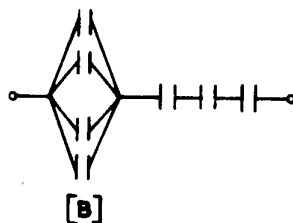
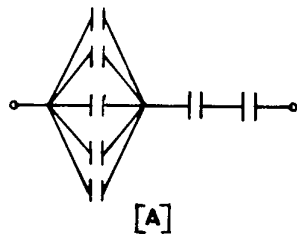
(a) $Q = \frac{\epsilon_0 AV}{Kd}$

(b) $Q = \frac{\epsilon_0 KAV}{d}$

(c) $E = \frac{V}{Kd}$

(d) $W = \epsilon_0 \frac{AV^2}{2d} \left(1 - \frac{1}{K}\right)$

7. In the given figures, four combinations of capacitors, each of capacity $2 \mu\text{F}$, are shown. The combination which has an effective capacitance of $(10/11) \mu\text{F}$ is



8. Two parallel plate capacitors of capacitance C and $2C$ are connected in parallel and charged to a potential difference V . The battery is then disconnected and the region between the plates of the capacitor C is completely filled with a material of dielectric constant K . The potential difference across the capacitors now becomes

(a) $\frac{2V}{K}$

(b) $\frac{3V}{K}$

(c) $\frac{3V}{K+2}$

(d) $\frac{2V}{K+3}$

9. A parallel plate capacitor is charged and the charging battery is disconnected. If the plates of the capacitor are moved farther apart by means of insulating handles,

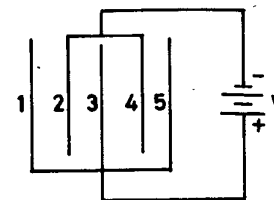
- (a) the charge on the capacitor increases
 (b) the voltage across the plates increases
 (c) the capacitance increases
 (d) the electrostatic energy stored in the capacitor decreases
10. A parallel plate capacitor (air medium) is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are Q_0 , V_0 , E_0 and U_0 respectively. A dielectric slab is now introduced to fill the space between the plates with the battery still in connection. The corresponding quantities now given by Q , V , E and U are related to the previous ones as
- (a) $Q > Q_0$ (b) $V > V_0$
 (c) $E > E_0$ (d) $U < U_0$

11. Five identical capacitor plates, each of area A , are arranged such that the adjacent plates are a distance d apart, the plates are connected to a source of emf V as shown. The charges on the plates 1 and 4 respectively are

(a) $-\left(\frac{2\epsilon_0 A}{d}\right)V$ and $\left(\frac{\epsilon_0 A}{d}\right)V$

(b) $-\frac{\epsilon_0 A}{d}V$ and $\left(\frac{2\epsilon_0 A}{d}\right)V$

(c) $\left(\frac{\epsilon_0 A}{d}\right)V$ and $-\left(\frac{2\epsilon_0 A}{d}\right)V$ (d) $\left(\frac{2\epsilon_0 A}{d}\right)V$ and $-\left(\frac{\epsilon_0 A}{d}\right)V$



12. A parallel-plate capacitor has a plate area of 100 cm^2 , and the separation between the plates is 1.0 cm . A potential difference of 100 V is applied and then the battery is disconnected. A dielectric slab of dielectric constant 7.0 and width 0.50 cm is introduced. The following conclusions are drawn (mark the one which is wrong):

(a) the electric field strength in the gap is $1.0 \times 10^4 \text{ V/m}$

(b) the free charge is $8.9 \times 10^{-10} \text{ C}$

(c) the potential difference between the plates is 57 V

(d) the capacitance with the slab in place is $12 \mu\text{F}$

13. A capacitor of capacity $5 \mu\text{F}$ is charged to a potential difference of 100 V . The battery is disconnected, and then connected in parallel to an uncharged capacitor of capacitance C . The potential difference measured across this combination is 25 V . The capacitance C is

(a) $15 \mu\text{F}$

(b) $10 \mu\text{F}$

(c) $5 \mu\text{F}$

(d) $20 \mu\text{F}$

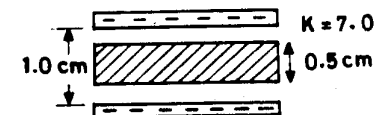
14. In the above problem, the ratio of stored energy U_0 in the capacitor of $5 \mu\text{F}$ and U when it is connected in parallel to an uncharged capacitor of capacitance C is given by

(a) $\frac{U}{U_0} = \frac{3}{4}$

(b) $\frac{U}{U_0} = \frac{4}{5}$

(c) $\frac{U}{U_0} = \frac{2}{5}$

(d) $\frac{U}{U_0} = \frac{1}{4}$



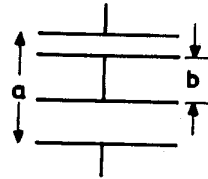
15. A capacitor of capacity $20 \mu\text{F}$ is charged to a potential difference of 200 V , and thereafter the battery is removed. This capacitor is then connected in parallel to an uncharged capacitor of capacity $5 \mu\text{F}$. The heat energy developed in the connecting wire is

(a) 0.32 J (b) 0.08 J
(c) 0.40 J (d) 0.04 J

16. A parallel plate capacitor of capacitance $50 \mu\text{F}$ is charged to a potential of 200 V and the battery is then disconnected. A dielectric slab of $K = 5$ is then introduced between the plates. The person who introduces the slab would do negative work on the (capacitor + slab) system which equals

(a) $0.4 \times 10^{-2} \text{ J}$ (b) 0.8 J
(c) 0.4 J (d) 0.04 J

17. Two capacitors are connected as shown in the figure and the centre section can be moved vertically up or down. The equivalent capacitance for any position of the centre section is



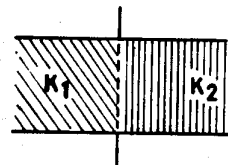
(a) $\frac{\epsilon_0 A}{a+b}$

(b) $\frac{\epsilon_0 A}{a^2 - b^2}$

(c) $\frac{\epsilon_0 A}{a-b}$

(d) $\frac{2\epsilon_0 A}{b+a}$

18. A parallel plate capacitor is filled with two dielectrics, (dielectric constants being K_1 and K_2 as shown in the figure). The capacitance is given by



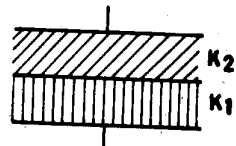
(a) $\frac{\epsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2} \right)$

(b) $\frac{\epsilon_0 A}{d} \left(\frac{K_1 + K_2}{K_1 K_2} \right)$

(c) $\frac{\epsilon_0 A}{d} \left(\frac{2}{K_1 + K_2} \right)$

(d) $\frac{\epsilon_0 A}{d} \left(\frac{K_1 + K_2}{2} \right)$

19. A parallel plate capacitor is filled with two dielectrics (K_1 and K_2) as shown in the figure. The capacitance is given by



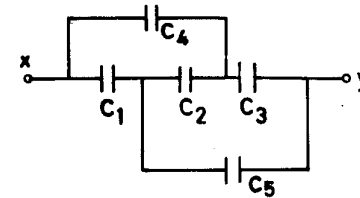
(a) $\frac{2\epsilon_0 A}{d} \left(\frac{K_1 K_2}{K_1 + K_2} \right)$

(b) $\frac{\epsilon_0 A}{d} \left(\frac{K_1 + K_2}{2} \right)$

(c) $\frac{\epsilon_0 A}{d} \left(\frac{K_1 + K_2}{K_1 K_2} \right)$

(d) $\frac{\epsilon_0 A}{d} \left(\frac{2}{K_1 + K_2} \right)$

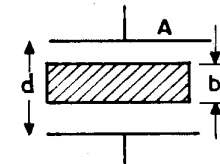
- 20*. Five capacitors form a network as shown in the figure here. $C_1 = C_3 = C_4 = C_5 = 5 \mu\text{F}$ and $C_2 = 15 \mu\text{F}$. The equivalent capacitance between the points x and y is



(a) $35 \mu\text{F}$
(c) $16.25 \mu\text{F}$

(b) $5 \mu\text{F}$
(d) $0.2 \mu\text{F}$

21. A copper slab of thickness b is symmetrically introduced between the plates of a parallel plate capacitor. The capacitance after the introduction of the slab is



(a) $\frac{2\epsilon_0 A}{(d-b)}$

(b) $\frac{\epsilon_0 A}{(d-b)}$

(c) $\frac{\epsilon_0 A}{\left(\frac{d}{2} - b \right)}$

(d) $\frac{\epsilon_0 A}{\left(d - \frac{b}{2} \right)}$

22. Two oppositely charged metal spheres of the same radius (R) are placed far apart. The capacitance of these two spheres is

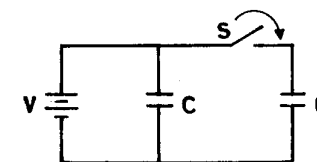
(a) $4\pi\epsilon_0 R$ (b) $8\pi\epsilon_0 R$
(c) $2\pi\epsilon_0 R$ (d) $\pi\epsilon_0 R$

23. Two metal spheres of radii a and b are connected by a thin wire. Their separation is large compared with their dimensions. A charge Q is introduced into this system. The capacitance of this system is

(a) $4\pi\epsilon_0 (ab)$ (b) $2\pi\epsilon_0 (a+b)$

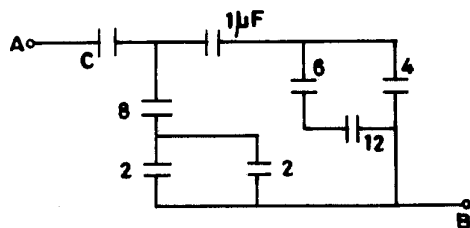
(c) $4\pi\epsilon_0 (a+b)$ (d) $4\pi\epsilon_0 \left(\frac{a^2 + b^2}{2} \right)$

24. The figure shows two identical parallel plate capacitors connected to a battery. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of $K = 3$. The ratio of the total electrostatic energy stored in both the capacitors before and after the introduction of the dielectric is



(a) $3/4$ (b) $4/5$
(c) $2/3$ (d) $3/5$

- 25*. In the given figure, a network of capacitors is shown. Each capacitance is in microfarads. If the equivalent capacitance between the points A and B is to be $1 \mu\text{F}$, the value of C should be



- (a) $\frac{13}{6} \mu\text{F}$ (b) $\frac{32}{23} \mu\text{F}$
 (c) $\frac{32}{19} \mu\text{F}$ (d) $\frac{13}{5} \mu\text{F}$

26. A dielectric slab of thickness b is inserted between the plates of a parallel plate capacitor of plate separation d . The capacitance of this capacitor is

- (a) $\frac{K\epsilon_0 A}{Kd - b(K-1)}$ (b) $\frac{\epsilon_0 A}{Kd + b(K-1)}$
 (c) $\frac{K\epsilon_0 A}{Kd + (K-1)b}$ (d) $\frac{K\epsilon_0 A}{Kd - b(K+1)}$

27. A parallel plate capacitor of plate area A is given a charge q and $-q$ on its plates. The two plates exert a force of attraction given by

- (a) $\frac{1}{4\pi\epsilon_0} \frac{q^2}{A}$ (b) $\frac{\epsilon_0 q^2}{2\pi A}$
 (c) $\frac{q^2}{2\epsilon_0 A}$ (d) $\frac{q^2}{\epsilon_0 A}$

28. A parallel plate capacitor of capacitance $500 \mu\text{F}$ and plate area 0.05 m^2 is filled with porcelain ($K = 6.5$). A potential difference of 200 V is applied. The following observations are recorded. Mark the one which is not correct.

- (a) the free charge on the plates is $0.1 \mu\text{C}$
 (b) the electric field in the porcelain is $3.4 \times 10^4 \text{ V/m}$
 (c) the induced surface charge is $8.4 \mu\text{C}$
 (d) the capacitance of the capacitor is $3.2 \times 10^{-3} \mu\text{F}$

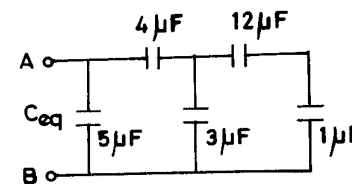
29. A parallel plate capacitor has plate area A and separation d and is charged to a potential difference V_0 . The charging battery is disconnected and the plates are pulled apart to three times the initial separation. The work required to separate the plates is

- (a) $\frac{3\epsilon_0 A V_0^2}{d}$ (b) $\frac{\epsilon_0 A V_0^2}{2d}$
 (c) $\frac{\epsilon_0 A V_0^2}{3d}$ (d) $\frac{\epsilon_0 A V_0^2}{d}$

30. A parallel plate capacitor is filled with a dielectric whose dielectric constant is 5 and has a dielectric strength of $20 \times 10^6 \text{ V/m}$. If the capacitance of this capacitor is $8.0 \times 10^{-2} \mu\text{F}$ and it could withstand a potential difference of 5000 V , the minimum plate area of the capacitor is

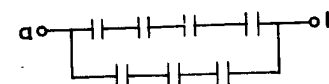
- (a) 0.45 m^2 (b) 0.62 m^2
 (c) 0.54 m^2 (d) 0.24 m^2

31. A network of five capacitors is shown in the figure here. The equivalent capacitance of this is

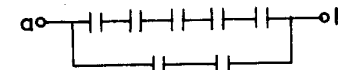


- (a) $1.913 \mu\text{F}$ (b) $3.66 \mu\text{F}$
 (c) $9.163 \mu\text{F}$ (d) $6.913 \mu\text{F}$

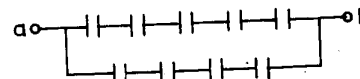
32. Given a bucketful of $1 \times 10^{-9} \text{ F}$ capacitors, you have to use a minimum number of capacitors and make a simple network of equivalent capacitance $0.7 \times 10^{-9} \text{ F}$. The possible circuits are:



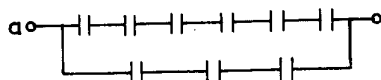
(a) a total of seven capacitors arranged as shown



(b) a total of seven capacitors arranged as shown



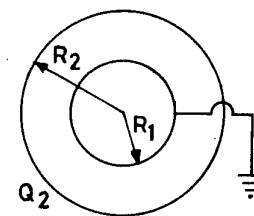
(c) a total of nine capacitors arranged as shown



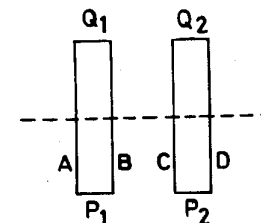
(d) a total of nine capacitors arranged as shown

33. An earthed sphere of radius R_1 is surrounded by another concentric sphere of radius R_2 ($R_2 > R_1$), having a charge Q_2 on it. The charge on the inner sphere is

- (a) $Q_1 = -Q_2$
 (b) $Q_1 = \frac{R_1}{R_2} Q_2$
 (c) $Q_1 = Q_2$
 (d) $Q_1 = \frac{-R_1}{R_2} Q_2$



34. Two infinite parallel conductors P_1 and P_2 have charges Q_1 and Q_2 on them. The charges on the sides A, B, C and D respectively are



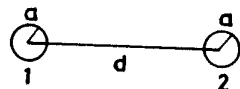
(a) $Q_A = Q_D = \frac{Q_1 + Q_2}{2}$ and $Q_B = -Q_C = \frac{Q_1 - Q_2}{2}$

(b) $Q_A = Q_C = \frac{Q_1 + Q_2}{2}$ and $Q_B = Q_D = \frac{Q_1 - Q_2}{2}$

(c) $Q_B = Q_C = \frac{Q_1 + Q_2}{2}$ and $Q_A = Q_D = \frac{Q_1 - Q_2}{2}$

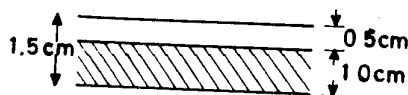
(d) $Q_A = Q_B = Q_C = Q_D = \frac{Q_1 + Q_2}{2}$

35. There are two uncharged identical metallic spheres of radius a , separated by a distance d . A charged metallic sphere (charge q) of same radius is brought and touches sphere 1. After some time it is moved away to a far off distance. After this, the sphere 2 is earthed. The charge on sphere 2 is



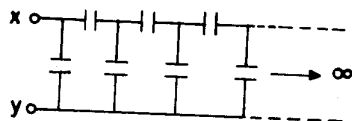
- (a) q (b) $-q$
(c) $-\frac{qa}{2d}$ (d) zero

36. A parallel plate capacitor consists of two square metal plates of size 5.0 cm, separated by 15 mm. A slab of pyrex glass ($K = 4.5$) 10 mm thick is placed on the lower plate. The capacitance of the capacitor is



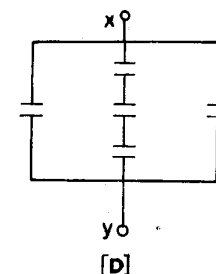
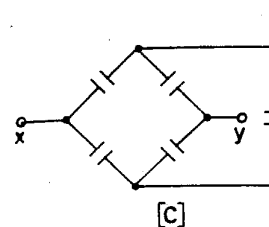
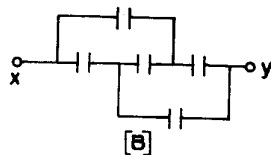
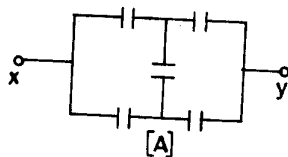
- (a) $4.5 \mu\text{F}$ (b) $60 \mu\text{F}$
(c) $6 \mu\text{F}$ (d) $3 \mu\text{F}$

- 37*. An infinite ladder of capacitors, each of capacitance $1 \mu\text{F}$, is shown here in the figure. The equivalent capacitance between points x and y is



- (a) $(1 + \sqrt{5}) \mu\text{F}$ (d) $(2 + \sqrt{5}) \mu\text{F}$
(b) $\left(\frac{1 + \sqrt{5}}{2}\right) \mu\text{F}$
(c) $\sqrt{5} \mu\text{F}$

38. Four ways of making a network of five capacitors of the same value are shown here. Three out of four are identical. The one which is different is



39. An air capacitor has plates of diameter 6 cm. The distance between the plates, so as to have the same capacitance as that of sphere of diameter 1 m, should be
- (a) 0.045 mm (b) 0.45 mm
(c) 4.5 mm (d) 0.54 mm

40. The thickness of the air layer between two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacity as a sphere of diameter 120 cm. The radius of its outer coating is

- (a) 12 cm (b) 10 cm
(c) 24 cm (d) 16 cm

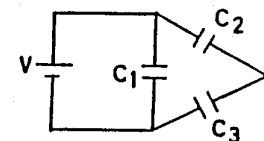
41. Two capacitors of capacitance C_1 and C_2 are charged separately to the same potential difference V , and subsequently, the positive terminal of one is connected to the negative terminal of the other. Then the two outermost connections are shorted together. The final charges on C_1 and C_2 respectively are

- (a) $\frac{(C_1 + C_2)VC_1}{C_1 - C_2}$ and $\frac{(C_1 - C_2)VC_1}{C_1 + C_2}$
(b) $\frac{(C_1 - C_2)VC_2}{C_1 + C_2}$ and $\frac{(C_1 + C_2)VC_1}{C_1 - C_2}$
(c) $\frac{(C_1 - C_2)VC_1}{C_1 + C_2}$ and $\frac{(C_1 - C_2)VC_2}{C_1 + C_2}$
(d) $\frac{(C_1 + C_2)VC_2}{C_1 - C_2}$ and $\frac{(C_1 + C_2)VC_1}{C_1 - C_2}$

42. In Problem 41, the loss in the electrostatic energy is

- (a) $\frac{2V^2C_1C_2}{C_1 - C_2}$ (b) $\frac{V^2C_1C_2}{C_1 + C_2}$
(c) $\frac{2V^2C_1C_2}{C_1 + C_2}$ (d) $\frac{V^2C_1C_2}{C_1 - C_2}$

43. C_1 , C_2 and C_3 respectively have capacitances 1, 2 and $3 \mu\text{F}$ and are connected to a battery of 100 V as shown in the figure. The electrical energy stored in the network is



- (a) 1.1 J (b) 1.1×10^{-4} J
(c) 11 J (d) 1.1×10^{-2} J

44. n identical capacitors are connected in parallel to a potential difference V . These

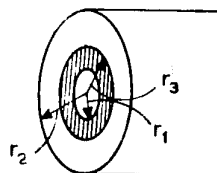
capacitors are then reconnected in series, their charges being left undisturbed. The potential difference obtained is

- (a) $\frac{V}{n}$ (b) nV
(c) n^2V (d) $(n-1)V$

45. An isolated metallic object is charged in vacuum to a potential V , its electrostatic energy being E . It is then disconnected from the source of potential, its charge being left unchanged, and is immersed in a large volume of dielectric with dielectric constant K . Its electrostatic energy is

- (a) $\frac{E}{K}$ (b) KE
(c) $\left(\frac{E}{V}\right)K$ (d) $\frac{V^2}{E}K$

- 46.* A capacitor is made of two concentric cylinders of radii r_1 and r_2 ($r_1 < r_2$) and length L ($L \gg r_2$). The region between r_1 and $r_3 = \sqrt{r_1 r_2}$ is filled with a circular cylinder of length L and dielectric constant K (the remaining volume is an air gap). The capacitance is

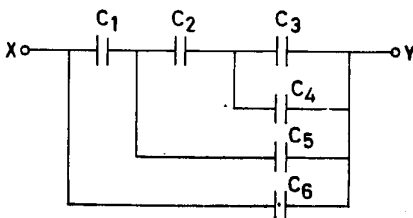


- (a) $\frac{\pi \epsilon_0 L}{\frac{1}{K} \ln \frac{r_3}{r_1} + \ln \frac{r_2}{r_3}}$ (b) $\frac{2\pi \epsilon_0 L}{K \ln \frac{r_3}{r_1} + \ln \frac{r_2}{r_1}}$
(c) $\frac{2\pi \epsilon_0 L}{\frac{1}{K} \ln \frac{r_3}{r_1} + \ln \frac{r_2}{r_3}}$ (d) $\frac{2\pi \epsilon_0 L}{\ln \frac{r_3}{r_1} + \frac{1}{K} \ln \frac{r_2}{r_3}}$

47. In the given network of capacitors as shown in the figure, the effective capacitance of the circuit between X and Y is

(given $C_1 = C_2 = C_3 = 400$ pF and $C_4 = C_5 = C_6 = 200$ pF)

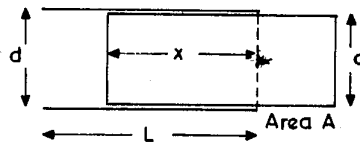
- (a) 810 pF
(b) 205 pF
(c) 600 pF
(d) 410 pF



48. In the above problem, if a voltage of 300 V is applied between X and Y, the potential difference across C_1 is

- (a) 157.7 V (b) 208.5 V
(c) 104.4 V (d) 84.6 V

49. In a capacitor with $C_0 = \epsilon_0 A/d$ (in vacuum) and a charge Q , a dielectric slab of dielectric constant K is inserted. If the inserted length is x and the edge effects are ignored the force on the slab is



- (a) attractive and equal to $\frac{Q}{2C_0 L} (K-1)$
(b) repulsive and equal to $\frac{Q}{2C_0 L} (K-1)$
(c) attractive and equal to $\frac{Q^2}{2C_0 L} \times \frac{K-1}{\left(1 + \frac{x}{L} (K-1)\right)^2}$
(d) repulsive and equal to $\frac{Q^2}{2C_0 L} \times \frac{K-1}{\left(1 + \frac{x}{L} (K-1)\right)^2}$

50. Fill in the blanks.

(a) The capacitance of a circuit element in which:

- (i) a voltage of 100 V yields an energy storage of an electric field of 0.05 J is _____
(ii) a voltage increases linearly from zero to 100 V in 0.2 s causing a current flow of 5 mA is _____
(b) The current which flows through a $1 \mu\text{F}$ capacitor when:
(i) the voltage increases linearly at the rate of 1000 V/s is _____
(ii) the energy storage remains fixed at 0.0005 J is _____

Answers

- | | | | | |
|------------------------------|-----------------------|--------------|-----------|---------|
| 1. (d) | 2. (c) | 3. (a) | 4. (d) | 5. (c) |
| 6. (c) | 7. (a) | 8. (c) | 9. (b) | 10. (a) |
| 11. (c) | 12. (d) | 13. (a) | 14. (d) | 15. (b) |
| 16. (b) | 17. (c) | 18. (d) | 19. (a) | 20. (b) |
| 21. (b) | 22. (c) | 23. (c) | 24. (d) | 25. (b) |
| 26. (a) | 27. (c) | 28. (c) | 29. (d) | 30. (a) |
| 31. (d) | 32. (b) | 33. (d) | 34. (a) | 35. (c) |
| 36. (d) | 37. (b) | 38. (d) | 39. (b) | 40. (a) |
| 41. (c) | 42. (c) | 43. (d) | 44. (b) | 45. (a) |
| 46. (c) | 47. (d) | 48. (a) | 49. (c) | |
| 50. (a) (i) $10 \mu\text{F}$ | (ii) $10 \mu\text{F}$ | (b) (i) 1 mA | (ii) zero | |

Note: Problems marked with a star should take about four minutes each and the rest should take two to three minutes.

21

Current, Resistance and Circuits

Current

Moving charges constitute current and the process whereby charges are transported is called conduction.

$$I = \frac{dQ}{dt}$$

I is expressed in amperes.

$$1A = \frac{1 \text{ coulomb}}{\text{second}}$$

In metals, the charge carriers are electrons while in electrolytes or in gaseous conductors they may be positive or negative ions or both. The sign convention is that all charge carriers are assumed to be positive, and current arrows are drawn in the direction in which such charges would move. If the charge carriers are negative, they simply move opposite to the direction of the current arrow.

Current Density

It is a microscopic quantity and is a vector. This vector $\vec{j}$ is in the direction that a positive charge carrier would move at that point. The magnitude of current density is $j = i/A$ where A is the cross-sectional area of the conductor. The relationship between $\vec{j}$ and I is

$$I = \int \vec{j} \cdot d\vec{s}$$

where $d\vec{s}$ is an element of surface area and the integral is taken over the surface in question.

Drift Velocity of charge carriers in a conductor is

$$V_d = \frac{I}{nAe} = \frac{j}{ne}$$

here n is the number of conduction electrons and e is its charge.

Resistance

If a potential difference V is applied between two points of a conductor and a current I flows in it, then a characteristic of the conductor, called resistance, is defined as

$$R = V/I \text{ (volt/ampere) (called ohm)}$$

Resistivity

It is the characteristic of a material rather than of a particular specimen of the material. It is defined as $\rho = E/j$ where E is the electric field and j is the current density, or $\rho = RA/l$, where A is the area of cross-section of the resistance R and l is the length of the resistance. The units of resistivity are ohm-metre.

Temperature Coefficient of Resistivity (α)

$$\alpha = \frac{1}{\rho} \cdot \frac{d\rho}{dT}$$

Here $\frac{d\rho}{dT}$ is the variation of resistivity with temperature T . The relation can also be expressed as

$$\rho = \rho_0 [1 + \bar{\alpha} (T - T_0)]$$

Here T_0 is 0°C , and ρ_0 is the resistivity at 0°C . The average temperature coefficient of resistivity $\bar{\alpha}$ is defined as

$$\bar{\alpha} = \frac{1}{\rho_0} \left[\frac{\rho - \rho_0}{T - T_0} \right]$$

Conductivity is the reciprocal of resistivity.

$$\sigma = \frac{1}{\rho}$$

and its units are mho/metre.

Variation of resistance with temperature

$$R = R_0 + \alpha R_0 (T - T_0)$$

The electric work (in joules) required to transfer a charge of Q coulombs through a potential difference of V volts is given by

$$W = Q \cdot V$$

The electric power (in watts) = $\frac{QV}{t} = VI$, where I is in amperes.

$$\text{Power Loss in a Resistor} = VI = I^2 R = \frac{V^2}{R}$$

Heat generated in a resistor = $I^2 R t$ where t is the time in seconds and heat is in joules.

Units of Electrical Power

One watt of work is done in an electric circuit when one ampere of current flows between two points having a potential difference of 1 V.

$$1 \text{ kW} = 10^3 \text{ W and } 1 \text{ MW} = 10^6 \text{ W}$$

and one horse power is equal to 746 W.

Electric Energy

The unit is the joule. A larger unit of energy is watt-hour (Wh).

$$1 \text{ Wh} = 3600 \text{ J}$$

$$\text{and } 1 \text{ kilowatt-hour (kWh)} = 1000 \times 3600 \text{ J} \\ = 36 \times 10^5 \text{ J}$$

1 kWh is also called Board of Trade Unit or simply 'Unit'. In terms of heat unit, $1 \text{ kWh} = (36 \times 10^5 \times 0.239) \text{ cal} = 8.6 \times 10^5 \text{ cal}$ ($1 \text{ J} = 0.239 \text{ cal}$)

Equivalent Resistance

$$(a) \text{ In series: } R = R_1 + R_2 + R_3 + \dots$$

$$(b) \text{ In parallel: } \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Kirchhoff's Laws

(a) Kirchhoff's point or junction rule

The sum of all currents coming into a point must equal the sum of all currents leaving the point.

(b) Kirchhoff's loop or circuit rule

On tracing a closed circuit, the algebraic sum of the potential changes encountered is zero. In this sum, a potential rise is positive and a potential drop is negative.

Current always flows from a higher potential to a lower potential through a resistor. As one traces through a resistor in the direction of the current, the potential change is negative because it is a potential drop.

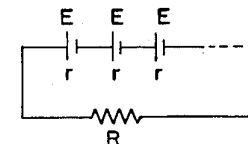
The positive terminal of the battery is regarded at higher potential, independent of the direction of the current through the emf source.

Grouping of cells

(a) All cells in series

Consider n cells, each of emf E and internal resistance r . Let R be the external resistance of the circuit. Then the current in the circuit is

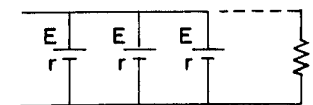
$$I = \frac{nE}{R + nr}$$



(b) All cells in parallel

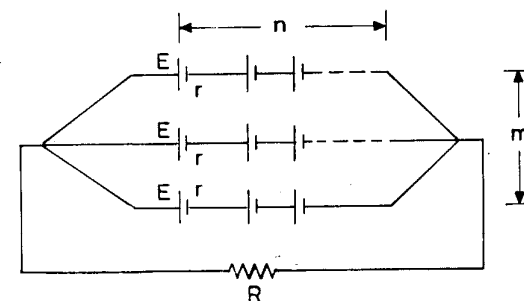
n cells each of internal resistance r are connected in parallel as shown

$$I = \frac{nE}{nR + r}$$



(c) Mixed grouping of cells

Let there be n cells in series and m be the number of such rows containing n cells each. Let r be the internal resistance of each cell and R , the external resistance of the circuit. The current in the circuit is



$$I = \frac{mnE}{mR + nr}$$

For maximum current,

$$mR = nr \text{ and } I_{\max} = \frac{E}{2} \sqrt{\frac{mn}{rR}}$$

ILLUSTRATIONS

1. A load of 1000 kg is to be lifted at a rate of 10 m/min with the help of a motor operating at 240 V and requiring a current of 10 A. Calculate the overall efficiency of the system.

440 Problems in Physics

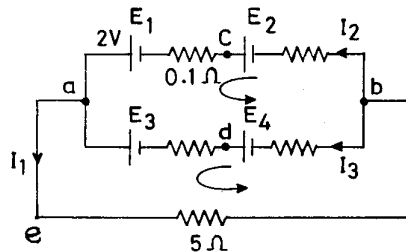
Solution

$$\text{Power input} = I \times V = 10 \text{ A} \times 240 \text{ V} = 2400 \text{ W}$$

$$\text{Power output} = F \times \text{velocity} = (1000) \times 9.8 \times \left(\frac{10}{60}\right) = 1633 \text{ W}$$

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{1633 \text{ W}}{2400 \text{ W}} = 0.68 = 68\%$$

2. Four cells, each of emf 2.0 V and internal resistance 0.1 Ω , are connected as shown in the figure. Find I_1 , I_2 and I_3 .



Solution

At point b , apply Kirchhoff's junction rule

$$I_1 = I_2 + I_3 \quad (\text{i})$$

Now apply Loop rule to $bcaeb$

$$-(0.1)I_2 + 2.0 - (0.1)I_2 + 2.0 - 5I_1 = 0$$

$$\text{or } -0.2I_2 + 4.0 - 5I_1 = 0 \text{ or } 5I_1 + 0.2I_2 = 4 \quad (\text{ii})$$

Now consider loop $bdaeb$

$$-(0.1) \times I_3 + 2 - (0.1)I_3 + 2 - 5I_1 = 0$$

$$-0.2I_3 + 4 - 5I_1 = 0 \text{ or } 5I_1 + 0.2I_3 = 4 \quad (\text{iii})$$

From Eqs (i), (ii) and (iii), we get $I_2 = I_3$ and $I_1 = 2I_2$

Substituting in (ii), we get

$$5 \times 2I_2 + 0.2I_2 = 4$$

$$10.2I_2 = 4 \Rightarrow I_2 = \frac{4}{10.2} = 0.392 \text{ A}$$

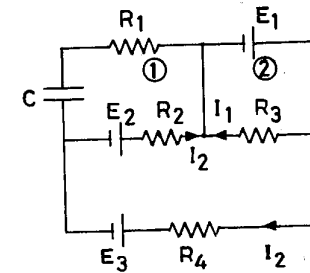
$$\therefore I_1 = 2 \times 0.392 = 0.784 \text{ A and } I_3 = 0.392 \text{ A}$$

3. In the given circuit, $E_1 = 3E_2 = 2E_3 = 6 \text{ V}$

$$R_1 = 2R_4 = 6 \Omega$$

$$R_3 = 2R_2 = 4 \Omega \text{ and } C = 5 \mu\text{F}$$

Find the current in R_3 and energy stored in C .



Solution

No current flows in the loop containing C (loop (1)). In loop (2),

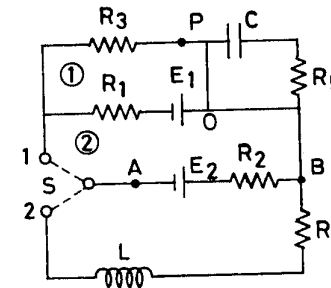
$$I_1 = \frac{E_1}{R_3} = \frac{6}{4} = 1.5 \text{ A}$$

$$I_2(R_2 + R_4) = E_1 + E_2 - E_3 = 6 + 2 - 3 = 5 \text{ V}$$

$$I_2 = \frac{5 \text{ V}}{5 \Omega} = 1.0 \text{ A; p.d. across } C = E_2 - R_2 I_2 = 2 - 2.0 = 0 \text{ V}$$

$$\text{Energy stored} = \frac{1}{2} CV^2 = 0$$

4. A circuit contains a two-position switch S as shown. The values of the elements used in the circuit are



$$R_1 = 2 \Omega, R_2 = 2 \Omega, R_3 = 2 \Omega,$$

$$R_4 = 3 \Omega, R_5 = 1 \Omega, C = 2 \mu\text{F}$$

$$E_1 = 12 \text{ V and } E_2 = 3 \text{ V, } L = 10 \text{ mH}$$

- The switch S is in position 1. Find the potential difference $V_A - V_B$.
- Find the rate of heat production in R_1 .
- The switch S is put in position 2 at $t = 0$. Find the steady current in R_4 .
- When switch S is in position 2, find the time when the current in R_4 is half the steady value.
- Calculate the energy stored in the inductor when the current is half the steady value.

Solution

No current flows in the loop containing C.

Applying Kirchhoff's loop law to (1)

$$-E_1 + I_1 R_1 + (I_1 + I_2) R_3 = 0 \text{ or } E_1 = I_1(R_1 + R_3) + I_2 R_3$$

$$\text{or } 12 = 4I_1 + 2I_2 \quad (1)$$

In loop (2):

$$I_2 R_2 - E_2 - I_1 R_1 + E_1 = 0 \text{ or } E_2 - E_1 = I_2 R_2 - I_1 R_1$$

$$\text{or } -9 = I_2 \times 2 - I_1 \times 2 \quad (2)$$

From (1) and (2), we get

$$I_2 = -1 \text{ A and } I_1 = \frac{7}{2} \text{ A}$$

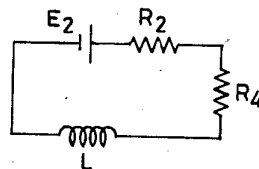
$$\therefore V_A - V_B = -E_2 + I_2 R_2 = -3 + (-1 \times 2) = -5 \text{ V}$$

$$(b) \text{ Joule's heat} = I_1^2 R_1 = \left(\frac{7}{2}\right)^2 \times 2 = \frac{49}{2} = 24.5 \text{ W}$$

(c) When switch (2) is closed, the circuit becomes as shown in figure here. At steady state,

$$\text{Current } I_s = \frac{V}{R_2 + R_4}$$

$$= \frac{3 \text{ V}}{5 \Omega} = 0.6 \text{ A}$$



Hence the current in R_4 in steady state is 0.6 A.

$$(d) I = I_s [1 - e^{-(R/L)t}]$$

$$\text{or } \frac{1}{2} = [1 - e^{-(R/L)t}]$$

$$\text{or } -\left(\frac{R}{L}\right)t = \ln \frac{1}{2} = -0.693$$

$$t = \frac{0.693 L}{R} = \frac{0.693 \times 10 \times 10^{-3}}{5} = 1.386 \text{ milli second}$$

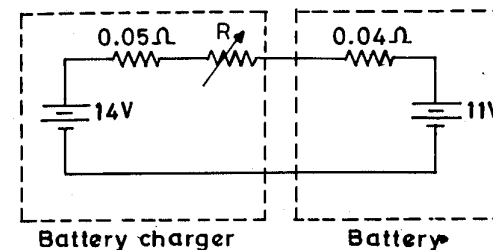
$$(e) \text{ Energy stored} = \frac{1}{2} Li^2$$

$$= \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{0.6}{2}\right)^2$$

$$= 4.5 \times 10^{-4} \text{ J}$$

5. For a battery charger as shown in the figure here:

(a) Find R so that a charging current of 2.5 A flows.



(b) Find R so that 25 W of power is delivered to the 11 V ideal voltage source.

(c) Find R so that 25 W is delivered to the combination of 0.04Ω and 11 V.

Solution

$$(a) I = 2.5 = \frac{14 - 11}{0.05 + R + 0.04}$$

$$\therefore R = 1.110 \Omega$$

$$(b) P_{11V} = 25$$

$$\therefore I = \frac{25}{11} = \frac{14 - 11}{0.05 + 0.04 + R} = \frac{3}{0.09 + R}$$

$$\Rightarrow R = 1.23 \Omega$$

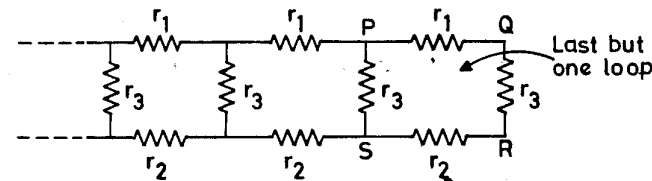
$$(c) 25 = 0.04 I^2 + 11 I$$

$$\therefore I = \frac{-11 \pm \sqrt{121 + 4}}{0.08} = 2.25 \text{ A}$$

$$2.25 = \frac{3}{0.09 + R}$$

$$\Rightarrow R = 1.24 \Omega$$

6. A network of resistances (called an infinite ladder) is shown in the figure here. If $r_1 = 2 \Omega$, $r_2 = 3 \Omega$ and $r_3 = 4 \Omega$, find the value of the resistance of this network.



Solution

The circuit is made up of an infinite number of repeated sections with the resistances r_1 , r_2 and r_3 as shown. Note that infinity will not change if we remove one element from it. Let us remove the last section on the right. What remains is still the same. Let the resistance of the arrangement be R . Considering the loop PQRS, the resistance across PS = $(r_1 + r_2 + R)$ and r_3 in parallel

$$= \frac{r_3(r_1 + r_2 + R)}{(r_1 + r_2 + r_3 + R)}$$

This value should be equal to R , so that we can write the last but one loop as shown in Fig. (a). By repeating this operation many times, the last loop network we are left with is of the type shown in Fig. (b). This means that

$$\frac{r_3(r_1 + r_2 + R)}{(r_1 + r_2 + r_3 + R)} = R$$

Putting the values of r_1 , r_2 and r_3 , we have

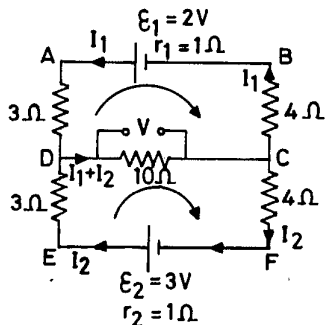
$$R = \frac{4(2 + 3 + R)}{(9 + R)}$$

$$\text{or } R^2 + 5R - 20 = 0$$

$$\text{or } R = \frac{-5 + \sqrt{105}}{2} = 2.6 \Omega$$

(negative value of R has been ignored.)

7. A network of five resistances and two batteries is shown in the figure here and their values are marked therein. Find the potential drop V , across the resistor of 10Ω .



Solution

Consider the loop ABCDA and apply Kirchhoff's voltage law:

$$3I_1 + 10(I_1 + I_2) + 4I_1 + 1I_1 = 2 \Rightarrow 9I_1 + 5I_2 = 1 \quad (1)$$

For loop DCFED:

$$10(I_1 + I_2) + 4I_2 + 1I_2 + 3I_2 = 3 \Rightarrow 10I_1 + 18I_2 = 3 \quad (2)$$

From (1) and (2), we get

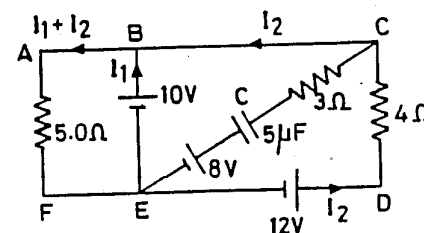
$$I_1 = \frac{3}{112} \text{ A and } I_2 = \frac{17}{112} \text{ A}$$

Total current flowing in the $10\text{-}\Omega$ resistor

$$I_1 + I_2 = \frac{3}{112} + \frac{17}{112} = \frac{20}{112} = \frac{5}{28} \text{ A}$$

$$\therefore \text{Voltage drop across the } 10\text{-}\Omega \text{ resistor} = \left(10 \times \frac{5}{28}\right) \text{ V} = \frac{50}{28} = 1.79 \text{ V}$$

8. A network consisting of three resistors, three batteries and a capacitor is shown in the figure here. Find the charge on the capacitor C in the steady state.



Solution

When a steady state is reached, no current passes through the capacitor and therefore, there is no current in the CE branch of the network.

Considering the loop ABEFA,

$$5(I_1 + I_2) = 10 \quad \text{or} \quad I_1 + I_2 = 2 \quad (1)$$

Considering the loop BCDEB,

$$4I_2 = 12 - 10 = 2 \Rightarrow I_2 = 0.5 \text{ A}$$

$$\therefore I_1 = 2 - 0.5 = 1.5 \text{ A}$$

To find the charge on the capacitor, we must know the potential difference across the plates.

Consider the closed loop CEDCE

$$-12 + 4I_2 + 3 \times (0) - V_c + 8 = 0$$

$$\text{or } -12 + 2 - V_c + 8 = 0$$

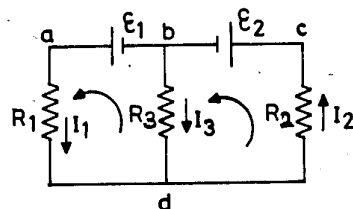
$$\text{or } V_c = -2 \text{ V}$$

(The negative sign indicates that the plate of the capacitor nearer to E is negative and the one nearer to C is positive.)

$$\therefore \text{Charge on the capacitor, } Q = CV = 5 \mu\text{F} \times 2.0 \text{ V} = 10 \mu\text{C}$$

9. Two batteries of emfs ϵ_1 and ϵ_2 and negligible internal resistance are

connected with three resistances R_1 , R_2 and R_3 as shown in the figure. Find the currents in the resistances R_1 , R_2 and R_3 .



Solution

There are two junctions (b and d) and three branches (bad, bd and bcd). Let us arbitrarily assume the current approaching to be positive and current leaving to be negative. Then at junction d, we have [the directions of current are marked at random]

$$I_1 + I_3 - I_2 = 0 \quad (1)$$

Applying Kirchhoff's loop theorem to loop abda:
Choosing a counterclockwise direction, we get

$$\varepsilon_1 - I_1 R_1 + I_3 R_3 = 0 \quad (2)$$

Similarly for the loop bdcb:

$$-I_3 R_3 - I_2 R_2 - \varepsilon_2 = 0 \quad (3)$$

There are three equations and three unknowns (I_1 , I_2 and I_3). Solving these equations, we get

$$I_1 = \frac{\varepsilon_1(R_2 + R_3) - \varepsilon_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \quad I_2 = \frac{\varepsilon_1 R_3 - \varepsilon_2(R_1 + R_3)}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$\text{and } I_3 = \frac{-\varepsilon_1 R_2 - \varepsilon_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

To find out whether the directions of current marked are correct or not, we shall have to put some values in the expressions obtained for the three currents.

Let us put $\varepsilon_1 = 8 \text{ V}$, $\varepsilon_2 = 12 \text{ V}$, $R_1 = 5 \Omega$, $R_2 = 8 \Omega$ and $R_3 = 12 \Omega$

$$I_1 = \frac{8(8 + 12) - 12 \times 12}{40 + 96 + 60} = \frac{160 - 144}{196} = \frac{16}{196} = 0.08 \text{ A}$$

in the direction marked in the circuit. (It turns out to be right.)

$$I_2 = \frac{8 \times 12 - 12(5 + 12)}{196} = \frac{96 - 204}{196} = -0.55 \text{ A}$$

The negative sign here shows that the direction of I_2 is the reverse of what is marked in the figure for this case.

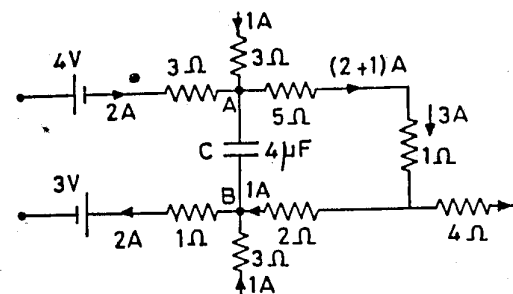
$$I_3 = \frac{-8 \times 8 - 12 \times 5}{196} = \frac{-64 - 60}{196} = \frac{-124}{196} = -0.63 \text{ A}$$

This direction also happens to be the reverse of what is marked in the figure for this case.

Note: Verify this by the junction theorem now.
Consider junction b. I_1 and I_2 leave the junction while I_3 enters it.

$$\therefore I_1 + I_2 = I_3 \text{ or } 0.08 + 0.55 = 0.63 \text{ A}$$

10. A part of the circuit in a steady state along with the currents flowing in the branches is shown in the figure here. Calculate the energy stored in the capacitor.



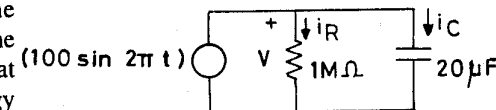
Solution

The current through 5Ω and 1Ω resistor is 3 A and that through the 2Ω resistor is 1 A .

$$\therefore \text{Potential difference across } C = 5 \times 3 + 1 \times 3 + 2 \times 1 = 20 \text{ V}$$

$$\text{Energy stored} = \frac{1}{2} CV^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (20)^2 = 800 \mu\text{J}$$

11. A sinusoidal voltage source is in parallel with a $1\text{-M}\Omega$ resistor and a $20 \mu\text{F}$ capacitor. The parallel resistor may be assumed to represent the resistance of the dielectric between the plates of the physical capacitor. What is the percentage of the total energy stored which is dissipated in the resistor?



Solution

$$\begin{aligned} \text{Current through resistor} &= \frac{V}{R} = \frac{100 \sin 2\pi t}{10^6 \Omega} \\ &= 10^{-4} \sin 2\pi t \end{aligned}$$

$$\begin{aligned} \text{and Current through capacitor} &= i_c = C \frac{dV}{dt} \\ &= 20 \times 10^{-6} \times \frac{d}{dt} (100 \sin 2\pi t) \\ &= 4\pi \times 10^{-3} \cos 2\pi t \end{aligned}$$

$$\text{Energy stored in capacitor, } U_c = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 20 \times 10^{-6} \times (100 \sin 2\pi)^2 = \frac{1}{10} \sin^2 2\pi$$

At $t = 0$, $U = 0$ and at $t = (1/4)$ s,

U is maximum and equals 0.1 J.

The energy again decreases to zero in another $(1/4)$ s. During the $(1/2)$ s interval, the energy dissipated in the resistor

$$U_R = \int_0^{0.5} P_R \cdot dt = \int_0^{0.5} \frac{V^2}{R} dt = \int_0^{0.5} \frac{(100 \sin 2\pi)^2}{10^6} dt$$

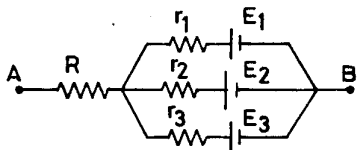
$$= \int_0^{0.5} 10^{-2} \sin^2 2\pi dt = 2.5 \text{ mJ}$$

Maximum stored energy = 0.1 J

Energy dissipated = 2.5 mJ

$$\text{Percentage lost} = \frac{2.5 \times 10^{-3}}{0.1} \times 100 = 2.5$$

12. In the given circuit, $E_1 = 3$ V, $E_2 = 2$ V, $E_3 = 1$ V, $R = r_1 = r_2 = r_3 = 1 \Omega$.



(i) Find the potential difference between points A and B and the currents through each branch.

(ii) If r_2 is short-circuited and point A is connected to point B, find the currents through E_1 , E_2 , E_3 and resistor R .

Solution

$$E_1 - I_1 r_1 = E_2 + I_2 r_2 = E_3 + I_3 r_3 \quad (1)$$

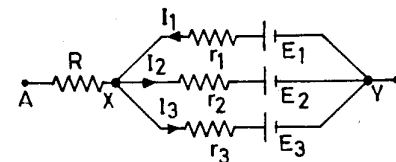
$$I_1 = I_2 + I_3 \quad (2)$$

$$3 - I_1 = 2 + I_2 = 1 + I_3; I_2 + I_1 = 1; I_2 - I_3 = -1$$

$$I_1 + I_3 = 2$$

$$\therefore I_1 = I_3 = 1 \text{ A} \quad \text{and} \quad I_2 = 0$$

Potential difference across AB = 2 V. The potential difference across XY is always 2 V. Hence, the current through $R = 2$ A.

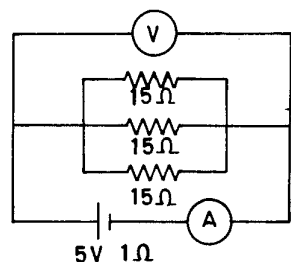


The current through E_1 and E_3 does not change since potential difference across XY remains unchanged.

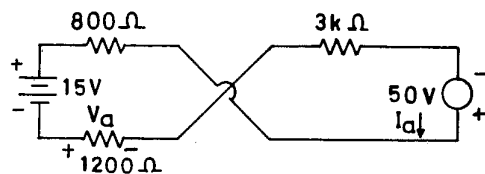
$\therefore$ Current through $E_2 = 2$ A.

EXERCISES

- A voltmeter connected to the terminals of a battery reads 6 V. When the lamps are lighted with the battery, the voltmeter reads 4 V. If the resistance of the lamps is 10Ω , the internal resistance of the battery is
(a) 0.5 Ω (b) 10 Ω
(c) 3 Ω (d) 5 Ω
- A wire with a resistance of 10.0Ω is drawn out so that the new length is three times its original length. If the resistivity and density remain constant during the drawing process, the resistance of the elongated wire is
(a) 80 Ω (b) 90 Ω
(c) 60 Ω (d) 40 Ω
- The temperature at which the resistance of a copper conductor would be twice its resistance at 0°C is (temperature coefficient of resistivity for copper = $3.9 \times 10^{-3}/^\circ\text{C}$)
(a) 256°C (b) 320°C
(c) 180°C (d) 400°C
- A current of 3 A exists in a 20Ω resistor for 5 minutes. The number of electrons that pass through any cross-section of the resistor in this time is
(a) 6.5×10^{17} (b) 6.5×10^{28}
(c) 5.6×10^{21} (d) 5.6×10^{24}
- A 1000 W heating unit is designed to operate on a 120 V line. The line voltage drops to 110 V. The percentage of heat output drops by
(a) 9% (b) 27%
(c) 16% (d) 30%
- The terminals of a battery of emf 12 V and negligible internal resistance are connected to two coils, each of resistance 50Ω joined together in series. A voltmeter of resistance 1000Ω is connected first across the coils and then across the terminals of the battery. The readings in the three cases are
(a) 6.15 V across each coil and 9.2 V across the battery
(b) 6.15 V across each coil and 12 V across the battery
(c) 3.5 V across each coil and 8 V across the battery
(d) 3.5 V across each coil and 12 V across the battery.
- Three 15Ω resistors are connected in parallel and a source of current of 5 V and internal resistance 1Ω is put in the circuit. The voltmeter and the ammeter readings respectively are

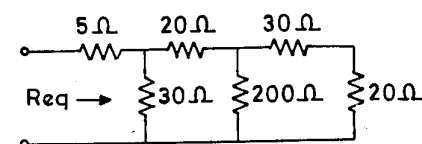


- (a) 4.16 V and 0.83 A
(b) 4 V and 0.83 A
(c) 0.83 V and 4 A
(d) 2.08 V and 0.41 A
8. Three batteries of negligible internal resistance and three resistors of 4, 8 and $12\ \Omega$ are connected as shown in the figure here. The current through the $12\ \Omega$ resistor is
- (a) 0.57 A
(b) 1.09 A
(c) 0.04 A
(d) 1.14 A
- 9*. In the given network of five batteries shown in the figure here, the batteries getting charged are
- (a) 1 and 3
(b) 1, 3 and 5
(c) 1 and 4
(d) 1, 2 and 5
10. A network of resistors and voltage sources is shown in the given figure. The potential difference V_x across the points a and b is
- (a) 4 V
(b) 6 V
(c) 8 V
(d) 12 V
11. In the given circuit, I_a , V_a and the power supplied by the 15 V battery respectively are

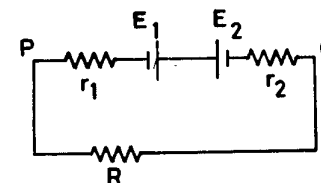


- (a) 7 mA, 8.4 V and 58.8 mW (b) 0.01 mA, 2 V and 0.02 mW
(c) 7 mA, 8.4 V and 105 mW (d) 0.01 mA, 8.4 V and 105 mW

12. In the given network of resistors, the equivalent resistance of the circuit is

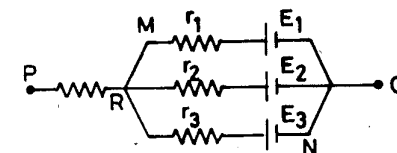


- (a) $20\ \Omega$
(b) $25\ \Omega$
(c) $60\ \Omega$
(d) $40\ \Omega$
13. Two batteries of emf 8 and 12 V and internal resistance $r_1 = 1\ \Omega$ and $r_2 = 2\ \Omega$ respectively are connected in a circuit with a resistance R of $9\ \Omega$ as shown in the figure. The current in the circuit and the potential difference between points P and Q respectively are
- 
- $R_{eq} \rightarrow$ $30\ \Omega$ $200\ \Omega$ $20\ \Omega$



- (a) $\frac{1}{3}$ A and 3 V (b) $\frac{2}{3}$ A and 11.33 V
(c) $\frac{1}{3}$ A and 11.0 V (d) $\frac{2}{3}$ A and 11.0 V

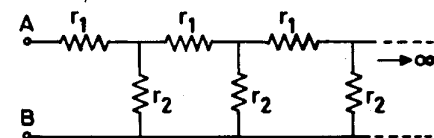
- 14*. In the given network of one resistor R and three batteries each of internal resistance $1\ \Omega$, shown in the given figure, the potential difference across the points P and Q is



[Given: $E_1 = 3 \text{ V}$, $E_2 = 2 \text{ V}$ and $E_3 = 1 \text{ V}$ and $r_1 = r_2 = r_3 = 1 \Omega$]

- (a) 4 V (b) 3 V
(c) 1 V (d) 2 V

- 15*. An infinite number of resistances r_1 and r_2 are connected to form a network of an infinite number of sections as shown in the figure here. The equivalent resistance of this network is



- (a) $r_1 \left(1 + \sqrt{\frac{r_2}{r_1}} \right)$

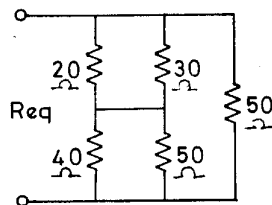
(c) $\frac{r_1}{4} \left(1 + \sqrt{\frac{r_2}{r_1}} \right)$

(b) $\frac{r_1}{2} \left(1 + \sqrt{1 + 4 \frac{r_2}{r_1}} \right)$

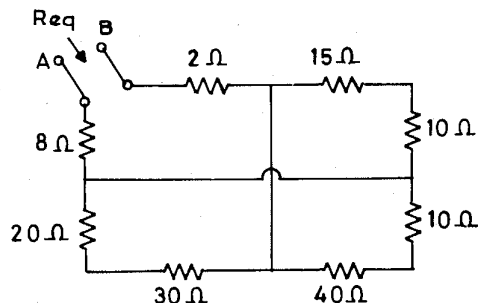
(d) $\frac{r_1}{2} \left(1 + \sqrt{2 \frac{r_2}{r_1}} \right)$

16. In the given network shown in the figure here, the equivalent resistance is

(a) 20.32Ω
 (b) 32.20Ω
 (c) 10.16Ω
 (d) 16.10Ω

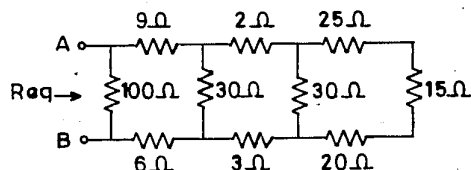


17. The equivalent resistance of the circuit across points A and B is



(a) 32.5Ω (b) 22.5Ω
 (c) 12.5Ω (d) 42.5Ω

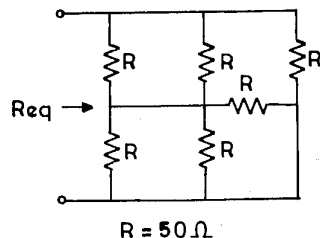
18. In the given three section network, the equivalent resistance between the points A and B is



(a) 42.64Ω (b) 32.42Ω
 (c) 22.26Ω (d) 12.13Ω

19. If $R = 50 \Omega$, the equivalent resistance of the network shown in the figure here is

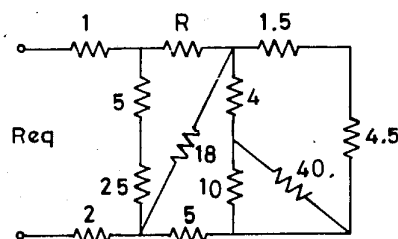
(a) 12.13Ω
 (b) 32.42Ω
 (c) 62.64Ω
 (d) 22.73Ω



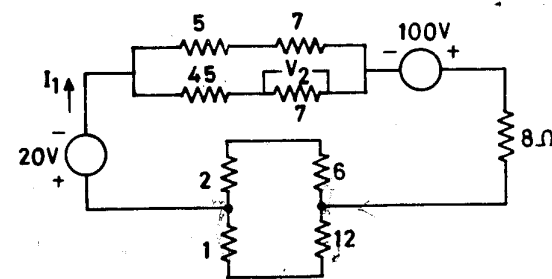
$R = 50 \Omega$

20. A network of twelve resistors is shown in the given figure here. The equivalent resistance of the network is 14Ω . The value of the resistor R is

(a) 11.37Ω
 (b) 22.74Ω
 (c) 34.11Ω
 (d) 17.05Ω

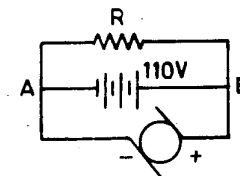


21. A combination of sources and resistances is shown in the figure here. The current I_1 and the potential difference across 7Ω resistor, V_2 , respectively are



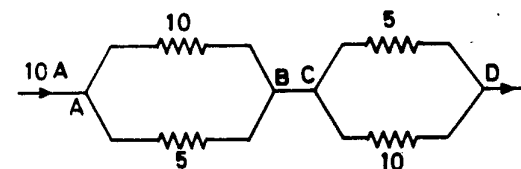
(a) 5 A and 12.5 V (b) 5 A and 17.5 V
 (c) 3.5 A and 12.5 V (d) 3.5 A and 17.5 V

22. A dc generator with an emf of 120 V and internal resistance of 1Ω , a resistance R and an accumulator of 110 V with negligible internal resistance are connected as shown in the figure



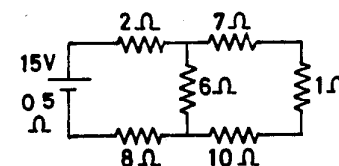
(a) current will flow from the accumulator if $R > 11 \Omega$
 (b) current will flow into the accumulator if $R > 11 \Omega$
 (c) no current will flow through the accumulator if $R < 11 \Omega$
 (d) no current will flow through the accumulator if $R > 11 \Omega$

23. Four resistances carrying current as shown are immersed in a box containing ice at 0°C . The quantity of ice that must be put in the box every 10 minutes to keep the average quantity of ice in the box constant is ($L_{\text{ice}} = 80 \text{ cal/g}$, $J = 418 \text{ J/cal}$)



(a) 0.916 kg (b) 0.691 kg
 (c) 1.691 kg (d) 1.196 kg

24. In the circuit shown here in the figure, the internal resistance of the battery is 0.5Ω . The current through the battery and the power dissipated in the 6Ω resistor respectively are



(a) 0.5 A and $\frac{24}{5} \text{ W}$

(b) 1 A and $\frac{24}{5} \text{ W}$

(c) 1 A and $\frac{27}{8} \text{ W}$

(d) 0.5 A and $\frac{27}{8} \text{ W}$

25. A wire of resistance of $0.1 \Omega/\text{cm}$ is bent to form a square ABCD of side 10 cm . A similar wire is connected between the corners B and D to form the diagonal BD. If a 2 V battery of negligible internal resistance is connected across A and C, the total power dissipated is

(a) 2 W

(b) 1.5 W

(c) 8 W

(d) 4 W

26. A galvanometer of resistance 100Ω gives full scale deflection when a current of 1 mA is passed through it. In order to have a full scale deflection for a current of 10 A , the resistance required is

(a) 0.01Ω

(b) 10.000Ω

(c) 0.100Ω

(d) 1.000Ω

27. The above modified galvanometer is connected across the terminals of a battery and shows a current of 4 A . The current drops to 1 A when a resistance of 1.5Ω is connected in series with the modified galvanometer. The emf and the internal resistance of the battery respectively are

(a) 1.8 V and 0.1Ω

(b) 1.8 V and 0.5Ω

(c) 2.04 V and 0.5Ω

(d) 2.04 V and 1Ω

28. In the given circuit shown here, $E_1 = 4 \text{ V}$, $E_2 = 8 \text{ V}$ and their internal resistances are 0.5Ω and 1Ω respectively. The potential differences across E_1 and E_2 respectively are

(a) 4.25 V and 7.5 V

(b) 4.25 V and 7.82 V

(c) 3.82 V and 8.20 V

(d) 3.82 V and 7.5 V

29. In the circuit shown, heat is generated at the rate of 10.24 cal/s in the 5Ω resistance develops due to the current flowing through it. The heat generated in the 2Ω resistor and the potential difference across 6Ω resistor respectively are

(a) 10.5 cal/s and 6.0 V

(b) 3.81 cal/s and 3.0 V

(c) 7.13 cal/s and 5.64 V

(d) 6.0 cal/s and 7.62 V

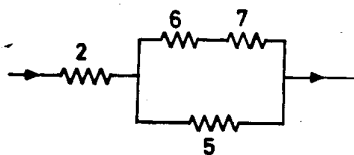
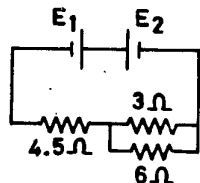
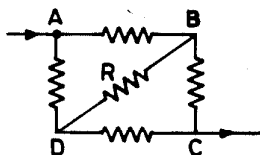
30. An electric kettle has two heating coils. When one of the coils is switched on, the water in the kettle begins to boil in 6 minutes . When the other coil is switched on, the water begins to boil in 8 minutes . Now the two coils are connected simultaneously, first in *series* and then in *parallel*. The time required for the water to boil in the two cases respectively are

(a) 7.0 min and 1.7 min

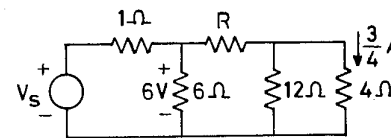
(b) 21.0 min and 6.86 min

(c) 10.0 min and 4.0 min

(d) 14 min and 3.43 min



- 31*. In the circuit shown here, the values of R and V_s respectively are



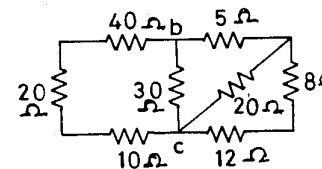
(a) 3Ω and 8 V

(b) 6Ω and 7 V

(c) 3Ω and 6 V

(d) 6Ω and 3 V

32. For the network of resistances shown here, the equivalent resistance of the circuit as seen between points b and c is



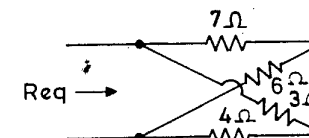
(a) 30Ω

(b) 25Ω

(c) 20Ω

(d) 40Ω

33. In the given network of four resistances, the equivalent resistance is



(a) 20Ω

(b) 5.4Ω

(c) 12Ω

(d) 4.5Ω

34. Six identical resistors (R) are joined to form a hexagon. Six more resistors (again of same value, R) are connected between the six vertices and the centre of the hexagon. The equivalent resistance between opposite vertices is

(a) $\frac{5}{4} R$

(b) $\frac{4}{5} R$

(c) $\frac{3}{5} R$

(d) $\frac{4}{3} R$

35. In the above problem, the equivalent resistance between any two adjacent vertices is

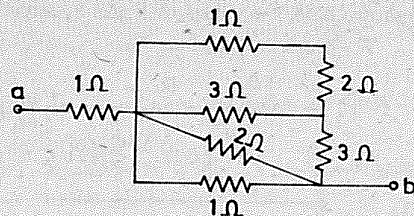
(a) $\frac{20}{11} R$

(b) $\frac{9}{20} R$

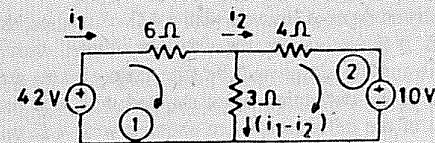
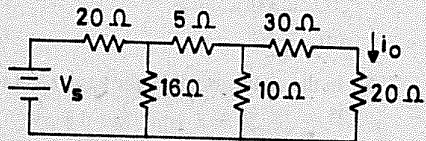
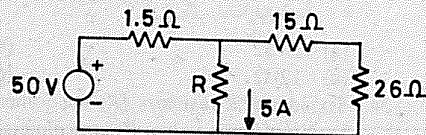
(c) $\frac{7}{20} R$

(d) $\frac{11}{20} R$

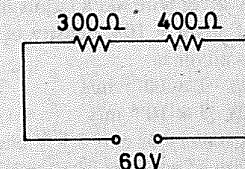
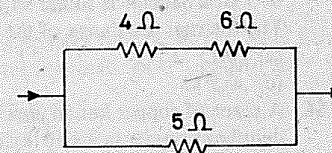
36. A current of 5 A flows into the circuit as shown in the figure. The potential difference between points a and b is



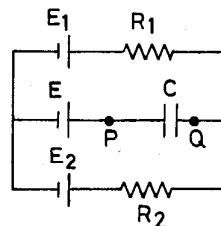
- (a) 9.7 V (b) 3.4 V
(c) 7.9 V (d) 6.4 V
37. A coil develops 1000 cal/s when 50 V is supplied across its ends. The resistance of the coil is
(a) 1.6 Ω (b) 0.06 Ω
(c) 6.6 Ω (d) 0.6 Ω
38. A line having a total resistance of 0.5 Ω delivers 15 kW at 240 V to a small factory. The efficiency of transmission is
(a) 78% (b) 89%
(c) 68% (d) 97%
- 39*. Given three resistors of 1 Ω, 2 Ω and 3 Ω, suitable combinations of these can give the following equivalent resistances:
(a) 6, 6/11, 4/3, 5/7, 3/4, 11/3, 11/6, 11/7
(b) 6, 6/11, 4/5, 4/7, 5/6, 11/3, 11/6, 11/5
(c) 6, 6/11, 3/4, 5/6, 3/2, 11/7, 11/2, 11/3
(d) 6, 6/11, 4/3, 5/6, 3/2, 11/5, 11/4, 11/3
40. A 0.4 W bulb is designed for operation with 2 V across its terminals. A resistance R is placed in parallel with the bulb and the combination is connected in series with a 3 Ω resistor and a 3 V battery (of internal resistance 1/3 Ω). If the bulb is to operate at designed voltage, the value of R should be
(a) 10 Ω (b) 12 Ω
(c) 20 Ω (d) 25 Ω
41. In the given circuit, a constant source of voltage 50 V along with four resistors is connected as shown in the figure. If a current of 5 A flows through the resistor R , its value is
(a) 2.8 Ω (b) 3.8 Ω
(c) 5.6 Ω (d) 8.2 Ω
42. In the three-section resistive ladder, assume that $i_0 = 1$ A. The value of V_s is
(a) 75 V (b) 300 V
(c) 60 V (d) 220 V
43. In the above problem, if $V_s = 100$ V, the current i_0 is
(a) 0.03 A (b) 3.3 A
(c) 0.66 A (d) 0.33 A
44. Two constant sources of voltages 42 V and 10 V are connected to a resistive circuit as shown in the figure. The current in the 3 Ω resistor is



- (a) 2 A (b) 3 A
(c) 0.2 A (d) 1.2 A
45. A microammeter has a resistance of 100 Ω and a full-scale deflection of 50 μA. It can be used as a voltmeter or as a high range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s):
(a) 50 V with 10 kΩ in series
(b) 5 mA range with 1 Ω in parallel
(c) 10 V with 100 kΩ in series
(d) 10 mA range with 1 Ω in parallel
46. In the circuit shown here, the heat produced in the 5 Ω resistor due to current flowing in it is 10 cal/s. The heat generated in 4 Ω resistor is
(a) 1 cal/s
(b) 2 cal/s
(c) 3 cal/s
(d) 4 cal/s
47. In the circuit shown here, a voltmeter reads 30 V when it is connected across the 400 Ω resistance. The same voltmeter is now connected across the 300 Ω resistance. Its reading will be
(a) 20.6 V (b) 22.5 V
(c) 24.6 V (d) 27.4 V
48. A constant voltage is applied between the two ends of a uniform metallic wire. Some heat is developed in it. The heat developed is doubled if
(a) both the length and radius of the wire are halved
(b) both the length and radius of the wire are doubled
(c) the radius of the wire is doubled
(d) the length of the wire is doubled and the radius of the wire is halved
49. A potentiometer wire of length 100 cm has a resistance of 10 Ω. It is connected in series with a resistance and a cell of emf 2 V and of negligible internal resistance. A source of emf of 10 millivolts is balanced against a length of 40 cm of potentiometer wire. The value of the resistance is
(a) 970 Ω (b) 490 Ω
(c) 395 Ω (d) 790 Ω
- 50*. A galvanometer of resistance 20 Ω gives a full-scale deflection when a current of 0.04 A is passed through it. It is desired to convert it into an ammeter reading 20 A in full scale. The only shunt available is a 0.05 Ω resistance. The resistance that must be connected in series with the coil of the galvanometer is
(a) 4.95 Ω (b) 5.94 Ω
(c) 9.45 Ω (d) 12.62 Ω



- 51*. When a galvanometer is shunted with a $4\ \Omega$ resistance, the deflection is reduced to one-fifth. If the galvanometer is further shunted with a $2\ \Omega$ wire, the further reduction in the deflection will be (the main current remains the same.)
 (a) $(8/13)$ of the deflection when shunted with $4\ \Omega$ only
 (b) $(5/13)$ of the deflection when shunted with $4\ \Omega$ only
 (c) $(3/4)$ of the deflection when shunted with $4\ \Omega$ only
 (d) $(3/13)$ of the deflection when shunted with $4\ \Omega$ only.
52. Two identical cells of the same emf and internal resistance give the same current through an external resistance of $2\ \Omega$, regardless of whether the cells are connected in series or parallel. The internal resistance of each cell is
 (a) $0.5\ \Omega$ (b) $1\ \Omega$
 (c) $3\ \Omega$ (d) $2\ \Omega$
- 53*. In an experiment to measure the internal resistance of a cell, it is found that no current passes through the galvanometer when the terminals of a cell are connected across $52\ \text{cm}$ of the potentiometer wire. If the cell is shunted by a resistance of $5\ \Omega$, a balance is found when the cell is connected across $40\ \text{cm}$ of the wire. The internal resistance of the cell is
 (a) $0.15\ \Omega$ (b) $1.5\ \Omega$
 (c) $0.5\ \Omega$ (d) $0.05\ \Omega$
54. A piece of copper has an area of cross-section of $10^{-3}\ \text{m}^2$ and length $1\ \text{m}$. If the density of copper is $9 \times 10^3\ \text{kg/m}^3$ and its atomic weight is $63\ \text{g/mole}$, the number of free electrons in this piece is (assuming there is one free electron per atom)
 (a) 8.6×10^{23} (b) 4.3×10^{25}
 (c) 8.6×10^{25} (d) 4.3×10^{23}
55. A silver wire has a radius of $0.1\ \text{cm}$ and carries a current of $2\ \text{A}$. [Atomic weight of silver = 108 , density of silver is $10.5\ \text{g/cc}$]. The drift velocity of electrons in silver is
 (a) $7 \times 10^{-10}\ \text{m/s}$ (b) $7 \times 10^{10}\ \text{m/s}$
 (c) $7 \times 10^{-4}\ \text{m/s}$ (d) $7 \times 10^4\ \text{m/s}$
56. A battery of emf $2.1\ \text{V}$ and internal resistance $0.05\ \Omega$ is shunted for $5\ \text{s}$ by a wire of constant resistance $0.02\ \Omega$, mass $1\ \text{g}$ and specific heat 0.1 . The rise in the temperature of the wire is
 (a) 10.7°C (b) 21.4°C
 (c) 107°C (d) 214°C
57. The armature of a shunt wound motor has a resistance of $1\ \Omega$. When the motor is running at normal speed and delivering $2\ \text{hp}$, the current in the armature is $7\ \text{A}$ when the voltage of the mains is $220\ \text{V}$. The rate of heat that is developed in the armature and the back emf of the motor respectively are
 (a) $11.6\ \text{cal/s}$ and $13.6\ \text{V}$ (b) $5.8\ \text{cal/s}$ and $6.8\ \text{V}$
 (c) $11.6\ \text{cal/s}$ and $6.8\ \text{V}$ (d) $23.2\ \text{cal/s}$ and $13.6\ \text{V}$
58. The temperature coefficient of resistance of a wire is $0.00125/^\circ\text{C}$. At $300\ \text{K}$, its resistance is $1\ \Omega$. The resistance of the wire will be $2\ \Omega$ at
 (a) $1127\ \text{K}$
 (b) $1400\ \text{K}$
 (c) $1154\ \text{K}$
 (d) $1200\ \text{K}$
59. In the circuit given here, the batteries $E = E_1 = 1.0\ \text{V}$, $E_2 = 2.5\ \text{V}$ and resistances $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$. The potential difference across the capacitor C is



- (a) $0.2\ \text{V}$ (b) $0.4\ \text{V}$
 (c) $1.2\ \text{V}$ (d) $0.5\ \text{V}$
60. A straight wire of length $1\ \text{km}$ carries a current of $50\ \text{A}$. If e/m for electrons is $0.17 \times 10^{12}\ \text{C/kg}$, the total momentum of electrons in this wire is
 (a) $0.92\ \mu\text{N.s}$ (b) $0.29\ \mu\text{N.s}$
 (c) $2.9\ \mu\text{N.s}$ (d) $9.2\ \mu\text{N.s}$
61. A resistance R carries a current I . The heat loss to the surroundings is $\lambda(T - T_0)$ where λ is a constant, T is the temperature of the resistance, and T_0 is the temperature of the atmosphere. If the coefficient of linear expansion is α , the strain in the resistance is
 (a) proportional to the length of the resistance wire
 (b) equal to $\frac{\alpha}{\lambda} I^2 R$
 (c) equal to $\frac{1}{2} \frac{\alpha}{\lambda} I^2 R$
 (d) equal to $\alpha \lambda (IR)$
62. The space between two concentric metallic spheres of radii R_1 and R_2 is filled with a material of conductivity σ . The total resistance of the system is
 (a) $\frac{R_2 - R_1}{4\pi R_1 R_2 \sigma}$ (b) $\frac{R_1 + R_2}{4\pi R_1 R_2 \sigma}$
 (c) $\frac{R_1 + R_2}{4\pi R_1 R_2 \sigma^2}$ (d) $\frac{R_2 - R_1}{R_1 R_2 \sigma}$

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (b) | 3. (a) | 4. (c) | 5. (c) |
| 6. (b) | 7. (a) | 8. (d) | 9. (c) | 10. (b) |
| 11. (c) | 12. (b) | 13. (a) | 14. (d) | 15. (b) |
| 16. (a) | 17. (b) | 18. (c) | 19. (d) | 20. (a) |
| 21. (b) | 22. (b) | 23. (d) | 24. (c) | 25. (d) |
| 26. (a) | 27. (c) | 28. (a) | 29. (c) | 30. (d) |
| 31. (a) | 32. (c) | 33. (d) | 34. (b) | 35. (d) |
| 36. (c) | 37. (d) | 38. (b) | 39. (d) | 40. (c) |
| 41. (d) | 42. (b) | 43. (d) | 44. (a) | 45. (b) |
| 46. (b) | 47. (b) | 48. (a) | 49. (d) | 50. (a) |
| 51. (a) | 52. (d) | 53. (b) | 54. (c) | 55. (c) |
| 56. (d) | 57. (c) | 58. (a) | 59. (d) | 60. (b) |
| 61. (b) | 62. (a) | | | |

Note: Problems marked with an asterisk should take about four minutes each and the rest should take about two to three minutes each.

22

Electromagnetics

Magnetic Field

A magnetic field is defined as the space around a magnet or a current carrying conductor as the site of a magnetic field. The basic magnetic field vector $\vec{B}$ is called magnetic induction. The magnetic flux ϕ_B for a magnetic field is defined as

$$\phi_B = \int \vec{B} \cdot d\vec{s}$$

in which the integral is taken over the surface (closed or open) for which ϕ_B is defined.

Force on a moving charge

$$\vec{F} = q_0 \vec{V} \times \vec{B}$$

If a positive test charge q_0 is fired with velocity $\vec{V}$ through a point where a magnetic induction $\vec{B}$ is present, then the force experienced by the test charge $\vec{F}$ is given by the cross product of $(q_0 \vec{V})$ and $\vec{B}$.

$$|\vec{F}| = q_0 V B \sin \theta$$

The direction of the force on a charge $+q$ in a magnetic field can be found from a right hand rule:

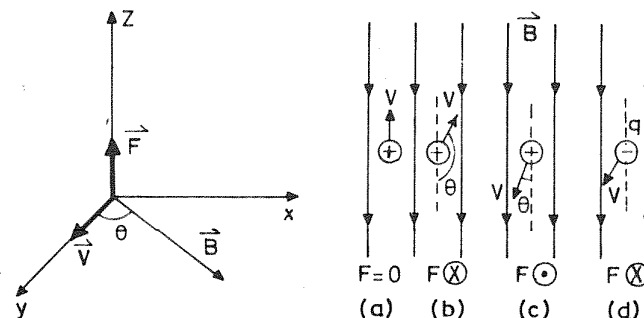
Hold the right hand flat with palm up. Point its fingers in the direction of the field. Orient the thumb along the direction of the velocity of the positive charge. Then the palm of the hand pushes in the direction of the force on the charge. The direction of force on a negative charge is opposite to that on a positive charge.

Some illustrations to determine the direction of force on a moving charge are shown here:

(a) A positive charge moves opposite to the direction of magnetic field. The force experienced by the charge is zero as shown in Fig. (a).

(b) A positive charge moves with velocity V at an angle θ with $\vec{B}$ as shown in Fig. (b). The force on the charge is directed into the plane of the page.

(c) A positive charge moves with a velocity V making an angle θ with $\vec{B}$ as shown in Fig. (c). The force experienced by the charge is out of the plane of the page.



⊗ means into the page

⊙ means out of the page

(d) A negative charge moves with a velocity V making an angle θ with the direction of $\vec{B}$ as shown in Fig. (d). The force experienced by the moving charge is directed into the plane of the page.

The unit of magnetic induction is $\frac{1 \text{ N}}{\text{A} \cdot \text{m}}$

and is equal to

$$\begin{aligned} \frac{1 \text{ Weber}}{\text{metre}^2} &= 1 \text{ Tesla} \\ &= 10^4 \text{ gauss} \end{aligned}$$

Force on a Charge Moving in an Electric and a Magnetic Field

$$\vec{F} = q_0 \vec{E} + q_0 \vec{V} \times \vec{B} \quad (\text{This is known as Lorentz force.})$$

Magnetic Force on a Current-Carrying Wire

If a wire of length l , carrying a current i , is placed in a magnetic field of strength $\vec{B}$, it experiences a force given by

$$\vec{F} = i \vec{l} \times \vec{B}$$

where $\vec{l}$ is a vector (displacement) that points along the straight wire in the direction of the current. For a differential element of a conductor of length $d\vec{l}$, the force $d\vec{F}$ acting on it can be found using the relation

$$d\vec{F} = i d\vec{l} \times \vec{B}$$

Torque on a Current Loop

If a loop of wire of length a and width b is placed in a uniform field of induction $\vec{B}$, a torque τ acts on it. This is given by

$$\tau = iabB \sin \theta$$

where θ is the angle which the normal to the plane of the loop makes with the direction $\vec{B}$. If there are N turns in the loop, then the torque,

$$\tau = NiAB \sin \theta = NiAB \sin \theta$$

where A is the area of the coil. This equation holds good for all plane loops of any shape.

Relation between Torque and Magnetic Dipole Moment of Loop

A current loop behaves like a dipole. When a dipole is placed in a magnetic field, it experiences a torque,

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where $\vec{\mu}$ is the magnetic dipole moment.

$$|\vec{\mu}| = NiA$$

Magnetic potential energy in any position θ is defined as the work that an external agent must do to turn the dipole from its zero-energy position ($\theta = 90^\circ$) to the given position θ .

$$U = -\mu B \cos \theta$$

and in vector form,

$$U = -\vec{\mu} \cdot \vec{B}$$

Motion of a Charged Particle in a Magnetic Field

If a charged particle of charge q and mass m moves with a velocity V in a uniform magnetic field B , then the radius R of the circular orbit in which it moves is

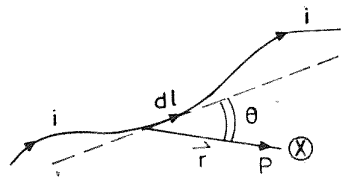
$$R = \frac{mV}{qB}$$

The Biot-Savart Law

An arbitrary current distribution consisting of a curved wire carrying a current i is shown here in the figure. A typical current element $d\vec{l}$ of the conductor is shown and its direction is at a tangent to the conductor (dashed line). The magnetic field at a point P is given by

$$dB = \frac{\mu_0 i dl \sin \theta}{4\pi r^2}$$

Here r is the displacement vector from the element to the point P and θ is the angle between this vector and $d\vec{l}$, μ_0 is permeability constant and has a value of $4\pi \times 10^{-7}$ Wb/A.m. The direction of $d\vec{B}$ is that of the vector $d\vec{l} \times \vec{r}$. In the case shown in the figure, $d\vec{B}$ is into the plane of the page.



Vector Form of Biot-Savart Law

$$d\vec{B} = \frac{\mu_0 i d\vec{l} \times \vec{r}}{4\pi r^3}$$

Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Faraday's law

$$\varepsilon = -\frac{d\phi}{dt}$$

or induced emf in the circuit is numerically equal to the rate of change of magnetic flux through it.

Lenz's Law

An induced emf always has such a direction as to oppose the change in magnetic flux that has produced it. For example, if the flux is increasing through a coil, the current produced by the induced emf will generate a flux that tends to cancel the increasing flux.

Motional emf

As the induced emf develops because of associated change of flux, a conductor when moving in a magnetic field develops induced emf and behaves like a source of emf. The induced emf in a straight conductor of length L moving with a velocity $\vec{V}$ perpendicular to a field $\vec{B}$ is given by

$$\varepsilon = B \cdot L \cdot V$$

where $\vec{B}$, $\vec{V}$ and the wire must be mutually perpendicular.

Permeability and Relative Permeability

Suppose a long solenoid carrying a current is placed in vacuum. The magnetic field at a point inside the solenoid is B_v . Now the solenoid is filled with a material and the magnetic field at that point becomes B . Then the relative permeability of the material is defined as

$$K_m = \frac{B}{B_v}$$

and the permeability of the material $\mu = K_m \cdot \mu_0$ where μ_0 is the permeability of free space.

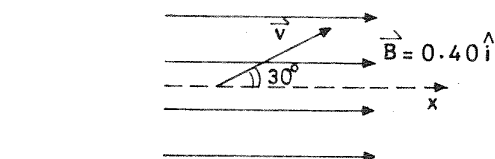
$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

ILLUSTRATIONS

1. A proton is fired with a speed of 5×10^7 m/s at an angle of 30° to a magnetic field $\vec{B} = 0.40 \hat{i}$ T. Describe fully the path followed by the proton.

Solution

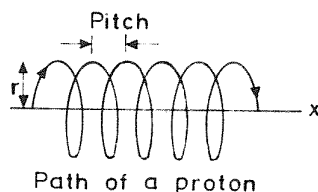
The proton will move on a circular path of radius



$$r = \frac{mV \sin \theta}{qB}$$

$$= \frac{(1.67 \times 10^{-27} \text{ kg})(5 \times 10^7 \text{ m/s}) \times 0.5}{(1.6 \times 10^{-19} \text{ C})(0.40 \text{ T})}$$

$$= \frac{1.67 \times 5 \times 0.5}{1.6 \times 0.4} \times 10^{-1} = 0.65 \text{ m}$$



The proton will spiral along the X-axis.

$$\text{Period} = \frac{2\pi r}{V_{\perp}} = \frac{2\pi r}{V \sin \theta} = \frac{2\pi(0.65)}{5 \times 10^7 \times 0.5} = 1.6 \times 10^{-7} \text{ s}$$

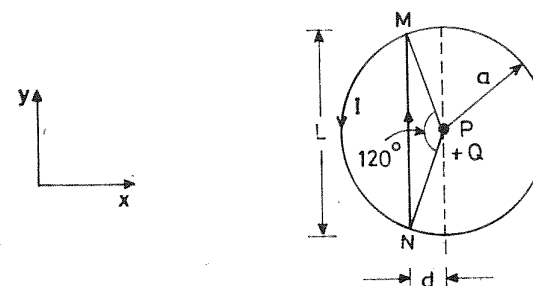
$$\text{Pitch} = (V_{\parallel}) (\text{period}) = (V \cos \theta) (T)$$

$$= \left(5 \times 10^7 \times \frac{\sqrt{3}}{2}\right) (1.6 \times 10^{-7}) = 6.9 \text{ m}$$

2. A wire loop carrying current I placed in that X-Y plane is shown in the figure here. If a particle of mass m and charge $+Q$ is placed at the centre P and given a velocity $\vec{V}$ along NP,

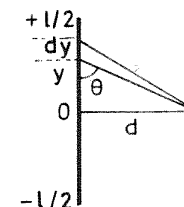
(a) Find its instantaneous acceleration.

(b) If an external uniform magnetic field $\vec{B} = B\hat{i}$ is applied, find the force and the torque acting on the loop due to this field.



Solution

(a) $\vec{B}_1$ due to arc MN will be directed out of the page $\odot$, while $\vec{B}_2$ due to line segment MN, will be directed into the page $\otimes$. From here we conclude that the total field at P will be directed into the page, i.e.



$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\otimes = \odot + \otimes$$

$$\vec{B}_1 \text{ at P} = \frac{\mu_0}{4\pi} I \left(\frac{2\pi a}{3} \right) \frac{a}{a^3} = \frac{\mu_0 I}{6a} \hat{k} \odot$$

$$\vec{B}_2 \text{ at P} = \frac{\mu_0}{4\pi} Id \int_{-L/2}^{+L/2} \frac{dy}{(y^2 + d^2)^{3/2}}$$

$$= \frac{\mu_0 IL}{2\pi d \sqrt{L^2 + 4d^2}}$$

$$\text{As } L = a/\sqrt{3} \text{ and } d = a/2,$$

$$\vec{B}_2 = \frac{\sqrt{3} \mu_0 I}{2\pi a} (-\hat{k}) \otimes$$

$$\text{Total field at P} = \left(\frac{\mu_0 I}{6a} - \frac{\sqrt{3} \mu_0 I}{2\pi a} \right) \hat{k} = -\frac{\mu_0 I}{2a} \left(\frac{\sqrt{3}}{\pi} - \frac{1}{3} \right) \hat{k}$$

$$\vec{F} = Q\vec{V} \times \vec{B} = m\vec{a}$$

$$\therefore \vec{a} = \frac{Q}{m} V \frac{\mu_0 I}{2a} \left(\frac{\sqrt{3}}{\pi} - \frac{1}{3} \right) \hat{k} \text{ at } 150^\circ \text{ from the positive X-axis.}$$

$$(b) \vec{B}_{\text{ext}} = B\hat{i} \text{ and is uniform, } \vec{F} = 0$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}_{\text{ext}}; \vec{\mu} = I \cdot A \cdot \hat{k}$$

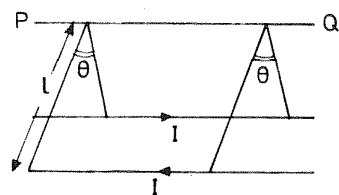
$$= IA\hat{k} \times B\hat{i} = I \cdot AB\hat{j}$$

$$(\hat{k} \times \hat{i} = \hat{j})$$

$$A = \frac{\pi a^2}{3} - \frac{1}{2} L \cdot d = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) a^2$$

$$\vec{\tau} = I \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) a^2 B_{\text{ext}} \hat{j} \text{ or } \vec{\tau} \text{ is along the } +Y\text{-axis.}$$

3. Two long parallel wires carry a current I of equal magnitude but flowing in opposite directions. These wires are suspended from a rod PQ by four chords of the same length l as shown in the figure. The mass per unit length of the wire is λ . Determine the value of θ , assuming it is small.



Solution

$$\text{Separation} = 2l \sin \frac{\theta}{2} = x$$

$$\frac{\text{Force}}{\text{length}} = \frac{\mu_0 I^2}{2\pi x} = \frac{\mu_0 I^2}{2\pi \times 2l \sin \theta/2}$$

$$\frac{\text{Downward force}}{\text{length}} = \lambda g = \frac{\mu_0 I^2}{2\pi \times 2l \sin \theta/2}$$

$$\tan \frac{\theta}{2} = \frac{\mu_0 I^2}{4\pi l \lambda g \sin \theta/2}$$

$$\frac{\sin^2 \theta/2}{\cos \theta/2} = \frac{\mu_0 I^2}{4\pi l \lambda g}$$

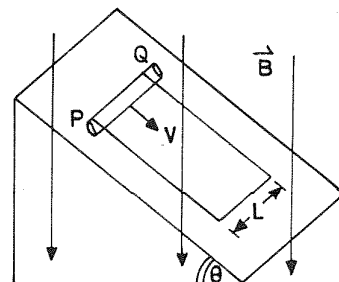
As θ is small,

$$\frac{(\theta/2)^2}{1} = \frac{\mu_0 I^2}{4\pi l \lambda g}$$

or

$$\theta = \sqrt{\frac{\mu_0}{\pi l \lambda g}} \cdot I$$

4. Metallic cylindrical rod PQ slides without friction on a rectangular circuit composed of perfectly conducting wires fixed on an inclined plane as shown in the figure. A vertical magnetic field $\vec{B}$ exists in the region of the above mentioned setup. Find the velocity of the rod PQ when it starts moving without any acceleration.



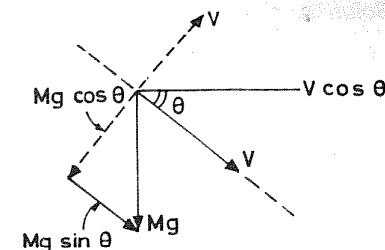
Solution

As the rod moves in the magnetic field, the induced emf in the rod is

$$\mathcal{E} = BL (V \cos \theta)$$

and the induced current

$$I = \frac{\mathcal{E}}{R} = \left(\frac{BLV}{R} \right) \cos \theta$$



A wire carrying current in a magnetic field experiences a force that is perpendicular to the plane formed by the wire and the magnetic field. The rod thus experiences a force F given by

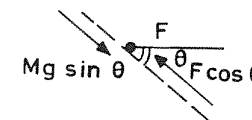
$$F = BIL = BL \left(\frac{BLV}{R} \right) \cos \theta = \left(\frac{B^2 L^2 V}{R} \right) \cos \theta$$

The force component along the plane is

$$F \cos \theta = \left(\frac{B^2 L^2 V}{R} \right) \cos^2 \theta$$

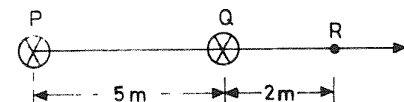
This force acts up the plane so as to prevent motion of the rod down the incline while the component of mg acts downward. The condition that the rod moves down with a constant velocity implies that the net force on the rod is zero or

$$\left(\frac{B^2 L^2 V}{R} \right) \cos^2 \theta = mg \sin \theta$$



$$\text{or } V = \frac{Rmg}{B^2 L^2} \left(\frac{\sin \theta}{\cos^2 \theta} \right)$$

5. Two long parallel wires carrying currents of 2.5 A and I A in the same direction (into the plane of the page) are held at P and Q respectively such that they are perpendicular to the plane of the paper. Two points P and Q are located at distances of 5 and 2 m respectively from a collinear point R as shown in the figure.



(i) An electron moving with a velocity 4×10^5 m/s along the positive X-direction experiences a force of magnitude 3.2×10^{-20} N at point R. Find the value of I .

(ii) Find all the positions at which a third long parallel wire carrying a current of magnitude 2.5 A may be placed so that the magnetic induction at R is zero.

Solution

$$(i) \quad \vec{F} = q |\vec{V}| |\vec{B}| = 3.2 \times 10^{-20} \text{ N}$$

$$B = \frac{3.2 \times 10^{-20}}{1.6 \times 10^{-19} \times 4 \times 10^5} = 5 \times 10^{-7} \text{ T}$$

$$B = \frac{\mu_0}{2\pi} \left(\frac{I_1}{r_1} + \frac{I_2}{r_2} \right) = \frac{\mu_0}{2\pi} \left(\frac{2.5}{5} + \frac{I}{2} \right) = \frac{4\pi \times 10^{-7}}{2\pi} \left(\frac{2.5}{5} + \frac{I}{2} \right)$$

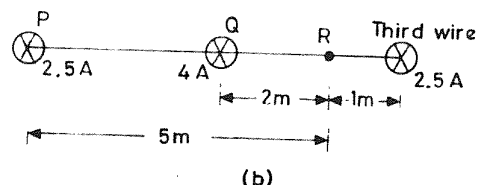
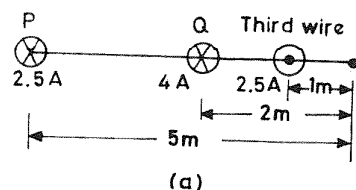
$$\therefore 5 \times 10^{-7} = \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{2.5}{5} + \frac{I}{2} \right] \Rightarrow I = 4 \text{ A}$$

$$(ii) \quad |\vec{B}| = \frac{\mu_0}{2\pi} \left(\frac{2.5}{5} + \frac{4.0}{2} + \frac{2.5}{x} \right) = 0$$

$$\therefore x = -1 \text{ m}, \quad I = 2.5 \text{ A out of the page } \odot$$

$$\text{and } x = 1 \text{ m}, \quad I = 2.5 \text{ A into the page } \otimes$$

Thus the position of the third wire could be shown like this.



6. A very small circular loop of radius a is initially coplanar and concentric with a much larger circular loop of radius b ($a \ll b$). A constant current I is passed in the large loop, which is kept fixed in space, and the small loop is rotated with angular velocity ω about a diameter. The resistance of the small loop is R and its inductive reactance is negligible.

(a) Find the current in the small loop as a function of time.

(b) Calculate how much torque must be exerted on the small loop to rotate it.

(c) Calculate the induced emf in the large loop as a function of time.

Solution

(a) The magnetic induction due to the large loop at its centre is

$$B = \frac{\mu_0 I_b}{2b}$$

where I_b is the current in the larger loop.

The total flux

$$\phi_B = \vec{B} \cdot \vec{A} = \left(\frac{\mu_0 I_b}{2b} \right) \pi a^2 \cos \theta \quad (1)$$

where θ is the angle between the two loops. If the small loop is rotating with an angular velocity ω , we have $\theta = \omega t$.

$$\text{Induced emf} = - \frac{d\phi}{dt} = \frac{\pi a^2 \mu_0 I_b \omega}{2b} \sin \omega t$$

$$\text{and the current} = \frac{\pi a^2 \mu_0 I_b \omega}{2bR} \sin \omega t$$

$$\text{Hence current in the small loop, } I_a = \left(\frac{\pi a^2 \mu_0 I_b \omega}{2bR} \right) \sin \omega t \quad (2)$$

$$(b) \text{ Input mechanical power} = \tau \frac{d\theta}{dt} = \text{induced electrical energy} \\ = I_a \frac{d\phi}{dt} \quad (\text{energy conservation})$$

$$\tau = I_a \frac{d\phi}{d\theta} = \left(\frac{\pi a^2 \mu_0 I_b}{2bR} \right) \sin \omega t \left(\omega \frac{d\phi}{d\theta} \right) \quad (3)$$

$$\therefore \tau = \left(\frac{\pi a^2 \mu_0 I_b}{2b} \right)^2 \frac{\omega \sin^2 \omega t}{R}$$

$$\text{From Eq. (1), we have } \left[\frac{d\phi}{d\theta} = - \left(\frac{\mu_0 I_b \pi a^2}{2b} \right) \sin \theta \right]$$

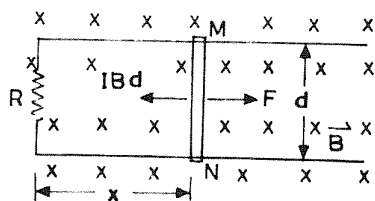
(c) Mutual inductance of the two loops is

$$M = \frac{d\phi}{dI} = \frac{\pi a^2 \mu_0}{2b} \cos \theta = \frac{\pi a^2 \mu_0}{2b} \cos \omega t$$

The induced emf in the large loop therefore is

$$\begin{aligned} \varepsilon &= - \frac{d\phi}{dt} = - \frac{d}{dt} (M I_a) \\ &= - \frac{d}{dt} \left[\frac{\pi a^2 \mu_0}{2b} \cos \omega t \cdot \frac{\pi a^2 \mu_0 I_b \omega}{2bR} \sin \omega t \right] \\ &= - \frac{I_b}{R} \left(\frac{\pi a^2 \mu_0}{2b} \right)^2 \frac{d}{dt} \left(\frac{\sin 2\omega t}{2} \right) \\ &= - \frac{\omega I_b}{R} \left(\frac{\pi a^2 \mu_0}{2b} \right)^2 2\omega \frac{\cos 2\omega t}{2} \\ &= - \frac{I_b}{R} \left(\frac{\pi a^2 \mu_0}{2b} \right)^2 \cos 2\omega t \end{aligned}$$

7. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R . A perfectly conducting rod MN of mass m is free to slide along the rails without friction. There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the plane of the paper. A variable force is applied to the rod MN such that, as the rod moves, a constant current flows through R .



- (a) Find the velocity of the rod and the applied force F as a function of distance x of the rod from R .
 (b) What fraction of the work done per second by F is converted into heat?

Solution

- (a) Let V be the velocity of the rod,

$$\text{Induced emf, } \varepsilon = VBd$$

$$\text{Induced current} = \frac{\varepsilon}{R}$$

and $R' = (R + 2\lambda x)$

$$\therefore I = \frac{VBd}{(R + 2\lambda x)} \quad \text{or} \quad V = \frac{1(R + 2\lambda x)}{B \cdot d} = \left(\frac{IR}{Bd} + \frac{2I\lambda}{Bd} x \right)$$

This is the required expression for the velocity of the rod as a function of x .

Force as a function of x

Acceleration of rod,

$$a = \frac{dV}{dt} = \left(\frac{2I\lambda}{Bd} \right) \frac{dx}{dt} = \left(\frac{2I\lambda}{Bd} \right) \left(\frac{R + 2\lambda x}{Bd} \right) I = \frac{2I^2\lambda}{B^2d^2} (R + 2\lambda x)$$

$$\therefore \text{Force} = ma = F - IBd$$

$$\text{or} \quad F = ma + IBd = \frac{2I^2\lambda m}{B^2d^2} (R + 2\lambda x) + IBd$$

(b) Work done in unit time by $F = \vec{F} \cdot \vec{v}$

$$= \left[\frac{2I^2\lambda m}{B^2d^2} (R + 2\lambda x) + IBd \right] \cdot \left[\frac{I(R + 2\lambda x)}{Bd} \right]$$

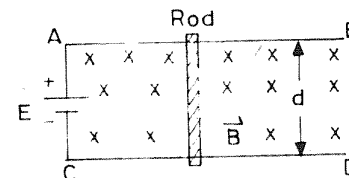
$$= \frac{2I^3\lambda m (R + 2\lambda x)^2}{B^3d^3} + I^2(R + 2\lambda x)$$

$$\text{Joule's heat} = I^2(R + 2\lambda x)$$

Fraction of the work done by F per second converted into Joule's heat

$$= \frac{1}{1 + \frac{2I^3\lambda m (R + 2\lambda x)^2}{B^3d^3 I^2(R + 2\lambda x)}}$$

8. Two long parallel guide wires AB and CD , separated by a distance d , are connected to a source of potential difference ε . A rod of resistance R is free to slide on these wires so as to remain perpendicular on the wires. A uniform magnetic field B exists perpendicular to the plane of the rod and the wires.



- (a) What is the steady state velocity of the rod (assume that there is no external mechanical load)?
 (b) If the mass of the rod is m , obtain an expression for the velocity of the rod as a function of time t , assuming that the velocity is zero at $t = 0$.
 (c) If a constant external force F is applied opposite to the direction of motion of the rod, find the new steady state velocity.
 (d) What fraction of the energy supplied by the battery is converted into mechanical work?

Solution

(a) As the induced emf opposes the potential difference of the battery, the system reaches the steady state when the two cancel each other. In such a case,

$$\varepsilon = BVd \quad \text{or} \quad V = \frac{\varepsilon}{Bd}$$

- (b) The current flowing in the bar at time t is

$$i = \frac{\varepsilon - BVd}{R}$$

and the force experienced by the rod is

$$F_x = iBd = \left(\frac{\varepsilon - BVd}{R} \right) Bd$$

$$F_x = \frac{mdV}{dt} = \left(\frac{\varepsilon - BVd}{R} \right) Bd$$

This is the required equation of motion of the rod. The solution is

$$V = \frac{\varepsilon}{Bd} \left[1 - \exp \left(- \frac{B^2d^2}{mR} t \right) \right]$$

(c) $m \frac{dV}{dt} = \left(\frac{\varepsilon - BVd}{R} \right) Bd - F$

In steady state, the condition is that $dV/dt = 0$.

$$\therefore \left(\frac{\epsilon - BVd}{R} \right) Bd - F = 0$$

$$\frac{\epsilon}{R} - \frac{BVd}{R} = \frac{F}{Bd} \text{ or } \frac{BVd}{R} = \frac{\epsilon}{R} - \frac{F}{Bd}$$

$$\therefore V = \frac{\epsilon}{Bd} - \frac{FR}{(Bd)^2}$$

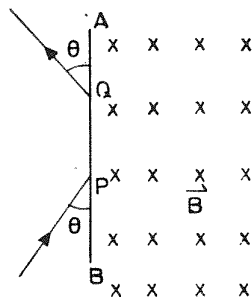
(d) The current under the condition set in part (c) is

$$i = \frac{\epsilon - BVd}{R} = \frac{F}{Bd}$$

The energy supplied by the battery is ei of which a power of $F.V$ is being converted into mechanical work

$$\therefore \frac{FV}{ei} = \frac{F \left[\frac{\epsilon}{Bd} - \frac{FR}{(Bd)^2} \right]}{F\epsilon/Bd} = 1 - \frac{FR}{\epsilon Bd}$$

9. A particle of mass 1.6×10^{-27} kg and charge 1.6×10^{-19} C enters a region of uniform magnetic field of strength 5 T at P along the direction shown. The speed of the particle is 2×10^7 m/s. The magnetic field is directed into the page and the particle leaves the region of the field at Q. Find the distance PQ and the angle θ .



Solution

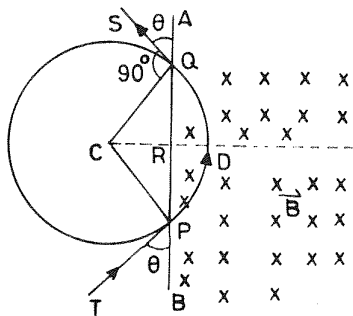
Given data

$$m = 1.6 \times 10^{-27} \text{ kg,}$$

$$q = +1.6 \times 10^{-19} \text{ C}$$

$$B = 5 \text{ T, } V = 2 \times 10^7 \text{ m/s.}$$

AB is the boundary of magnetic field. The particle in the magnetic field follows a circular path PDQ.



$$\text{Radius of the arc } PDQ = R = CP = CQ$$

TP and QS are tangents at P and Q respectively.

D is the middle point of the arc PQ and DC is perpendicular to PQ.

$$\angle DCP = 45^\circ$$

(The angle between the perpendicular PQ and TP is equal to the angle between PQ and TP.)

SQ is the tangent at Q. From considerations of symmetry,

$$\angle SQA = 45^\circ \text{ and } \angle QCD = 45^\circ$$

$$PQ = PR + RQ = 2R \cos 45^\circ \text{ (R is the radius of the circle)}$$

But

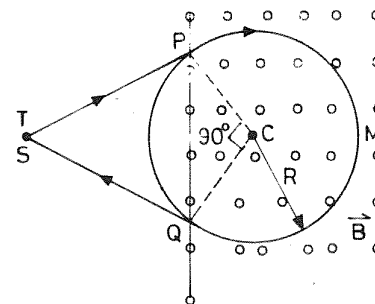
$$R = \frac{mV}{Bq} = \frac{1.6 \times 10^{-27} \times 2 \times 10^7}{5 \times 1.6 \times 10^{-19}} = \frac{2}{5} \times 10^{-1} = \frac{2}{50} = \frac{1}{25} \text{ m} = 4 \text{ cm}$$

$$\therefore PQ = 2 \times 4 \times \frac{1}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.66 \text{ cm}$$

10. In the above illustration, the charged particle enters a region where the magnetic field is directed out of the page, find how long does it take for the particle to travel in the region of the magnetic field.

Solution

As the direction of the magnetic field is reversed, all other parameters remaining the same, the direction of motion of the charged particle is reversed from anticlockwise to clockwise. The charged particle at P experiences a magnetic force directed along PC, perpendicular to TP and would again describe a circular path with TP as tangent. It would move clockwise and emerge at Q along QS. Since CP is perpendicular to PT and CQ is perpendicular to SQ, PC is perpendicular to QC. Thus we see that the chord PQ subtends an angle of 90° at the centre C. The length of the arc



$$PMQ = \frac{3}{4} (2\pi R)$$

(R is the radius of the circle)

∴ Time that the charged particle takes in moving from P to Q along PMQ

$$= \frac{\text{arc} \cdot \text{PMQ}}{\text{velocity}}$$

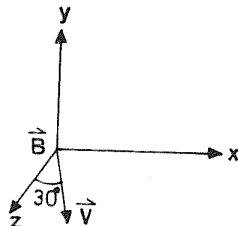
$$= \frac{(3/4) \times 2\pi R}{V} \quad (\text{the value of radius was obtained in the previous illustration})$$

$$= \frac{3 \times \pi \times 0.04}{2 \times 2 \times 10^7} = 0.09 \times 10^{-7} = 0.9 \times 10^{-8} \text{ s}$$

EXERCISES

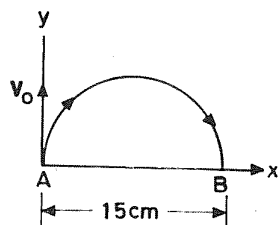
1. A proton beam moves along the positive Z-axis through a region of space where there exists a uniform magnetic field of magnitude 4.0 T. The protons have a velocity of 4×10^5 m/s in the X-Z plane at an angle of 30° to the positive Z-axis. The force on the proton is

- (a) $25.6 \times 10^{-14} \hat{j} \text{ N}$
 (b) $-25.6 \times 10^{-14} \hat{j} \text{ N}$
 (c) $-12.8 \times 10^{-14} \hat{j} \text{ N}$
 (d) $12.8 \times 10^{-14} \hat{j} \text{ N}$



2. A particle having a mass of 0.5 g carries a charge of 2.5×10^{-8} C. The particle is given an initial horizontal velocity (V) of 6×10^4 m/s. It is desired that the particle keeps moving on its horizontal track. The minimum magnetic field necessary for this purpose is
- (a) 3.27 T in the direction perpendicular to V
 (b) 3.27 T in the direction of V
 (c) 2.37 T in the direction of V
 (d) 2.37 T in the direction perpendicular to V

3. An electron is fired from the point A with a velocity $V_0 = 2 \times 10^8$ m/s. The magnitude and direction of magnetic field that will cause the electron to follow a semicircular path from A to B is
- (a) 1.5×10^{-4} T out of the page
 (b) 1.5×10^{-3} T into the page
 (c) 1.5×10^{-2} T into the page
 (d) 1.5×10^{-2} T out of the page



4. A singly charged Li^7 ion has a mass of 1.00×10^{-23} g. It is accelerated through a potential difference of 1000 V and then enters a magnetic field of 0.5 T, moving perpendicular to the field. The radius of its path in the magnetic field is
- (a) 2.2×10^{-3} m
 (b) 1.1×10^{-2} m
 (c) 2.2×10^{-4} m
 (d) 2.2 cm

5. A charged particle of mass m and charge q is accelerated through a potential difference of V volts. It then enters a region of uniform magnetic field B which is directed perpendicular to the direction of motion of the particle. The particle will move on a circular path of radius given by

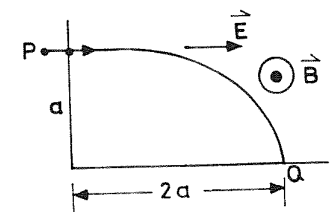
(a) $\sqrt{\frac{Vm}{qB^2}}$

(b) $\frac{2Vm}{qB^2}$

(c) $\sqrt{\frac{2Vm}{q}} \frac{1}{B}$

(d) $\sqrt{\frac{Vm}{q}} \frac{1}{B}$

6. A particle of charge $+q$ and mass m moving under the influence of a uniform field $E\hat{i}$ and a uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q. The velocities at P and Q are $V\hat{i}$ and $-2V\hat{j}$. Which of the following statements is correct?



(a) $E = \frac{mV^2}{4\pi\epsilon_0 a}$

(b) The change in kinetic energy in moving from P to Q is $(3/4) mV^2$

(c) The rate of work done by $\vec{E}$ at $P = \frac{3}{4} \left(\frac{mV^3}{a} \right)$

(d) Rate of work done at P is zero.

7. The magnitude of a current in a long straight conductor required to produce a flux density of 2 mT at a distance of 10 cm from the conductor is
- (a) 500 A
 (b) 1000 A
 (c) 250 A
 (d) 2000 A

8. A fixed horizontal wire carries a current of 250 A below which another wire of linear density 15 g/m carrying a current is kept balanced just parallel to the first one at a vertical separation of 2 cm. If the second wire is just hanging in air, the current that must flow through it is
- (a) 58.86 A
 (b) 85.68 A
 (c) 68.58 A
 (d) 48.56 A

9. In the above problem, the instantaneous acceleration experienced in the upward direction by the second wire when the current in the first wire is increased to 300 A is
- (a) 2.91 m/s^2
 (b) 0.092 m/s^2
 (c) 0.92 m/s^2
 (d) 1.92 m/s^2

10. A current I flows through a square loop of a wire of side a . The magnetic induction at the centre of the loop is

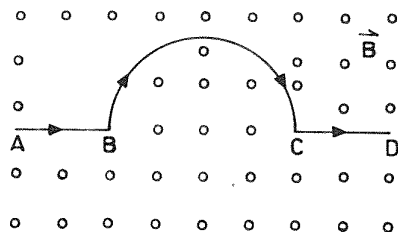
(a) $\frac{\sqrt{2} \mu_0 I}{\pi a}$

(b) $\frac{2\sqrt{2} \mu_0 I}{\pi a}$

(c) $\frac{\sqrt{2} \mu_0 I}{a}$

(d) $\frac{\sqrt{2} \pi \mu_0 I}{a}$

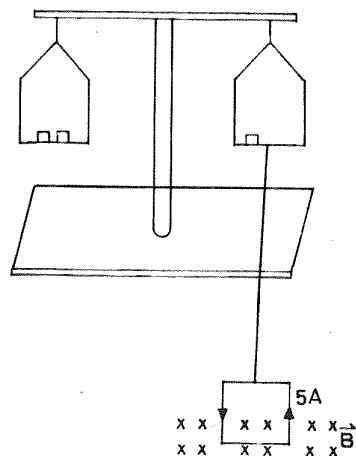
11. A wire ABCD is bent in the form shown here in the figure. Segments AB and CD are of length 1 m each while the semicircular loop is of radius 1 m. A current of 5 A flows from A towards the end D and the whole wire is placed in a magnetic field of 0.5 T directed out of the page. The force acting on the wire is



- (a) 40 N
(c) 10 N

12. A square shaped coil of 10 turns and area 100 cm^2 , made of metallic wire, is suspended from a pan of an accurate balance as shown in the figure. The weights are so adjusted that the balance remains in equilibrium position. A current of 5 A flows through the coil. An arrangement to immerse the coil in a uniform magnetic field $\vec{B}$ is shown and when the magnetic field is switched on (directed into the plane of the page), an additional weight of 50 g has to be placed on the right pan to reach equilibrium again. The strength of the magnetic field is

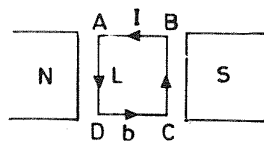
- (b) 5 N
(d) 20 N



- (a) $9.8 \times 10^{-1} \text{ T}$
(c) $9.8 \times 10^{-4} \text{ T}$

- (b) 9.8 T
(d) $9.8 \times 10^{-2} \text{ T}$

13. A 50 turns coil carrying a current of 4 A as shown in the figure is placed in a magnetic field $B = 0.50 \text{ T}$ ($L = 20 \text{ cm}$, $b = 15 \text{ cm}$). The torque acting on the coil and its direction of rotation are



- (a) 0.06 N.m and side AD coming out of the page.
(b) 0.06 N.m and side AD going into the page.
(c) 3 N.m and side AD coming out of the page.
(d) 3 N.m and side AD going into the page.

14. A circular coil of 100 turns and effective diameter 20 cm carries a current of 0.5 A. It is to be turned in a magnetic field $B = 2.0 \text{ T}$ from a position in which θ equals zero to one in which θ equals 180° . The work required in this process is

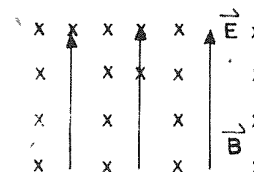
- (a) $\pi \text{ J}$
(c) $4\pi \text{ J}$

- (b) $2\pi \text{ J}$
(d) $8\pi \text{ J}$

15. A beam of particles of charge $+q$ enters a region of electric and magnetic fields acting simultaneously as shown. $\vec{E}$ is directed upwards and $\vec{B}$ is directed into the page. $E = 5 \times 10^4 \text{ V/m}$ and $B = 1.0 \text{ T}$. In order that the beam of charged

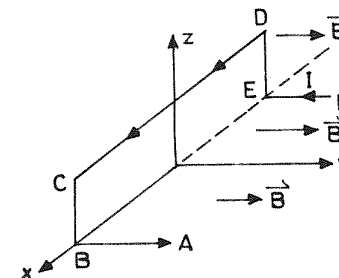
particles does not get deflected by the action of the two crossed fields, their velocity should be

- (a) $5 \times 10^4 \text{ m/s}$ and directed from right to left
(b) $5 \times 10^4 \text{ m/s}$ and directed from left to right
(c) $5 \times 10^6 \text{ m/s}$ and directed from left to right
(d) $5 \times 10^6 \text{ m/s}$ and directed from right to left



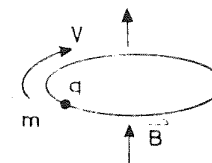
16. ABCDEF, each side of length L , bent as shown in the figure, carries a current I and is placed in a uniform magnetic induction B directed parallel to the positive Y-axis. The force experienced by the wire and its direction are

- (a) $2 IBL$ directed along the negative Z-axis
(b) $5 IBL$ directed along the positive Z-axis
(c) IBL directed along the positive Z-axis
(d) $2 IBL$ directed along the positive Z-axis



17. A magnetic field of induction $B = 0.5 \text{ T}$ is along the positive X-direction. A proton of mass $1.67 \times 10^{-27} \text{ kg}$ is shot with a speed of $5 \times 10^7 \text{ m/s}$ at an angle of 30° to $\vec{B}$. The radius of the helix and its pitch respectively are
(a) 1.04 m and 5.63 m
(b) 0.52 m and 5.63 m
(c) 0.52 m and 3.65 m
(d) 1.04 m and 2.81 m

18. A circular wire loop of radius r can withstand a radial force T before breaking. A particle of mass m and charge q ($q > 0$) is sliding over the wire. A magnetic field B is applied normal to the plane of the wire. The maximum speed V_{max} the particle can have before the loop breaks is



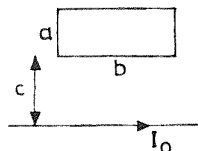
- (a) zero
(b) $\sqrt{\frac{rT}{m}}$
(c) $\frac{r}{2} \left[qB + \sqrt{q^2 B^2 + \frac{4T}{mr}} \right]$
(d) rqB

19. An infinitely long wire has a charge λ per unit length. The wire moves along its axis with a velocity V ($V \ll C$, the velocity of light). The ratio of the magnetic field to the electric field at a point r from the wire is
(a) proportional to r
(b) inversely proportional to r
(c) directly proportional to λ
(d) independent of r
20. A rectangular loop of sides a and b , has a resistance R and lies at a distance C from an infinite straight wire carrying current I_0 . The current decreases to zero in time τ

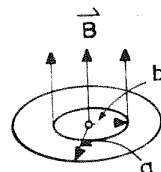
$$I(t) = I_0 \frac{(\tau - t)}{\tau}, \quad 0 < t < \tau$$

The charge flowing through the rectangular loop is

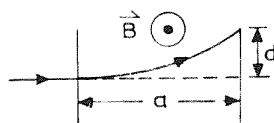
- (a) $\mu_0 I_0 \tau$
 (b) $\mu_0 I_0 \frac{ab}{C^2} \tau$
 (c) $\frac{\mu_0 b I_0}{2\pi R} \ln \frac{C+a}{C}$
 (d) $\frac{\mu_0 I \tau}{R} \frac{ba}{C^2}$



21. A line charge (σ per unit length) is in the form of a circular wheel of radius a and is free to rotate in a horizontal plane. There is a magnetic field inside $B = B_0 \hat{k}$ extending upto a radius b ($b < a$). If the magnetic field is switched off, the angular velocity ω of the wheel is given by (I is the moment of inertia)
- (a) $\pi a^2 b \sigma B / I$ clockwise as seen from above
 (b) $\pi a b^2 \sigma B / I$ clockwise as seen from above
 (c) $\pi a b^2 \sigma B / I$ anticlockwise as seen from above
 (d) $\pi a^2 b \sigma B / I$ anticlockwise as seen from above

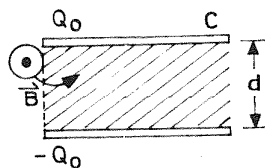


22. A charged particle q enters a region of uniform $\vec{B}$ (out of the page) and is deflected a distance d after travelling a horizontal distance a . The magnitude of the momentum of the particle is



- (a) $\frac{1}{2} \frac{q |B|}{2} \left[\frac{a^2}{d} + d \right]$
 (b) $\frac{1}{2} \frac{q |B| a}{2}$
 (c) zero
 (d) not possible to be determined as it keeps changing
23. A thin uniform ring of radius R carrying uniform charge Q and mass M rotates about its axis with angular velocity ω . The ratio of its magnetic moment and angular momentum is

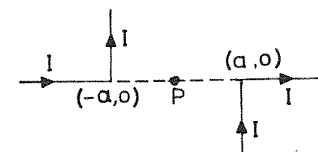
- (a) $\frac{Q}{M}$
 (b) $\frac{M}{Q}$
 (c) $\frac{Q}{2M}$
 (d) $\frac{M}{2Q}$



24. A slab of resistance R is inserted between two parallel plates of a capacitor charged to Q_0 . The capacitor is discharged through the solid. A magnetic field is present throughout and is out of the plane of the paper. The total momentum

given to the slab after complete discharge is (given that the capacitance of the capacitor is C and the plates are separated by a distance d)

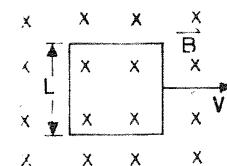
- (a) $CQ_0 B$
 (b) $d B Q_0$
 (c) zero
 (d) Infinity
25. Current I flows in the wires shown in the figure. The magnetic field at the origin (P) is given by
- (a) $\frac{\mu_0 I}{2\pi a}$ and pointing upwards
 (b) $\frac{\mu_0 I}{2\pi a}$ and pointing downwards
 (c) zero
 (d) Infinity



26. A circular loop of radius 15 cm carries a current 20 A. At its centre, a second loop of radius 2 cm, having 100 turns carrying a current of 2 A is placed such that the planes of the two loops are at right angles and the smaller loop remains in a uniform field B produced by the larger loop. The torque experienced by the smaller loop is

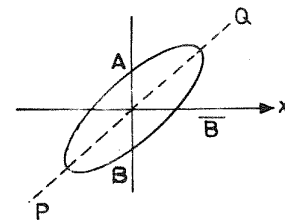
- (a) 2×10^{-7} N.m
 (b) 2×10^{-3} N.m
 (c) 0.02 N.m
 (d) 2×10^{-5} N.m

27. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity V perpendicular to one of its sides. A magnetic induction B , constant in time and space, pointing perpendicular and into the plane of the loop, exists everywhere. The current in the loop is



- (a) $\frac{BLV}{R}$ clockwise
 (b) $\frac{2BLV}{R}$ anticlockwise
 (c) $\frac{BLV}{R}$ anticlockwise
 (d) zero

28. A field of magnetic induction $B = 0.4$ T exists in the positive X direction. A loop of area 10 cm^2 rotates about an axis PQ in such a way that point A rotates towards positive- X values from the position shown in the figure. If line AB rotates through an angle of 45° from its position shown in a time interval of 0.2 s, the average induced emf is



- (a) 1×10^{-4} V
 (b) 0.57 mV
 (c) 5.7×10^{-3} V
 (d) 1×10^{-3} V

29. Two particles X and Y having equal charges, after being accelerated through the same potential, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

- (a) $\left(\frac{R_1}{R_2} \right)^{1/2}$
 (b) $\left(\frac{R_2}{R_1} \right)$

(c) $\left(\frac{R_1}{R_2}\right)^2$

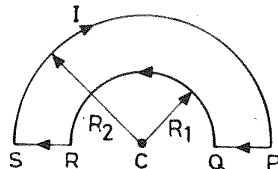
30. The wire loop PQRS formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in the figure. The magnetic field B at the point C is

(a) $\frac{\mu_0 I}{2\pi} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(b) $\frac{\mu_0 I}{4\pi} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(c) $\frac{\mu_0 I}{4} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(d) $\left(\frac{R_1}{R_2}\right)$



(d) $\frac{\mu_0 I}{2} \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

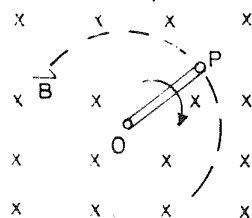
31. The rod OP rotates about the point O as pivot with a constant frequency of 10 revolutions/second in a uniform magnetic field of 0.5 T directed into the page. If the two ends of the rod are 1 m apart, the potential difference induced between O and P is

(a) 5.0 V

(b) 7.5 V

(c) 7.15 V

(d) 15.7 V



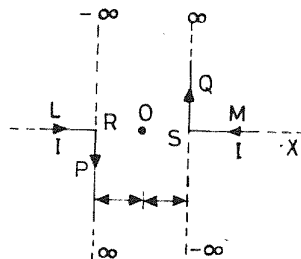
32. A pair of stationary and infinitely long bent wires are placed in the X-Y plane as shown in the figure. The wires carry currents of 10 A each. The segments L and M are along the X-axis. The segments P and Q are parallel to Y-axis such that OS = OR = 0.02 m. The magnetic field at O is

(a) 1×10^{-4} Wb/m² into the page

(b) 1×10^{-4} Wb/m² out of the page

(c) 0.5×10^{-4} Wb/m² into the page

(d) 0.5×10^{-4} Wb/m² out of the page



33. A wire of length L (metres) carries a current I (amperes), is bent in the form of a circle. The magnitude of its magnetic moment in MKS units is

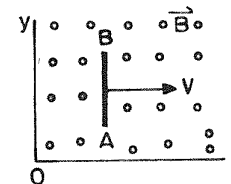
(a) $4\pi IL^2$

(b) $\frac{2IL^2}{\pi}$

(c) $\frac{IL^2}{\pi}$

(d) $\frac{IL^2}{4\pi}$

34. A conducting rod AB moves with a velocity V in a magnetic field $\vec{B}$, directed in positive Z-direction.



- (a) the end A of the rod gets positively charged.
(b) the end B of the rod gets positively charged.
(c) both do not get charged.
(d) both A and B get positively charged.

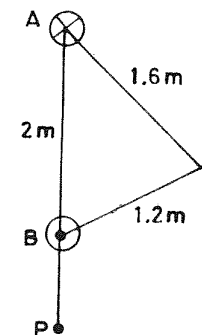
35. Two long straight parallel wires 2 m apart, are perpendicular to the plane of the paper. Wire A carries a current of 10 A directed into the plane of the paper. Wire B carries a current such that the magnetic field at P at a distance of 0.8 m from this wire is zero. The magnitude and direction of the current in wire B is

(a) 2.8 A into the page

(b) 2.8 A out of the page

(c) 2.8×10^{-6} A into the page

(d) 2.08 A out of the page



36. In the above problem, a point S is situated at a distance of 1.6 m from A and 1.2 m from B. The magnetic field at S due to the currents in wires A and B is

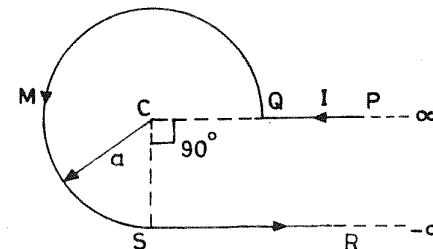
(a) 1.3×10^{-7} T

(b) 0.3×10^{-6} T

(c) 0.3×10^{-7} T

(d) 1.3×10^{-6} T

37. Two infinitely long wires PQ and RS, carrying a current I , are bent at one end in the form of three-quarters of a circle QMS of radius a as shown in the figure. The magnetic induction at C is



(a) $\frac{3\mu_0 I \pi}{4a}$

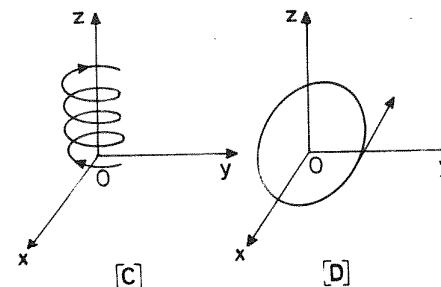
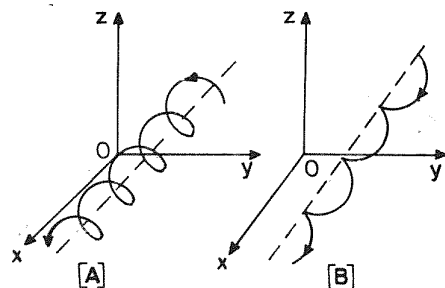
(b) $\frac{\mu_0 I}{4\pi a} (1 + \pi)$

(c) $\frac{\mu_0 I}{4\pi a}$

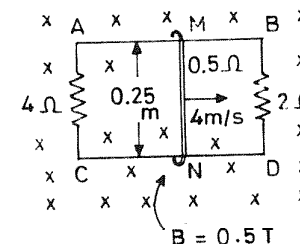
(d) $\frac{\mu_0 I}{4\pi a} \left(1 + \frac{3}{2} \pi\right)$

38. An all metal aeroplane drives down vertically at 150 km/s at a place where the horizontal component of the earth's magnetic field is 0.4×10^{-4} Wb/m². If the wing span is 25 m, the resulting potential difference between the tips is

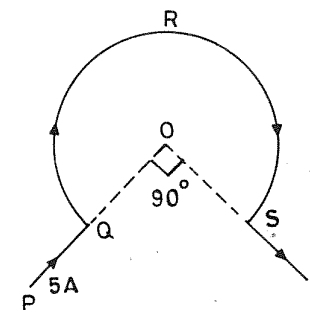
- (a) 150 V (b) 360 V
(c) 300 V (d) 260 V
39. A vertical copper disc of diameter 0.5 m makes 25 revolutions per second about a horizontal axis through its centre. A uniform magnetic field of 400 gauss acts normal to the plane of the disc. The potential difference between its centre and the rim is
(a) 3140 mV (b) 785 mV
(c) 78.5 mV (d) 196 mV
40. The rails of a railway track are 1.5 m apart and assumed to be insulated from one another. If a train is moving on these rails at a speed of 60 km/h and given that the angle of dip at the place is θ , $\tan \theta = 1.036$, and the earth's horizontal component $= 0.36 \times 10^{-4}$ Wb/m², the emf induced between the rails is
(a) 9.2 mV (b) 3.7 mV
(c) 0.92 mV (d) 0.46 mV
41. The ratio of the energy required to set up in a cube of side 10 cm a uniform magnetic field of 4 Wb/m² and a uniform electric field of 10^6 V/m is
(a) 1.4×10^7 (b) 1.4×10^5
(c) 1.4×10^6 (d) 1.4×10^3
42. Two parallel plates are kept in air 1 cm apart and a potential difference of 100 V is applied between them. If there is an electron resting on the plate at lower potential, the kinetic energy of this electron when it reaches the other plate is
(a) 1.6×10^{-20} J (b) 1.6×10^{-17} J
(c) 1.6×10^{-19} J (d) 1.6×10^{-23} J
43. A positive ion of charge q and mass m is accelerated by a potential difference V . It moves with a constant velocity through the space between the two plates separated by a distance d . The transit time of the ion between the plates is
(a) $\frac{2d}{(2qu/m)^{1/2}}$ (b) $\frac{2d}{(qu/m)^{1/2}}$
(c) $\frac{d}{(2qu/m)^{1/2}}$ (d) $\frac{d}{(qu/m)^{1/2}}$
44. A charged particle moves along the X-axis in crossed magnetic and electric fields given as $\vec{B} = B\hat{z}$ and $\vec{E} = E\hat{y}$. The motion of the particle as observed in the laboratory frame will look like:



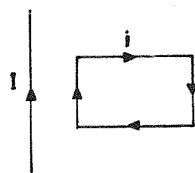
45. A sliding wire of length 0.25 m and having a resistance of 0.5Ω moves along conducting guiding rails AB and CD with a uniform speed of 4 m/s. A magnetic field of 0.5 T exists normal to the plane of ABCD directed into the page. The guides are short-circuited with resistances of 4 and 2Ω as shown. The current through the sliding wire is



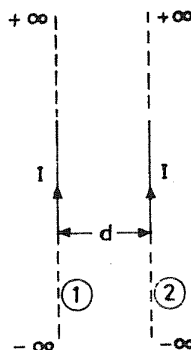
- (a) 0.27 A
(b) 0.37 A
(c) 1.0 A
(d) 0.72 A
46. According to the data given in Problem 45, the mechanical power needed for the motion of the sliding wire MN is
(a) 0.19 W (b) 1.4 W
(c) 0.14 W (d) 1.9 W
47. The conductor PQRST carries a current of 5 A. The magnetic field at the centre O of the circular part of it is [The radius of the circular conductor is 5 cm]
(a) $\frac{3}{4} \pi \times 10^{-7}$ T
(b) $\frac{3}{4} \times 10^{-7}$ T
(c) $2\pi \times 10^{-5}$ T
(d) $\frac{3}{2} \pi \times 10^{-5}$ T



48. A beam of protons with a velocity of 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. The pitch of the helical path traced is
(a) 4.38×10^{-2} m (b) 4.38×10^{-4} m
(c) 2.19×10^{-2} m (d) 2.19×10^{-4} m
49. A rectangular loop of wire carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current I is established in the wire, the loop will



- (a) rotate about an axis parallel to the wire
 (b) move away from the wire
 (c) move towards the wire
 (d) remain stationary
50. Two infinitely long conducting wires separated by a distance d carry equal currents flowing in the same direction. Among the given statements, mark the one which is incorrect.

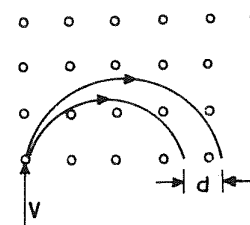


- (a) the direction of the magnetic field due to one of the wires at any point along the other wire is into the plane of the paper.
 (b) the direction of the force on one wire due to the other is towards wire 1 and normal to 2.
 (c) if the current in each is doubled, the force changes by a factor of 4.
 (d) the direction of the magnetic field at a point midway between the two wires is into the plane of the page.
51. A long wire is bent into a hairpin like shape as shown here. A current I flows in the wire, of which ABC is a circular portion with P as its centre. The magnetic field at P is
- (a) $\frac{\mu_0 I}{4\pi r} (2 + \pi)$
 (b) $\frac{\mu_0 I}{4\pi r} + \pi$
 (c) $\frac{1}{2} \frac{\mu_0 I}{\pi r}$
 (d) $\frac{\mu_0 I}{4\pi r} (4 + \pi)$
52. An electron moving with a velocity $\vec{V}_1 = 1\hat{i}$ m/s at a point in a magnetic field experiences a force $\vec{F}_1 = -e\hat{j}$ where e is the charge of the electron. If the electron

is moving with a velocity $\vec{V}_2 = (1\hat{j} + 1\hat{i})$ m/s at the same point, it experiences a force $\vec{F}_2 = e(\hat{i} - \hat{j})$. The force the electron would experience if it were moving with a velocity $\vec{V}_3 = \vec{V}_1 \times \vec{V}_2$ at the same point is

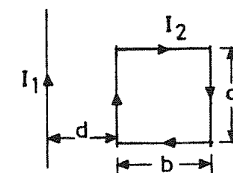
- (a) $-e(\hat{i} + \hat{j})$
 (b) $e(\hat{i} + \hat{j})$
 (c) zero
 (d) $e\hat{i}$

53. A mass spectrograph is a device for separating charged particles having different masses. Consider two particles of the same charge q but different masses m_1 and m_2 injected into the region of a uniform magnetic field $\vec{B}$ with a known velocity V normal to the magnetic field as shown in the figure. The particles are separated by a distance d , given by



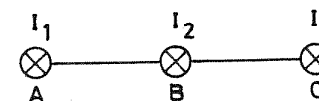
- (a) $d = \frac{|(m_2 - m_1)V|}{|qB|}$
 (b) $d = \frac{|2(m_2 - m_1)V|}{|qB|}$
 (c) $d = \frac{|(m_2 - m_1)V|}{|2qB|}$
 (d) $d = \frac{|(m_2 - m_1)V|}{|4qB|}$

54. An infinitely long straight wire carrying current I_1 A is situated in the plane of and parallel to one side of a rectangular loop of wire carrying current I_2 A as shown in the figure. The force experienced by the infinitely long wire due to the magnetic field of the rectangular loop of the wire is

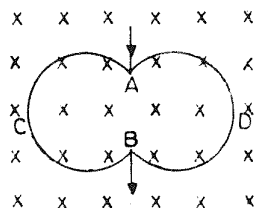


- (a) $\frac{\mu_0 I_1 I_2 a}{2\pi} \left(\frac{1}{d} - \frac{1}{d+b} \right)$
 (b) $\frac{2\mu_0 I_1 I_2 a}{\pi} \left(\frac{1}{d} - \frac{1}{d+b} \right)$
 (c) $\frac{\mu_0 I_1 I_2 a}{\pi} \left(\frac{1}{d} - \frac{1}{d+b} \right)$
 (d) $\frac{4\mu_0 I_1 I_2 a}{\pi} \left(\frac{1}{d} - \frac{1}{d+b} \right)$

55. A circular coil of radius 12 cm carrying a constant current has a magnetic field of 4×10^{-5} T at its centre. The magnetic field on its axis at a distance of 0.05 m from its centre is
- (a) 3×10^{-4} T
 (b) 3×10^{-5} T
 (c) 1×10^{-4} T
 (d) 1×10^{-5} T
56. A current of 2 A flows through a wire frame having the shape of a regular polygon. A magnetic field of intensity 33 A/m is formed at the centre of the frame. The length of the wire of the frame is
- (a) 0.5 m
 (b) 0.2 m
 (c) 0.4 m
 (d) 0.3 m
57. Three infinitely long conductors, A, B, C, carrying current are shown in the figure. $I_1 = I_2 = I$, $I_3 = 2I$ and $AB = BC = 5$ cm. The position of the point lying in the straight line AC where the magnetic field induced by currents I_1 , I_2 and I_3 is zero, is



- (a) between I_1 and I_2 at a distance of 3.2 cm from B
 (b) between I_2 and I_3 at a distance of 3.2 cm from A
 (c) between I_1 and I_2 at a distance of 1.3 cm from B
 (d) between I_2 and I_3 at a distance of 1.3 cm from B.
58. The electric field and the magnetic field in a region are given by $\vec{E} = \hat{i}E_0$ and $\vec{B} = \hat{j}B_0$. Consider a frame of reference moving with a velocity $v_0\hat{k}$. The electric field in this frame will be zero if v_0 is equal to
- (a) $\frac{E_0}{B_0}$ (b) $\frac{2E_0}{B_0}$
 (c) $\frac{E_0}{2B_0}$ (d) none of these
59. The figure shows a conducting loop ACBDA placed in the plane perpendicular to a uniform magnetic field B . The two parts ACB and ADB are circular arcs of radius a . The separation between the points A and B is l . The points A and B are connected to a battery which sends a current i . The magnetic force on the loop due to the field B is
- (a) ilB (b) $2ilB$
 (c) iaB (d) $2iaB$
60. A circular loop of radius r carries a charge q distributed uniformly on it. It is rotated at a frequency ν about its axis. A uniform magnetic field B exists along the axis of the loop. The torque on the loop due to the magnetic field is
- (a) $qv\pi r^2 B$ (b) $\frac{1}{2}qv\pi r^2 B$
 (c) $2qv\pi r^2 B$ (d) zero
61. A circular loop of radius r carries a charge distributed uniformly on it. It is rotated at a frequency ν about one of the diameters. A uniform magnetic field B exists along this diameter. The maximum and minimum torques acting on the loop due to the magnetic field are, respectively,
- (a) $qv\pi r^2 B, 0$ (b) $0, 0$
 (c) $2qv\pi r^2 B, qv\pi r^2 B$ (d) none of these
62. A positive electric charge Q is distributed over a circular ring of radius a . It is placed in a horizontal plane and is rotated about its axis at a uniform angular speed ω . A horizontal magnetic field B exists in the space. The torque acting on the ring due to the magnetic force is
- (a) $q\omega a^2 B$ (b) $\frac{1}{2}q\omega a^2 B$
 (c) $\frac{1}{\pi}q\omega a^2 B$ (d) $\frac{1}{2\pi}q\omega a^2 B$
63. An insulating rod of length l carries a charge q distributed uniformly on it. The rod is pivoted at an end and is rotated at a frequency ν about a fixed perpendicular axis. The magnetic moment of the system is
- (a) zero (b) $\pi q\nu l^2$
 (c) $\frac{1}{2}\pi q\nu l^2$ (d) $\frac{1}{3}\pi q\nu l^2$



64. A circular loop of radius 2 cm carries a current of 2 A. A long, straight wire is placed in the plane of loop at a distance of 4 cm from its centre. A current is passed through the straight wire and it is found that the magnetic field at the centre of the loop is zero. The current in the straight wire is about
- (a) 2 A (b) 3.14 A
 (c) 6.28 A (d) 12.6 A
65. A circular loop of radius 1 mm carries an electric current. Let P and Q be two points on the axis of the loop at distance 1 m and 2 m respectively from the centre. If the magnetic field at P due to the current is $16 \mu\text{T}$, that at Q will be about
- (a) $8 \mu\text{T}$ (b) $4 \mu\text{T}$
 (c) $2 \mu\text{T}$ (d) $1 \mu\text{T}$
66. Two solenoids A and B of the same length and made of the same wire are connected in parallel and the combination is joined to a battery. The solenoid A has 200 turns/cm whereas the solenoid B has 400 turns/cm. Assume that the magnetic field inside a solenoid is mainly due to the current in that solenoid. If the magnetic field at the centre of the solenoid A is $50 \mu\text{T}$, that at the centre of B will be
- (a) $100 \mu\text{T}$ (b) $50 \mu\text{T}$
 (c) $25 \mu\text{T}$ (d) none of these
67. A long, cylindrical wire of radius 10 cm carries an electric current. The magnetic field at a point outside the wire at a distance 5 cm from the surface is $4.0 \mu\text{T}$. The magnetic field at a point inside the wire at a distance 5 cm from the surface will be
- (a) $3 \mu\text{T}$ (b) $4 \mu\text{T}$
 (c) $5 \mu\text{T}$ (d) $12 \mu\text{T}$
68. Two long straight wires A and B are placed parallel to each other, each carrying a current of 5 A in the same direction. The separation between the wires is 10 cm. The area of cross-section of the wires A and B are 1 cm^2 and 2 cm^2 respectively. The force of attraction per unit length on the wire A due to the wire B is
- (a) $50 \mu\text{N}$ (b) $25 \mu\text{N}$
 (c) $42 \mu\text{N}$ (d) $56 \mu\text{N}$
69. A circular loop of radius 10 cm carries a current of 10 A. A particle having a charge 100 mC and moving along the planar axis of the loop crosses the plane of the loop at the centre with a speed 20 m/s. The force on the charged particle at the instant it passes through the centre is
- (a) zero (b) $4 \times 10^{-5} \text{ N}$
 (c) $4\pi \times 10^{-5} \text{ N}$ (d) $2 \times 10^{-5} \text{ N}$
70. Electric charge q is uniformly distributed over a rod of length l . The rod is placed parallel to a long wire carrying a current i . The separation between the rod and the wire is a . The force needed to move the rod along its length with a uniform velocity v is
- (a) $\frac{\mu_0 i q v}{2\pi a}$ (b) $\frac{\mu_0 i q v}{4\pi a}$
 (c) $\frac{\mu_0 i q v}{2\pi l}$ (d) $\frac{\mu_0 i q v}{4\pi l}$
71. Electric current passes through a circular loop of radius 10 cm. 10^{20} electrons pass through any cross-section of the loop in one second. The magnetic field at the centre of the loop is about

- (a) 10^{-4} T (b) 10^{-3} T
(c) 10^{-2} T (d) 10^{-1} T
72. Electric current i passes through a cylindrical wire of radius a . The current density at a point in the wire is proportional to the distance of the point from the axis of the wire. The magnetic field at the surface of the wire is
- (a) $\frac{\mu_0 i}{2\pi a}$ (b) $\frac{\mu_0 i}{4\pi a}$
(c) $\frac{2\mu_0 i}{\pi a}$ (d) none of these
73. Consider the situation in the previous problem. The magnetic field at a distance r ($< a$) from the axis of the wire is
- (a) $\frac{\mu_0 i}{2\pi r}$ (b) $\frac{\mu_0 i r}{2\pi a^2}$
(c) $\frac{\mu_0 i r^2}{2\pi a^3}$ (d) $\frac{\mu_0 i r^3}{2\pi a^4}$
74. Two coplanar long wires carry equal currents of 10 A. The angle between the wires is 30° . Consider a small portion of length 4.0 mm of a wire situated at a distance of 20 cm from the other wire. The force on this portion is about
- (a) 4.0×10^{-7} N (b) 2.0×10^{-7} N
(c) 3.5×10^{-7} N (d) zero
75. A wire is bent in the form of a long rectangle. An ideal battery is connected in a shorter arm of the rectangle to drive a current through it. The two longer arms are found to repel each other by a force of 1.0×10^{-4} N/m. If the battery is replaced by another ideal battery of double the emf, the longer arms will repel each other by a force per unit length
- (a) 1.0×10^{-4} N/m (b) 4.0×10^{-4} N/m
(c) 8.0×10^{-8} N/m (d) 16.0×10^{-8} N/m
76. Consider the situation in the previous problem. Suppose the wire is replaced by another having double its radius, and the original battery is used. The length and breadth of the rectangle are kept the same as in the previous problem. The longer arms will repel each other by a force per unit length
- (a) 1.0×10^{-4} N/m (b) 4.0×10^{-4} N/m
(c) 8.0×10^{-8} N/m (d) 16.0×10^{-4} N/m
77. A straight wire is kept along the axis of a tightly wound long solenoid. The solenoid has 1000 turns per metre. If the same current 2.0 A is passed through the solenoid and the wire, the magnetic force per unit length of the wire will be
- (a) zero (b) 3.2π mN/m
(c) 1.6π mN/m (d) 6.4π mN/m [mN is 10^{-3} N]
78. Three long parallel wires are kept in such a way that the separation between any two is 10 cm. Each of the wires carries a current of 10 A in the same direction. The magnetic force per unit length of each wire has a magnitude of about force per unit length of each wire has a magnitude of about
- (a) 1.0×10^{-4} N/m (b) 3.46×10^{-4} N/m
(c) 8.65×10^{-5} N/m (d) zero
79. Two parallel long wires separated by a distance d carry equal current i . If the currents are in the same direction, the magnetic field at a point midway between the wires is
- (a) $\frac{\mu_0 i}{4\pi d}$ (b) $\frac{\mu_0 i}{2\pi d}$
(c) $\frac{\mu_0 i}{\pi d}$ (d) zero
80. The magnetic field in a region is given by $\vec{B} = B_0 \left(1 + \frac{x}{a}\right) \hat{k}$. A square loop of edge-length d is placed with its edges along the X and Y-axes. The loop is moved with a constant velocity $\vec{v} = v_0 \hat{i}$. The emf induced in the loop is
- (a) zero (b) $v_0 B_0 d$
(c) $\frac{v_0 B_0 d^3}{a^2}$ (d) $\frac{v_0 B_0 d^2}{a}$
81. A coil having 100 turns and radius 10 cm is held in the vertical plane containing the north-south direction. The horizontal component of the earth's magnetic field is $20 \mu\text{T}$. The coil is rotated about its vertical diameter through 180° in 1 second. The average emf induced in this period is
- (a) zero (b) 3.14×10^{-3} V
(c) 6.28×10^{-15} V (d) none of these
82. Two circular conducting loops are placed in the same plane with their centres coinciding. The radii of the loops are 20 cm and 2 mm. A current $i = (10 \text{ A/min}) t$ is established in the outer loop. The emf induced in the inner loop is about
- (a) 6.7×10^{-12} V (b) 6.7×10^{-6} V
(c) 3.3×10^{-6} V (d) 3.3×10^{-12} V
83. An electron moves with a constant speed V along a circle of radius R . The magnitude of the magnetic moment of the circulating electron is
- (a) $\frac{eVR}{2}$ (b) $2 eVR$
(c) $\frac{eVR}{4}$ (d) $\frac{eVR}{4\pi}$
84. A conducting wire in the shape of a semicircle of radius 5 cm is translated in its plane at a uniform velocity 5 m/s. The velocity is perpendicular to the line joining the ends of the wire. A uniform magnetic field 20 mT exists perpendicular to the plane of the wire. The emf induced between the ends of the wire is
- (a) 0.01 V (b) 0.005 V
(c) 0.157 V (d) 0.0314 V
85. A long solenoid having 200 turns/cm carries an electric current which is increasing at a uniform rate 0.1 A/s. A conducting square loop edge of length 1 cm is placed inside the solenoid in such a way that two of its edges are parallel to the axis of the solenoid. The emf induced in the loop is
- (a) zero (b) $8\pi \times 10^{-6}$ V
(c) $4\pi \times 10^{-6}$ V (d) $2\pi \times 10^{-6}$ V
86. Suppose the square loop of the previous problem is rotated by 90° so that all the four edges are perpendicular to the axis of the solenoid. The emf induced in the loop is
- (a) zero (b) $8\pi \times 10^{-8}$ V
(c) $4\pi \times 10^{-8}$ V (d) $2\pi \times 10^{-8}$ V
87. A short magnet is allowed to fall along the axis of a horizontal metallic ring. Starting from rest, the distance fallen by the magnet in one second may be
- (a) 4.0 m (b) 5.0 m
(c) 6.0 m (d) 7.0 m
88. A magnet is taken towards a metallic ring in such a way that a constant current of 10 mA is induced in it. The total resistance of the ring is 0.25Ω . In 10 seconds, the flux of the magnetic field through the ring changes by
- (a) 25 μWb (b) 25 mWb
(c) 25 Wb (d) 25 kWb

89. A charge q is moving with a velocity $\vec{v}_1 = 1\hat{i}$ m/s at a point in a magnetic field and experiences a force $\vec{F}_1 = q(-1\hat{j} + 1\hat{k})$ N. If the charge is moving with a velocity $\vec{v}_2 = 1\hat{j}$ m/s at the same point, it experiences a force $\vec{F}_2 = q(1\hat{i} - 1\hat{k})$ N. The magnetic induction $\vec{B}$ at that point is
- (a) $(1\hat{i} + 1\hat{j} + 1\hat{k})$ Wb/m² (b) $(1\hat{i} - 1\hat{j} + 1\hat{k})$ Wb/m²
 (c) $(-1\hat{i} + 1\hat{j} - 1\hat{k})$ Wb/m² (d) $(1\hat{i} + 1\hat{j} - 1\hat{k})$ Wb/m²
90. A square loop of edge-length a is placed in the same plane as a long straight wire carrying a current i . The centre of the loop is at a distance r from the wire where $r \gg a$. The loop is moved away from the wire with a constant velocity v . The induced emf in the loop is
- (a) $\frac{\mu_0 i v}{2\pi}$ (b) $\frac{\mu_0 i a v}{2\pi r}$
 (c) $\frac{\mu_0 i a^2 v}{2\pi r^2}$ (d) $\frac{\mu_0 i a^3 v}{2\pi r^3}$

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (c) | 4. (d) | 5. (c) |
| 6. (c) | 7. (b) | 8. (a) | 9. (d) | 10. (b) |
| 11. (c) | 12. (d) | 13. (c) | 14. (b) | 15. (a) |
| 16. (c) | 17. (b) | 18. (c) | 19. (d) | 20. (c) |
| 21. (c) | 22. (a) | 23. (c) | 24. (b) | 25. (c) |
| 26. (d) | 27. (d) | 28. (b) | 29. (c) | 30. (c) |
| 31. (d) | 32. (b) | 33. (d) | 34. (a) | 35. (b) |
| 36. (d) | 37. (d) | 38. (a) | 39. (b) | 40. (c) |
| 41. (c) | 42. (b) | 43. (c) | 44. (b) | 45. (a) |
| 46. (c) | 47. (d) | 48. (a) | 49. (c) | 50. (d) |
| 51. (a) | 52. (c) | 53. (b) | 54. (a) | 55. (b) |
| 56. (b) | 57. (a) | 58. (a) | 59. (a) | 60. (d) |
| 61. (b) | 62. (b) | 63. (d) | 64. (d) | 65. (c) |
| 66. (b) | 67. (a) | 68. (a) | 69. (c) | 70. (a) |
| 71. (a) | 72. (a) | 73. (c) | 74. (c) | 75. (b) |
| 76. (d) | 77. (a) | 78. (b) | 79. (d) | 80. (d) |
| 81. (a) | 82. (a) | 83. (a) | 84. (a) | 85. (a) |
| 86. (b) | 87. (a) | 88. (b) | 89. (a) | 90. (c) |

Note: All problems are of the same standard and should not take more than three minutes each.

23

Varying Currents and Alternating Currents

Induced Emf

$$E = - \frac{d(N\phi_B)}{dt}$$

where $N\phi_B$ is the flux linkage.

$$E = -L \frac{di}{dt}$$

where L is the inductance of the device. The unit of inductance is

$$1 \text{ henry} = \frac{1 \text{ volt} \cdot \text{second}}{\text{ampere}}$$

For a close-packed coil with no iron nearby,

$$L = \frac{N\phi_B}{i}$$

Inductance of a Solenoid

Consider a solenoid of area of cross-section A , length l , having n turns per unit length, and permeability constant μ_0 . Its inductance is $\mu_0 n^2 l A$

$$\left(\mu_0 = 4\pi \times 10^{-7} \frac{\text{tesla-meter}}{\text{ampere}} \right)$$

Inductance of a Toroid

The inductance of a toroid of rectangular cross-section is given by $(\mu_0 N^2 h / 2\pi) \ln(b/a)$ where a is the inner radius, b is the outer radius, h is the height and N is the number of turns.

Inductive Circuit

The differential equation involving the variable i and its first derivative di/dt is $L di/dt + iR = E$ and its solution is given by

$$i = \frac{E}{R} (1 - e^{-Rt/L}), \text{ the initial conditions are to be defined properly.}$$

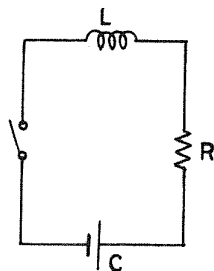
Inductive Time Constant

$\tau_L = L/R$ (This has the dimensions of time.)

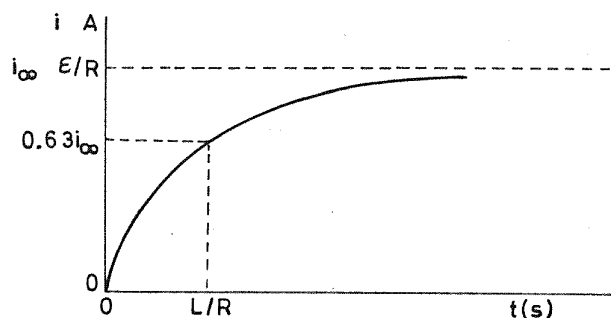
$$i = \frac{E}{R} (1 - e^{-t/\tau_L})$$

and if $t = \tau_L$,

$$i = \frac{E}{R} \left(1 - \frac{1}{e}\right) = 0.63 \frac{E}{R}$$



(a)



(b)

Consider a circuit as shown in the figure here. When the switch is first closed, the current in the circuit rises and it does not jump to its final value as it happens in a purely resistive circuit. At $t = L/R$, it attains a current 63% of its final value. The maximum value of current is attained after a very long time and at that instant of time, $i = i_\infty = E/R$.

Energy Stored in Magnetic Field

$$U_B = \frac{1}{2} Li^2$$

and Energy density $u_B = \frac{1}{2} \frac{B^2}{\mu_0}$ (Energy density = $\frac{\text{energy}}{\text{total volume}}$)

This expression is true for all magnetic field configurations.

Mutual Inductance

For two coils 1 and 2, the mutual inductance

$$M_{21} = \frac{N_2 \phi_{21}}{i_1}$$

where coil 1 has N_1 turns and coil 2 has N_2 turns.

The induced emf appearing in coil 2 due to the changing current in coil 1 is

$$E_2 = -M_{21} \frac{di_1}{dt}$$

and similarly

$$E_1 = -M_{12} \frac{di_2}{dt}$$

M_{21} and M_{12} are equal and we have

$$E_2 = -M \frac{di_1}{dt} \quad \text{and} \quad E_1 = -M \frac{di_2}{dt}$$

As M is the mutual inductance, its unit is henry.

RC Circuits

In purely resistive circuits, the currents do not vary with time. An introduction of a capacitor in the circuit leads to a concept of *time-varying currents*.

$$E = R \frac{dq}{dt} + \frac{q}{C}$$

is the differential equation and its solution is

$$q = CE (1 - e^{-t/RC})$$

Capacitive time constant

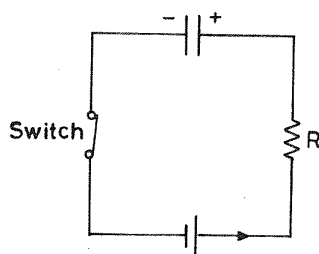
$$\tau_C = RC \quad (\text{This too has the dimensions of time.})$$

$$q = CE (1 - e^{-t/\tau_C})$$

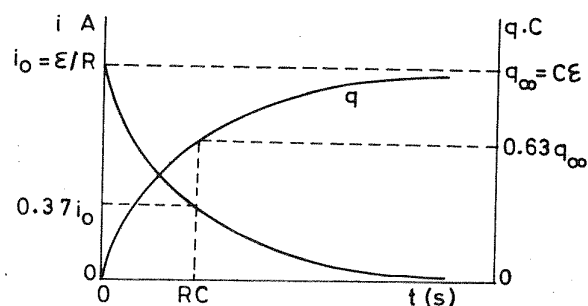
and if $t = \tau_C$

$$q = CE \left(1 - \frac{1}{e}\right) = 0.63 CE.$$

Consider an RC circuit as shown in Fig. (a) here. The capacitor is initially uncharged. The switch is now closed and the current i in the circuit and the charge q on the capacitor vary as shown in Fig. (b) here. With the passage of time, i decreases and when $t = RC$, the current decreases to 37% of its initial value. The variation of q , the charge on the capacitor, is also shown in Fig. (b). At $t = RC$, the charge attained by the capacitor is 63% of its final value.



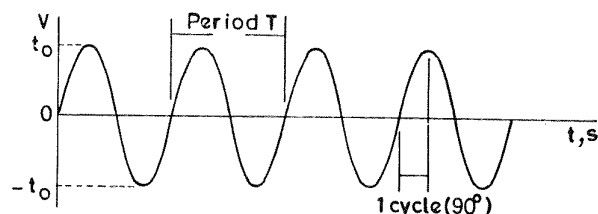
(a)



(b)

Alternating Current

When a coil is rotated in a magnetic field, the emf generated is shown in figure here. This is called ac voltage and the corresponding current is called alternating current.



$$V = V_0 \sin \omega t = V_0 \sin 2\pi \nu t$$

Here $\omega = 2\pi \nu$ is the angular velocity and ν is the frequency in hertz (cps).

(a) **A Resistive Circuit** If a circuit contains a resistive element only, acted on by the alternating emf ($V = V_0 \sin \omega t$), then the current is given by

$$i_R = \left(\frac{V}{R}\right) \sin \omega t$$

An important conclusion is that the time varying quantities V_R and i_R are in phase, and reach their maximum values at the same time.

(b) **A Capacitive Circuit** A circuit containing capacitive element only has the following relations

$$q = E_m C \sin \omega t$$

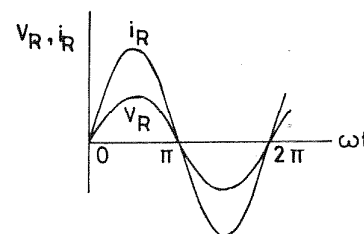
$$i_C = \frac{dq}{dt} = \omega C E_m \cos \omega t$$

and $V_C = E_m \sin \omega t$

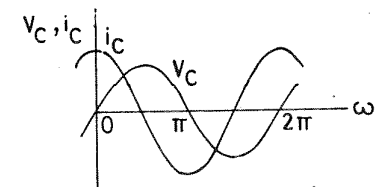
The time varying quantities V_C and i_C are one quarter cycle out of phase

$$i_C = \left(\frac{E_m}{X_C}\right) \cos \omega t \quad \text{where} \quad X_C = \frac{1}{\omega C}$$

X_C is called capacitive reactance.



(a)



(b)

(c) **An Inductive Circuit** This circuit contains an inductive element only and the relations are

$$V_L = E_m \sin \omega t$$

$$i_L = -\left(\frac{E_m}{\omega L}\right) \cos \omega t$$

In this case also, the time varying quantities V_L and i_L are a quarter cycle out of phase.

$$i_L = -\left(\frac{E_m}{X_L}\right) \cos \omega t$$

where $X_L = \omega L$, called inductive reactance.

Single Loop LCR Circuit

The emf is given by

$$E = E_m \sin \omega t$$

Current in the circuit is $i = i_m \sin(\omega t - \phi)$ and $E = V_R + V_C + V_L$. These are all sinusoidally time-varying quantities.

$$E_m = i_m \sqrt{R^2 + (X_L - X_C)^2}$$

and $\sqrt{R^2 + (X_L - X_C)^2}$ is called impedance, Z , of the circuit.

$$\therefore i_m = \frac{E_m}{Z}$$

We can also write

$$i_m = \frac{E_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

The phase angle ϕ between i and E in this case is given by

$$\tan \phi = \frac{X_L - X_C}{R}$$

Power in AC Circuits

$$P_{av} = E_{rms} i_{rms} \cos \phi$$

Here E_{rms} is the root mean square value of E and is given by $\frac{E_m}{\sqrt{2}}$

$$i_{rms} = \frac{i_m}{\sqrt{2}}$$

and $\cos \phi$ is known as the *power factor*.

Resonance in AC Circuits

If the factors E_m , R , C and L are kept fixed and the angular frequency ω is varied to see its effect on i_{rms} then the maximum value of i_{rms} occurs when $X_L = X_C$ or $\omega L = 1/\omega C$.

Therefore,

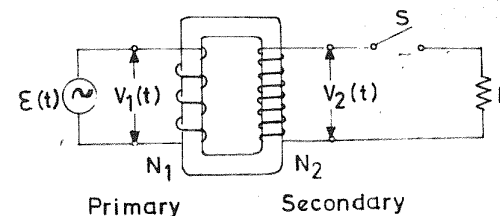
$$\omega = \frac{1}{\sqrt{LC}}$$

This condition is known as the condition of *resonance*.

The maximum value of i_{rms} occurs when the frequency ω of the driving force $[E(t)]$ is exactly equal to the natural frequency ω_0 of the undamped ($R = 0$) circuit.

Transformer

An alternating current transformer is a device to convert very large potential differences to small ones or vice versa.



$$V_{2,rms} = V_{1,rms} \left(\frac{N_2}{N_1} \right)$$

If $N_2 > N_1$, it is a *step-up transformer*.

If $N_2 < N_1$, we call it a *step-down transformer*.

ILLUSTRATIONS

1. Two parallel wires whose centres are at a distance d apart carry equal currents in opposite directions. Neglecting the flux within the wires themselves, find the inductance of a length of such a pair of wires, given that the radius of the wires is a .

Solution

The magnetic induction due to the current in the two wires is

$$B = \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right)$$

at a distance x from one wire.

Consider an element of width dx and length l .

$$\text{Flux } \phi_B = \int B l \cdot dx = \frac{\mu_0 i l}{2\pi} \int_a^{d-a} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx$$

$$= \frac{\mu_0 i l}{2\pi} \left[\ln x - \ln(d-x) \right]_a^{d-a}$$

$$= \frac{\mu_0 i l}{2\pi} \left[\ln \left(\frac{d-a}{a} \right) - \ln \left(\frac{a}{d-a} \right) \right]$$

$$= \frac{\mu_0 i l}{2\pi} \ln \left(\frac{d-a}{a} \right)^2 = \frac{\mu_0 i l}{\pi} \ln \frac{d-a}{a}$$

$$\therefore L = \frac{\phi_B}{i} = \frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$$

2. A 100 V potential difference is suddenly applied to a coil of inductance 100 mH and resistance 50 Ω . At what rate is the current increasing after one millisecond?

Solution

$$\text{Inductive time constant } \tau_L = \frac{L}{R} = \frac{100 \times 10^{-3} \text{ H}}{50 \Omega} = 2 \times 10^{-3} \text{ s}$$

$$I = I_0 (1 - e^{-t/\tau}) \text{ and } I_0 = \frac{E}{R} = \frac{100 \text{ V}}{50 \Omega} = 2 \text{ A}$$

$$\frac{dI}{dt} = \frac{I_0}{\tau} e^{-t/\tau} = \frac{2}{2 \times 10^{-3}} \exp\left(-\frac{0.001}{2 \times 10^{-3}}\right)$$

$$= 10^3 e^{-0.5} = 10^3 \times 0.606 = 606 \text{ A/s}$$

3. Find the inductance of two coaxial cylinders, the inner one of radius a and the outer one of radius b , carrying the same current per unit length but in the opposite direction.

Solution

The field B due to the current i in the inner cylinder at a distance r is given by

$$B = \frac{\mu_0 i}{2\pi r}$$

The flux through an elemental cylinder of thickness dr is

$$d\phi_B = \frac{\mu_0 i}{2\pi r} dr$$

$$\text{and } \phi_B = \int_a^b \frac{\mu_0 i}{2\pi r} dr = \frac{\mu_0 i}{2\pi} \left[\ln r \right]_a^b = \frac{\mu_0 i}{2\pi} \ln \frac{b}{a}$$

$$E = -\frac{d\phi_B}{dt} = -\frac{\mu_0}{2\pi} \ln \frac{b}{a} \left(\frac{di}{dt} \right) = -L \frac{di}{dt}$$

$$\therefore L = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

4. Find whether the discharge of a capacitor through the following inductive circuit is oscillatory. If it is, find the frequency of oscillation. $C = 0.5 \mu\text{F}$, $L = 15 \text{ mH}$ and $R = 250 \Omega$.

Solution

The frequency of oscillation is given by

$$\nu = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

The circuit will be oscillatory when $R^2/4L^2 < 1/LC$ or $R^2 < 4L/C$.

$$\frac{4L}{C} = \frac{4 \times 15 \times 10^{-3}}{0.5 \times 10^{-6}} = 120 \times 10^3 = 12 \times 10^4$$

$$\text{and } R^2 = 62500$$

We find that $R^2 < 4L/C$.

Hence, the circuit is oscillatory and its frequency is given by

$$\begin{aligned} \nu &= \frac{1}{2\pi} \sqrt{\frac{1}{15 \times 10^{-3} \times 0.5 \times 10^{-6}} - \frac{(250)^2}{4 \times (15 \times 10^{-3})^2}} \\ &= \frac{1}{2\pi} \sqrt{0.13 \times 10^9 - 69.4 \times 10^6} = \frac{10^3}{2\pi} \sqrt{0.13 \times 10^3 - 69.4} \\ &= \frac{10^3}{2\pi} \sqrt{60.6} \\ &= \frac{10^3}{2\pi} \times 7.78 \\ &= 1239 \text{ cps} \end{aligned}$$

5. The capacitor of an oscillatory circuit of negligible resistance is enclosed in a container. When the container is evacuated, the frequency of the circuit is 150k cycles per second, and when the container is filled with a gas, the frequency changes by 100 cycles per second. Find the dielectric constant of the gas.

Solution

$$\nu = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

In the first case, when there is vacuum,

$$\nu = 150 \times 10^3 \text{ cps}$$

$$150 \times 10^3 = \frac{1}{2\pi} \sqrt{\frac{1}{LC}} \quad (1)$$

In the second case, when the medium is a gas,

$$(150 \times 10^3 - 100) = \frac{1}{2\pi} \sqrt{\frac{1}{LKC}} \quad (2)$$

K is the dielectric constant of the gas.

Dividing (1) by (2), we get,

$$\frac{150 \times 10^3}{149900} = \sqrt{K} = 1.0006 \Rightarrow K = 1.0012$$

6. An ac voltage source of $E = 150 \sin 100t$ is used to run a device which offers a resistance of 20Ω and restricts the flow of current in one direction only. Calculate the average and rms values of the current in the circuit.

Solution

$$E = E_0 \sin \omega t$$

Here we have

$$E = 150 \sin 100t$$

$$\therefore E_0 = 150 \text{ V} \quad \omega = 100 \quad R = 20 \Omega$$

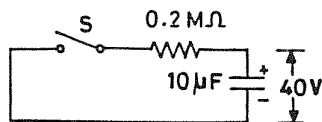
$$\therefore I_{\max} = \frac{E_0}{R} = \frac{150}{20} = 7.5 \text{ A}$$

Since the current flows only in one direction,

$$I_{\text{rms}} = \frac{I_{\max}}{2} = \frac{7.5 \text{ A}}{2} = 3.75 \text{ A}$$

$$\text{and } I_{\text{av}} = \frac{I_{\max}}{\pi} = \frac{7.5 \text{ A}}{\pi} = 2.38 \text{ A}$$

7. In the circuit shown here, the capacitor has an initial charge of such value that the voltage across its plates is 40 V. For the values of the parameters indicated, find the expression for the current which flows when the switch S is closed. Also find the total energy dissipated in the resistor during the transient state and compare it with the energy initially stored in the capacitor.



Solution

Taking voltage drops in a clockwise direction as positive, Kirchhoff's voltage law applied to the circuit with S closed yields:

$$0 = Ri + \frac{1}{C} \int i \, dt$$

$$i = -\frac{q_0}{RC} e^{-t/RC} = -\frac{40}{R} e^{-t/RC}$$

$$\text{Time constant } \tau_c = RC = 0.2 \times 10^6 \times 10 \times 10^{-6} = 2 \text{ s}$$

The solution for current may therefore be expressed as

$$i(t) = -\frac{40}{0.2 \times 10^6} e^{-t/2} = -2 \times 10^{-4} e^{-t/2} \text{ A} \quad (1)$$

The negative sign indicates that the actual current flow is counterclockwise rather than clockwise as initially assumed. The amount of energy dissipated in the resistor is readily found from the following equation.

$$\begin{aligned} W_{\text{diss.}} &= \int_0^\infty i^2(t) R \, dt = \int_0^\infty (4 \times 10^{-8}) \times 2 \times 10^5 e^{-t} \, dt \\ &= 8 \times 10^{-3} \left(e^{-t} \right)_0^\infty = 8 \times 10^{-3} \text{ J} \end{aligned} \quad (2)$$

$$W_i = \frac{1}{2} CV_0^2 = \frac{1}{2} \times 10^{-5} \times (40)^2 = 8 \times 10^{-3} \text{ J} \quad (3)$$

A comparison of (2) and (3) shows that all the capacitor energy is dissipated as heat after the switch is closed.

8. An LCR circuit consists of $L = 0.05 \text{ H}$, $C = 1 \mu\text{F}$ and $R = 500 \Omega$, all connected in series across a 220 V, 50 Hz power source. Find: (a) the current, (b) the power loss, (c) the phase angle between the current and the source voltage, and (d) the voltmeter readings across L , C and R .

Solution

The impedance of the circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}; \text{ here } R = 500 \Omega$$

$$X_L = 2\pi\nu L = 2\pi \times 50 \times 0.05 = 15.7 \Omega$$

$$\begin{aligned} X_C &= \frac{1}{2\pi\nu C} = \frac{1}{2 \times \pi \times 50 \times 1 \times 10^{-6}} = \frac{10^6}{314} \\ &= 0.003 \times 10^6 \Omega = 3 \times 10^3 \Omega \end{aligned}$$

$$Z = \sqrt{(500)^2 + (15.7 - 3000)^2} = \sqrt{(500)^2 + (-3000)^2}$$

[As an approximation neglect 15.7 as compared to 3000]

$$\approx 100\sqrt{925} \approx 3000 \Omega$$

$$(a) \quad \text{Current} = \frac{220}{3000} \text{ A} = 0.07 \text{ A}$$

$$(b) \quad \text{Power loss} = I^2 R = (0.07)^2 \times 500 = 2.45 \text{ W}$$

$$(c) \quad \tan \phi = \frac{X_L - X_C}{R} = \frac{15.7 - 3000}{500} = -5.99 \approx -6.0$$

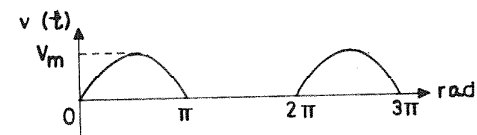
$$\phi = \tan^{-1}(-6) = -80.5^\circ \quad (\text{voltage lags behind the current})$$

$$(d) \quad V_R = IR = 0.07 \times 500 = 35 \text{ V}$$

$$V_C = IX_C = 0.07 \times 3000 = 210 \text{ V}$$

$$V_L = IX_L = 0.07 \times 15.7 = 1.09 \text{ V}$$

9. In the sketch here is shown a half sine wave. Find the average and effective value of the wave.

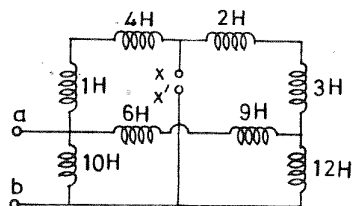


Solution

$$V_{av} = \frac{1}{2\pi} \left[\int_0^\pi V_m \sin \alpha \, d\alpha \right] = \frac{V_m}{2\pi} \left[-\cos \alpha \right]_0^\pi = \frac{V_m}{\pi}$$

$$V_{eff}^2 = \frac{1}{2\pi} \left[\int_0^\pi (V_m \sin \alpha)^2 \, d\alpha + 0 \right] = \frac{V_m^2}{4} \Rightarrow V_{eff} = \frac{V_m}{2}$$

10. A network of inductances is shown here in the figure. Find the equivalent inductance offered between a and b if terminals $x-x'$ are: (a) open, and (b) short-circuited.



Solution

(a) $x-x'$ open

$$1 + 4 + 2 + 3 = 10; \quad 6 + 9 = 15; \quad 10 \parallel 15 = 6$$

$$6 + 12 = 18; \quad 18 \parallel 10 = \frac{180}{28}$$

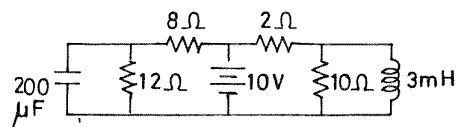
$$\therefore L_{eq} = 6.429 \text{ H}$$

(b) $x-x'$ short circuited

$$2 + 3 = 5; \quad 5 \parallel 12 = \frac{60}{17}; \quad 6 + 9 = 15$$

$$15 + \frac{60}{17} = \frac{315}{17}; \quad 5 \parallel 10 = \frac{10}{3}; \quad \frac{10}{3} \parallel \frac{315}{17} = 2.825 \text{ H}$$

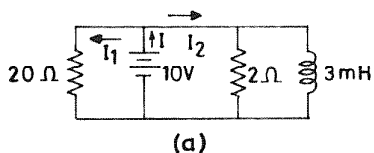
11. For the given circuit here, find the



- Energy stored in C,
- Energy stored in L,
- Current in each circuit element, and
- Voltage across each circuit element.

Solution

The voltage across 12Ω resistor is 6 V and the same potential difference will be there across the capacitor. This can be seen from a reduced circuit shown here. As no current will flow through



the capacitor, the circuit will have a resistance of 20Ω across the battery on one side and 2Ω on the other side.

$$I = I_1 + I_2$$

and $2I_2 = 10; \quad 20I_1 = 10 \Rightarrow$ current through 12Ω resistor $= \frac{1}{2} \text{ A}$

$\therefore$ Voltage drop across 12Ω resistor $= 6 \text{ V}$

$\therefore$ Energy stored in the capacitor

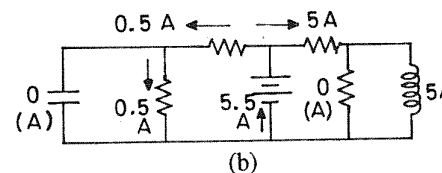
$$= \frac{1}{2} C \cdot V_c^2 = \frac{1}{2} \times 200 \times 10^{-6} \times (6)^2 = 3.6 \text{ mJ}$$

(b) Current through the inductance $= i_L = 5 \text{ A}$

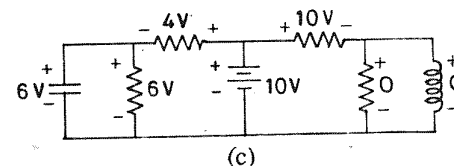
Energy stored in the inductance

$$= \frac{1}{2} L i_L^2 = \frac{1}{2} \times 3 \times 10^{-3} \times 25 = 37.5 \text{ mJ}$$

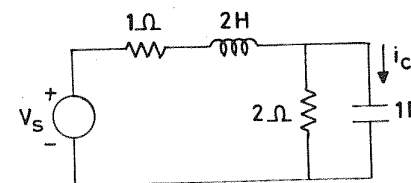
(c) Note that, here, inductance has zero resistance. Therefore the equivalent resistance of 10Ω in parallel to the inductance is zero. As a result, total resistance of the right loop is 2Ω . The currents are shown here in Fig. (b).



(d) The voltage drops across each element are shown in Fig. (c)



12. A circuit shown here consists of a voltage source V_s , an inductance, two resistances and a capacitance. If, for this circuit, $i_C = -2e^{-t}$, find V_s .



Solution

$$V_C = \frac{1}{C} \int_0^t i_C \, dt = - \int_0^t 2e^{-t} \, dt = 2[e^{-t} - 1] \text{ V}; \quad V_C(0) = 0$$

This voltage appears also across the $2\text{-}\Omega$ resistance (R and C are in parallel). Therefore,

$$i_{(2\Omega)} = \frac{2(e^{-t} - 1)}{2} = (e^{-t} - 1) \text{ A}$$

By applying Kirchhoff's current law through RL series,

$$i = (-2e^{-t} + e^{-t} - 1) = (-e^{-t} - 1) \text{ A}$$

$$V_L = L \frac{di}{dt} = 2 \frac{d}{dt} (-e^{-t}) = -e^{-t} \text{ V}$$

$$V_R = Ri = 1 \times (-e^{-t} - 1) \text{ V}$$

Applying Kirchhoff's voltage law round the mesh,

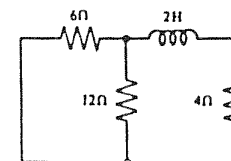
$$V_s - [-e^{-t} - 1] - [-e^{-t}] - [2e^{-t} - 2] = 0$$

$$\text{or } V_s + 3 = 0 \Rightarrow V_s = -3.0 \text{ V}$$

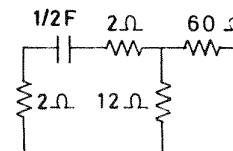
EXERCISES

- The inductance of a solenoid of 2500 turns wound uniformly over a length of 50 cm on a cylindrical cardboard tube 5 cm in diameter when the medium is air is
(a) 308.0 mH (b) 30.8 mH
(c) 15.4 mH (d) 154.0 mH
- A circuit contains an inductance L , a resistance R and a battery of emf E . The circuit is switched on at $t = 0$. The charge flown through the battery in one time constant (τ) is
(a) $\frac{2E\tau}{R \cdot e}$ (b) $\frac{E}{R} \cdot \frac{\tau}{e}$
(c) $\frac{E\tau}{2Re}$ (d) zero
- A coil has an inductance of 10 H and resistance of 25Ω . If a 100 V emf is applied, the energy stored in the magnetic field after the current has risen to its maximum value is
(a) 80 J (b) 40 J
(c) 160 J (d) 60 J
- The ratio of the energies required to set up in a cube of side 10 cm a uniform electric field of 10^5 V/m and a uniform magnetic field of 0.5 Wb/m^2 is
(a) 4.4×10^{-7} (b) 4.4×10^{-5}
(c) 4.4×10^7 (d) 4.4×10^5
- Two coils, one primary of 500 turns and one secondary of 25 turns, are wound on an iron ring of mean diameter 20 cm and cross-sectional area 12 cm^2 . If the permeability of iron is 800, the mutual inductance is
(a) 0.48 H (b) 2.4 H
(c) 0.12 H (d) 0.24 H
- The time constant of a circuit containing resistance and inductance, when the current rises to 63.2% of its steady value in one second, is

- 0.15 s (b) 0.5 s
 - 1 s (d) 2 s
- A capacitor of capacitance $2 \mu\text{F}$ is first charged and then discharged through a resistance of $1 \text{ M}\Omega$. The time in which the charge on the capacitor will fall to 50% of its initial value is
(a) 0.693 s (b) 1.38 s
(c) 0.35 s (d) 1.04 s
 - When an oscillatory circuit of negligible resistance is enclosed in a vacuum chamber its frequency is 10 k cycles per second. When the same circuit is immersed in a suitable dielectric medium, its frequency reduces by 20%. The dielectric constant of the medium is
(a) 2.08 (b) 1.84
(c) 1.56 (d) 1.056
 - A capacitor of capacitance $2 \mu\text{F}$ is first charged and then discharged through a resistance of $5 \times 10^4 \Omega$. The time in which the charge on the capacitor will fall to 36.8% of its initial value is
(a) 0.099 s (b) 0.99 s
(c) 0.49 s (d) 0.74 s
 - In an oscillatory circuit, $L = 0.4 \text{ H}$, $C = 0.0024 \mu\text{F}$. The maximum value of the resistance that should be included is
(a) $2.58 \times 10^4 \Omega$ (b) $2.58 \times 10^3 \Omega$
(c) $5.82 \times 10^4 \Omega$ (d) $5.82 \times 10^3 \Omega$
 - The equation of an alternating current is $i = 40 \sin 314t$. Its rms and average value respectively are
(a) 28.46 A and 28.46 A (b) 28.46 A and 14.26 A
(c) 14.2 A and 28.42 A (d) 28.46 A and 25.46 A
 - In the given circuit containing an inductor and resistors, the time constant of the circuit is



- $\frac{1}{4} \text{ s}$
 - $\frac{1}{2} \text{ s}$
 - $\frac{1}{3} \text{ s}$
 - $\frac{1}{5} \text{ s}$
- The given circuit consists of a capacitor and resistors. The time constant of the circuit is



- $\frac{1}{4} \text{ s}$
- 4 s
- 2 s
- 1 s

- The current through an inductor of 1 H is given by

$$i = 3t \sin t$$

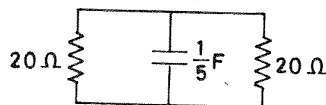
The voltage across the inductor of 1 H is

- $3 \sin t + 3 \cos t$ (b) $3 \cos t + t \sin t$
 - $3 \sin t + 3t \cos t$ (d) $3t \cos t + \sin t$
- If the value of series connected R and L is reduced to $R/2$ and $L/2$, the time constant is

- (a) half that of the RL circuit
 (b) twice that of RL circuit
 (c) same as that of the RL circuit
 (d) one fourth that of the RL circuit

16. Two resistors and a capacitor are connected in a circuit as shown in the figure here. The time constant of the circuit is

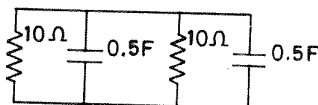
- (a) 2 s
 (b) 4 s
 (c) 8 s



- (d) 1/2 s

17. The time constant of the circuit shown in this figure is

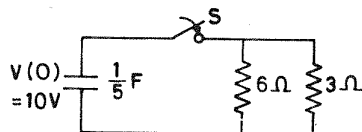
- (a) 0.1 s
 (b) 10 s
 (c) 1/5 s
 (d) 5 s



18. A capacitor of capacitance $1/5$ F is initially charged to a potential difference of 10 V, and then the switch S is closed. The potential difference across the capacitor at $t = 1$ s is

- (a) 2.0 V
 (c) 0.8 V

- (b) 4.0 V
 (d) 0.4 V



19. In the above problem, the expression for the decay of voltage across the capacitor for $t > 0$ (switch S is closed at $t = 0$) when the initial voltage $V(0) = 10$ is

- (a) $10e^{-3t/2}$
 (c) $10e^{-2t/5}$

- (b) $10e^{-5t/2}$
 (d) $10e^{-t/2}$

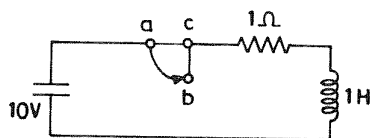
20. In the given circuit here, the switch is closed to the position bc from the earlier position of ac at $t = 0$. The current in the inductor after 2 s of closing the switch between b and c is

- (a) $\frac{1}{10e^2}$

- (b) $10e^{-2}$

- (c) $10e$

- (d) $10e^2$



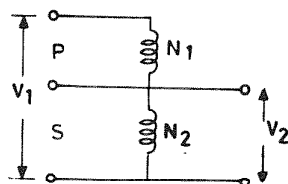
21. The primary (P) of a transformer has N_1 turns while the secondary (S) has N_2 turns. The connections of the two windings are made as shown in the figure and the voltages measured are V_1 and V_2 . The ratio V_1/V_2 is

- (a) $\frac{N_2 - N_1}{N_2}$

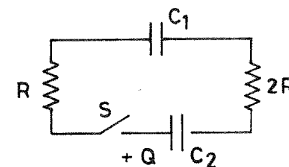
- (b) $\frac{N_2 - N_1}{N_1 + N_2}$

- (c) $\frac{N_2 + N_1}{N_2 - N_1}$

- (d) $\frac{N_2 + N_1}{N_2}$



22. In the circuit shown, the switch S is closed at $t = 0$. After a very long time,



- (a) the charges on both the capacitors will be equal but not zero.
 (b) the charges on both the capacitors will be non-zero and in the ratio of $C_1 : C_2$
 (c) charges in both the capacitors will be non-zero and in the ratio of $\frac{1}{C_1} : \frac{1}{C_2}$
 (d) charges will be zero

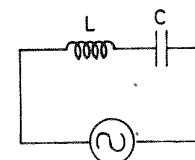
23. The voltage applied is of the form $V = A \sin \omega t + B \sin 2\omega t$. The voltage drop across the inductance is

- (a) $\frac{A\omega L}{\omega L - 1/\omega C}$

- (b) $\frac{2B\omega L}{2\omega L - 1/2\omega C}$

- (c) $\frac{A\omega L}{\omega L - 1/\omega C} + \frac{2B\omega L}{2\omega L - 1/2\omega C}$

- (d) $\frac{A\omega L}{\omega L - 1/\omega C} - \frac{2B\omega L}{2\omega L - 1/2\omega C}$



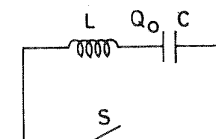
24. At $t < 0$, the capacitor is charged and the switch is open. At $t = 0$ the switch is closed. The shortest time T at which the charge on the capacitor will be zero is given by

- (a) $\pi \sqrt{LC}$

- (b) $\frac{3}{2} \pi \sqrt{LC}$

- (c) $\frac{\pi}{2} \sqrt{LC}$

- (d) $2\pi \sqrt{LC}$



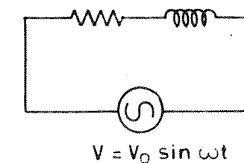
25. In the given circuit, the energy lost in one cycle is given by

- (a) $\frac{V_0^2}{2R}$

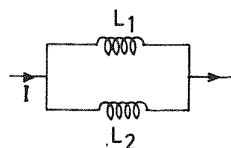
- (b) $\frac{V_0^2}{R}$

- (c) $\frac{V_0^2 R}{2(R^2 + \omega^2 L^2)}$

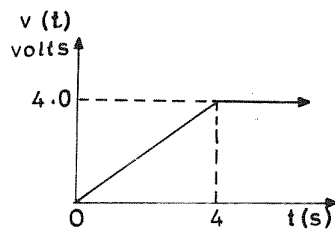
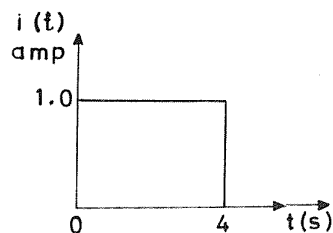
- (d) $\frac{V_0^2 R}{(R^2 + \omega^2 L^2)}$



26. Two inductors L_1 and L_2 are connected in parallel and a current $I = I_0 e^{-t} + K_0 e^{-2t}$ flows as shown. The ratio of currents I_1 and I_2 is given by



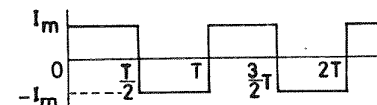
- (a) $\frac{L_1}{L_2}$ (b) $\frac{L_2}{L_1}$
 (c) $\frac{L_1}{L_2} e^{-t}$ (d) $\frac{L_2}{L_1} e^{-t}$
27. The inductance of a coil in which a current of 0.1 A yields an energy storage of 0.05 J is
 (a) 2 H (b) 10 H
 (c) 8 H (d) 20 H
28. The inductance of a coil in which a current increases linearly from zero to 0.1 A in 0.2 s, producing a voltage of 5 V is
 (a) 10 H (b) 2 H
 (c) 8 H (d) 20 H
29. The inductance of a coil in which a current of 0.1 A increasing at the rate of 0.5 A/s represents a power flow of 1/2 W is
 (a) 2 H (b) 8 H
 (c) 20 H (d) 10 H
30. When a dc voltage is applied to a capacitor, the voltage across its terminals is found to build up in accordance with $V_c = 150(1 - e^{-20t})$. If the current is 1.14 mA after 0.05 s, the capacitance of the capacitor is
 (a) $3.01 \mu\text{F}$ (b) $0.03 \mu\text{F}$
 (c) $1.03 \mu\text{F}$ (d) $10.3 \mu\text{F}$
31. In the above problem, the energy stored at this time is
 (a) 0.0046 J (b) 0.046 J
 (c) 0.64 J (d) 0.460 J
32. A circuit element is placed in a black box. At $t = 0$, a switch is closed and the current flowing through the circuit element and the voltage across its terminals are recorded to have the waveshapes shown in the figure here. The type of element and its magnitude are:



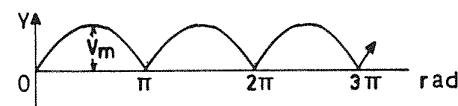
- (a) inductance of 4 H (b) resistance of 4 Ω
 (c) capacitance of 1 F (d) a voltage source of emf 4 V
33. A voltage waveshape of the form $V(t) = Kt^3$ is applied to a circuit element at time $t = 0$. The current through a resistor, an inductor and a capacitor respectively are

- (a) $\frac{K}{R} t^3$, $\frac{K}{L} \frac{t^3}{3}$ and $\frac{3CKt^4}{4}$ (b) $\frac{K}{4R} t^4$, $\frac{K}{L} \frac{t^4}{4}$ and $\frac{3KC^2t^2}{2}$
 (c) $\frac{K}{R} t^3$, $\frac{K}{L} \frac{t^4}{4}$ and $3CKt^2$ (d) $\frac{K}{R} t^3$, $\frac{K}{L} \frac{t^3}{3}$ and CKt^3

34. A waveshape is shown in the figure here. The average and effective values respectively are

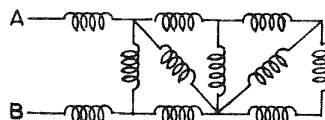


- (a) $\frac{I_m}{2}$ and $\frac{I_m}{\sqrt{2}}$ (b) zero and I_m
 (c) I_m and $\frac{I_m}{\sqrt{2}}$ (d) zero and $\frac{I_m}{\sqrt{2}}$
35. The average and effective values for the waveshape shown here in the figure are

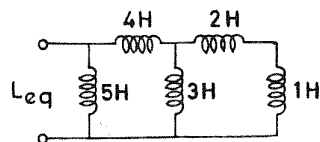


- (a) $\frac{2}{\pi} V_m$ and $\frac{V_m}{2}$ (b) $\frac{V_m}{\pi}$ and $\frac{V_m}{\sqrt{2}}$
 (c) $\frac{2}{\pi} V_m$ and $\frac{V_m}{\sqrt{2}}$ (d) $\frac{V_m}{\sqrt{2}\pi}$ and $\frac{V_m}{\sqrt{2}}$
36. A sinusoidal voltage $V(t) = 170 \sin(377t + \pi/3)$ is applied to a 0.1 H inductor. The effective value of the steady state current in amperes is
 (a) 37.7 (b) 3.18
 (c) 8.31 (d) 3.81
37. In the above problem, the expression for the instantaneous value of current is
 (a) $170 \sin(377t - \pi/6)$ (b) $3.18 \sin(377t - \pi/6)$
 (c) $6.36 \sin(377t - \pi/3)$ (d) $4.5 \sin(377t - \pi/6)$
- 38*. A capacitor of capacitance $4.5 \mu\text{F}$ and having an initial energy of 1.0 J is discharged through a resistor of $2 \text{ M}\Omega$. The following conclusions are drawn. Mark the one which is not correct.
 (a) The initial charge on the capacitor is 3 mC
 (b) The current through the resistor when the discharge starts is $1/3 \text{ mA}$
 (c) The voltage across the capacitor as a function of time is given as $V_C = 666 \times e^{-t/9}$
 (d) The rate of joule heating in the resistor as a function of time is $0.22 \times e^{-t/9}$
39. An RC circuit is discharged by closing a switch at time $t = 0$. The initial potential difference across the capacitor is 150 V. The potential difference decreases to 10 V after 10 s. The potential difference 20 s after $t = 0$ is
 (a) 0.677 V (b) 0.166 V
 (c) 1.660 V (d) 6.160 V
- 40*. A series circuit consists of a 200Ω , noninductive resistor, an inductor of 0.50 H of negligible resistance, and a capacitor of $100 \mu\text{F}$ capacitance across a 150-V, 60-Hz power source. The voltmeter readings across the three elements, R, L and C, respectively, are

- (a) 135 V, 106 V and 15 V (b) 116 V, 109 V and 15 V
 (c) 116 V, 106 V and 25 V (d) 109 V, 116 V and 25 V
41. A network of inductances, each of value 1 H, is shown here in the figure. The equivalent inductance of the circuit between points A and B is



- (a) 6.218 H (b) 2.618 H
 (c) 8.162 H (d) 0.268 H
42. The equivalent inductance of the network shown here is



- (a) 1.296 H (b) 9.126 H
 (c) 6.219 H (d) 2.619 H

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (b) | 3. (a) | 4. (a) | 5. (d) |
| 6. (c) | 7. (b) | 8. (c) | 9. (a) | 10. (a) |
| 11. (d) | 12. (a) | 13. (b) | 14. (c) | 15. (c) |
| 16. (a) | 17. (d) | 18. (c) | 19. (b) | 20. (b) |
| 21. (d) | 22. (c) | 23. (c) | 24. (c) | 25. (c) |
| 26. (b) | 27. (b) | 28. (a) | 29. (d) | 30. (c) |
| 31. (a) | 32. (c) | 33. (c) | 34. (b) | 35. (c) |
| 36. (b) | 37. (d) | 38. (d) | 39. (a) | 40. (b) |
| 41. (b) | 42. (d) | | | |

Note: Problems marked with an asterisk should take about four minutes each and the rest should take about two to three minutes each.

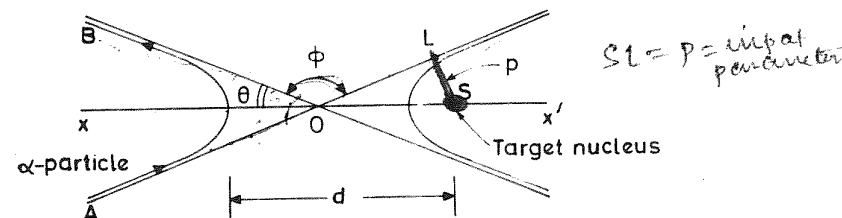
24 The Atom

A. Atomic Model and Spectra

Rutherford Model

According to this model, the atom consists mainly of a nucleus occupying a very small region of space and containing all the positive charges. The nucleus is surrounded by negatively charged electrons. The space between the nucleus and the electrons is empty.

Scattering of α -Particle Based on classical mechanics, an α -particle is scattered and follows a path of a hyperbola with the scattering nucleus at the external focus S. The α -particle, being incident in the direction of AO, shown in the figure, gets deflected through an angle ϕ (called the angle of scattering). If there were no force acting on the α -particle, it would pass by the nucleus at



the minimum distance equal to p (SL) which is the length of the perpendicular from S on the asymptote AO. This perpendicular distance is called *impact parameter*. If d is the distance of nearest approach (as shown in the figure) and θ is the angle which the asymptotic direction AO of the α -particle makes with the X-axis, then the relation between d , p and θ is as follows:

$$d = p \cdot \cot \frac{\theta}{2}$$

The relation between impact parameter p and the angle of scattering ϕ is given by

$$p = \frac{b}{2} \cot \frac{\phi}{2}$$

where b is a constant for the scattering set-up, equal to $Ze^2/\pi\epsilon_0 mV^2$. (Ze , being

the charge on the nucleus where Z is the atomic number, $2e$ is the charge on the α -particle.)

This equation shows that:

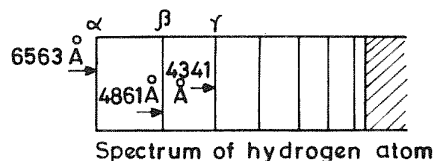
- as p increases, ϕ must decrease, i.e. the angle of scattering decreases with the increase in the impact parameter,
- an α -particle approaching a target nucleus with an impact parameter ranging from zero to p will be scattered through an angle ϕ or more. In other words, an α -particle which is directed anywhere within an area (πp^2) around the nucleus will be scattered through ϕ or more. The area (πp^2) is called *cross-section of interaction* (σ).

The distance of closest approach can also be expressed as

$$d = \frac{Ze^2}{2\pi\epsilon_0 mV^2} (1 + \operatorname{cosec} \phi/2)$$

Hydrogen Spectrum; Balmer's Formula

Balmer observed a set of spectral lines; the spacing between these lines gradually decreases towards the shorter wavelength and the set approaches a limit near ultraviolet. The series is known as Balmer Series. Balmer's expression for wavelength is:



$$\frac{1}{\lambda} = 10970600 \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

The reciprocal of wavelength is known as wave number $\bar{\nu}$

$$\therefore \bar{\nu} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where R_H is a constant and is equal to 10970600 m^{-1} .

Rydberg modified Balmer's formula and the value of R_H was given as 10967800 m^{-1} and is known as Rydberg's constant ($1.09678 \times 10^7 \text{ m}^{-1}$). The other series that have been discovered are known as Lyman series (in the UV region) and Paschen, Brackett and Pfund series (all in the infrared region)

$$\bar{\nu} = R_H \left(\frac{1}{1^2} - \frac{1}{n^2} \right), n = 2, 3, 4, \dots \text{ (Lyman series) } \} \text{ UV region}$$

$$\left. \begin{aligned} \bar{\nu} &= R_H \left(\frac{1}{3^2} - \frac{1}{n^2} \right), n = 4, 5, 6, \dots \text{ (Paschen series)} \\ \bar{\nu} &= R_H \left(\frac{1}{4^2} - \frac{1}{n^2} \right), n = 5, 6, 7, \dots \text{ (Brackett series)} \\ \bar{\nu} &= R_H \left(\frac{1}{5^2} - \frac{1}{n^2} \right), n = 6, 7, 8, \dots \text{ (Pfund series)} \end{aligned} \right\} \text{ IR region}$$

Bohr's Model of Hydrogen Atom

Bohr's Postulates

- The electron in a hydrogen atom moves in a circular orbit around the nucleus, the centripetal force being provided by the Coulomb attraction between the electron and the nucleus.
- An allowed orbit of the electron is one in which the orbital angular momentum L satisfies the relation $L = nh/2\pi$ where h is Planck's constant and n is a positive integer. (This amounts to the conclusion that the orbits are quantized.)
- The electron does not radiate as long as it is in an orbit satisfying the relation $L = nh/2\pi$. It emits radiation only when it jumps from an allowed orbit with an energy E_i to another allowed orbit with an energy E_f . The frequency ν of the emitted radiation is given by

$$h\nu = E_i - E_f$$

Radius of Bohr's n th orbit,

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$$

Kinetic energy of the electron in the orbit of radius $r = \frac{e^2}{8\pi\epsilon_0 r}$

Potential energy of the electron in the orbit of radius $r = \frac{-e^2}{4\pi\epsilon_0 r}$

Total energy of an electron in the n th orbit $= -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$

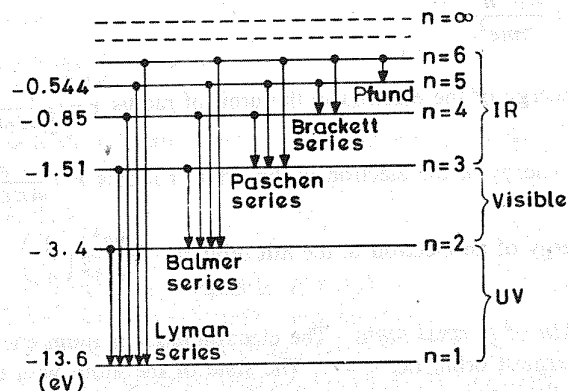
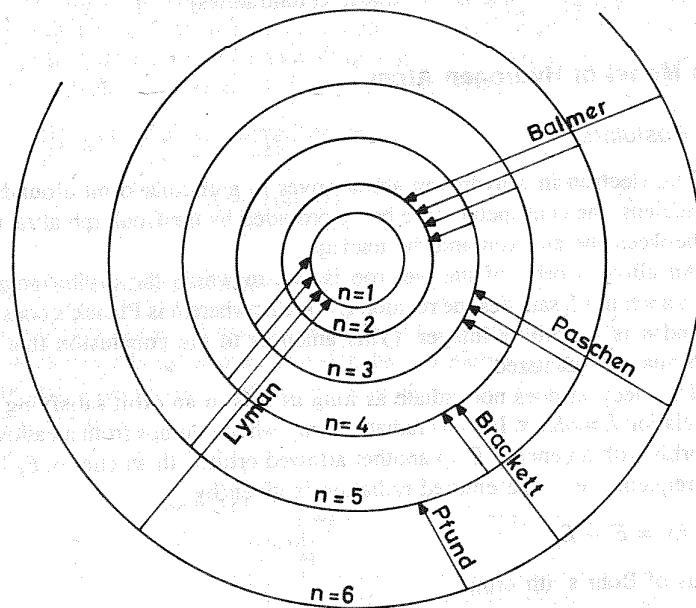
Ground state or normal state The electron has minimum energy when it is in its innermost orbit, i.e. $n = 1$. The state of the atom with the electron revolving in the innermost orbit is called the *ground state* or *normal state* and is obviously the most stable state of the atom.

Excited state The state of the atom when one of its electrons is forced to revolve in an outer orbit is called an excited state. (This is an unstable state.) The frequency of the emitted radiation is given by

$$v = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where R is a constant equal to $me^4/8\epsilon_0^2h^3$.

Energy Levels of Hydrogen



The ground state energy of the hydrogen atom

$$E_1 = -\frac{me^4}{8\epsilon_0^2h^2} = -21.76 \times 10^{-19} \text{ J} = -13.6 \text{ eV}$$

This is also called the ionization energy of the hydrogen atom. Similarly, $E_2 = -3.4 \text{ eV}$, $E_3 = -1.51 \text{ eV}$, and so on.

Excitation potential is that accelerating potential which gives to a bombarding electron sufficient energy to excite the target atom by raising one of its electrons from the inner to an outer orbit.

Ionization potential is that accelerating potential which gives to the bombarding electron enough energy to ionise the target atom by knocking one of its electrons completely out of the atom. The energy associated with the n th orbit is given by

$$E_n = -\frac{me^4}{8\epsilon_0^2h^2} \left(\frac{1}{n^2} \right) = -\frac{13.6 \text{ eV}}{n^2}$$

The energy required to raise the hydrogen atom from its ground state to its first excited state is $[-3.4 - (-13.6)] \text{ eV} = 10.2 \text{ eV}$; and that required to raise it to the second excited state is $[-1.51 - (-13.6)] \text{ eV} = 12.09 \text{ eV}$. The energy required to ionize the atom is $[0 - (-13.6)] \text{ eV} = 13.6 \text{ eV}$.

Hydrogen-like Atom

An atom of atomic number Z contains Z electrons moving around the nucleus with a charge Ze . If $(Z-1)$ electrons are removed, then, as in the hydrogen atom, there will only be a single electron moving around the nucleus in the atom except that the nuclear charge would be Ze instead of e . The energy values $E_{Z,n}$ are given by the expression

$$E_{Z,n} = -Z^2 E_n$$

where E_n are the allowed energy values of the hydrogen atom. The wavelength of the spectrum of an ionized atom is

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

where $m = 1, n = 2, 3, 4, \dots$

$m = 2, n = 3, 4, 5, \dots$

$m = 3, n = 4, 5, 6, \dots$ and so on.

ILLUSTRATIONS

1. Calculate the frequency of the electron in the n th orbit of a hydrogen-like atom with a nuclear charge Ze .

Solution

$$\frac{mV^2}{r} = \frac{Ze^2}{4\pi\epsilon_0 r^2}, \quad mVr = n\hbar \quad \left(\frac{h}{2\pi} = \hbar \right)$$

$$\therefore r_n = \frac{4\pi\epsilon_0 \hbar^2}{me^2} \frac{n^2}{Z} = \frac{a_0 n^2}{Z} \quad (\text{where } a_0 \text{ is the Bohr radius})$$

$$\text{and } V_n = \frac{Ze^2}{4\pi\epsilon_0 \hbar} \frac{1}{n} = \frac{Z\alpha c}{n} \quad \left(\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} \right) \approx \frac{1}{137}$$

The frequency of the electron in the n th orbit then is

$$\nu_n = \frac{V_n}{2\pi r_n} = \frac{Z^2 \alpha c}{2\pi a_0} \left(\frac{1}{n^3} \right)$$

Using $\frac{\alpha}{a_0} = \left(\frac{e^2}{4\pi\epsilon_0 c \hbar} \right) \left(\frac{me^2}{4\pi\epsilon_0 \hbar^2} \right) = \frac{mc\alpha^2}{\hbar}$

we get

$$\nu_n = \frac{Z^2 mc^2 \alpha^2}{2\pi \hbar} \left(\frac{1}{n^3} \right)$$

2. The energy in a Bohr orbit is given by $-13.6 (1/n^2)$ eV. Calculate the frequency of radiation when the electron jumps from the third state to the second.

Solution

$$E_n = -13.6 \frac{1}{n^2}; \quad E_3 = -\frac{13.6}{9} \text{ eV and } E_2 = -\frac{13.6}{4} \text{ eV}$$

$$E_3 - E_2 = +13.6 \left(-\frac{1}{9} + \frac{1}{4} \right) \text{ eV} = 13.6 \left(\frac{5}{36} \right) \text{ eV}$$

$$= 13.6 \times \frac{5}{36} \times 1.6 \times 10^{-19} \text{ J}$$

$$E_3 - E_2 = h\nu$$

$$\therefore \nu = \frac{E_3 - E_2}{h} = \frac{13.6 \times 5 \times 1.6}{36 \times 6.62 \times 10^{-34}} \times 10^{-19}$$

$$= 0.46 \times 10^{15} = 4.6 \times 10^{14} \text{ s}^{-1}$$

3. What are the three longest wavelengths of the spectral lines emitted by the hydrogen atom as it falls to the orbit with $n = 1$ from higher energy orbits?

Solution

The longest wavelengths correspond to the transitions:

(i) $E_4 - E_1$, i.e. from $n = 4$ to $n = 1 \Rightarrow \Delta E = -0.85 - (-13.6) = 12.75 \text{ eV}$

(ii) $E_3 - E_1$, i.e. from $n = 3$ to $n = 1 \Rightarrow \Delta E = -1.5 - (-13.6) = 12.1 \text{ eV}$

(iii) $E_2 - E_1$, i.e. from $n = 2$ to $n = 1 \Rightarrow \Delta E = -3.4 - (-13.6) = 10.2 \text{ eV}$

$$\lambda_{4,1} = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34} \text{ J.s})(3 \times 10^8 \text{ m/s})}{(12.75 \text{ eV} \times 1.6 \times 10^{-19} \text{ J/eV})} = 970 \text{ \AA}$$

$$\lambda_{3,1} = \frac{hc}{\Delta E} = \frac{hc}{12.1 \text{ eV}} = 1020 \text{ \AA}$$

$$\lambda_{2,1} = \frac{hc}{\Delta E} = \frac{hc}{10.2 \text{ eV}} = 1220 \text{ \AA}$$

4. Consider an electron moving around a nucleus of charge $2e$ in a circular orbit of radius 10^{-10} m .

(a) What would be the initial frequency of light emitted by the electron?

(b) How does the frequency of the emitted light vary with the orbit radius.

Solution

Centripetal force for the circular motion is provided by the Coulomb attraction of the nucleus. If V is the speed of the electron and r is the radius of the orbit, then

$$\frac{mV^2}{r} = \frac{2e^2}{4\pi\epsilon_0 r^2}$$

or $V = (2e^2/4\pi\epsilon_0 mr)^{1/2}$ and the frequency of electron moving around the nucleus is

$$f = \frac{V}{2\pi r} = \left(\frac{2e^2}{4\pi\epsilon_0 mr} \right)^{1/2} \frac{1}{2\pi r} \quad (\text{A})$$

(a) The initial frequency ν of the light emitted is

$$\left(\frac{2}{4\pi\epsilon_0 m} \right)^{1/2} \frac{e}{2\pi r^{3/2}} = \frac{1.414 \times (9 \times 10^9)^{1/2} \times 1.6 \times 10^{-19}}{(9.1 \times 10^{-31})^{1/2} \times 2\pi (10^{-10})^{3/2}}$$

$$= 3.6 \times 10^{15} \text{ Hz}$$

(b) From the expression (A), we see that the frequency varies as $1/r^{3/2}$.

5. Electrons in hydrogen-like atoms ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. Find the energies of the resulting radiations ($R = 1.094 \times 10^7 \text{ m}^{-1}$). Which transition will emit shorter wavelength?

Solution

$$E_n = -R \frac{Z^2 hc}{n^2} = \frac{-R \times 9 \times hc}{n^2}$$

$$= \frac{-1.094 \times 10^7 \times 9 \times 6.63 \times 10^{-34} \times 3 \times 10^8}{n^2}$$

$$= -\frac{122.4}{n^2} \text{ eV}$$

$$E_5 = -\frac{122.4}{25} \text{ eV} = -4.896 \text{ eV}$$

$$E_4 = -\frac{122.4}{16} \text{ eV} = -7.65 \text{ eV}$$

$$E_3 = -\frac{122.4}{9} \text{ eV} = -13.6 \text{ eV}$$

$$E_5 - E_4 = [-4.896 - (-7.65)] \text{ eV} = 2.754 \text{ eV}$$

$$E_4 - E_3 = [-7.65 - (-13.6)] \text{ eV} = 5.95 \text{ eV}$$

It follows that the radiation corresponding to the $4 \rightarrow 3$ transition is the one of shorter wavelength.

6. The average lifetime of an excited state of the hydrogen atom is of the order of 10^{-8} s. An electron is revolving in an orbit corresponding to $n = 3$. How many revolutions will it undergo before making a transition to the orbit with $n = 2$ (Bohr radius = 5.3×10^{-11} m)?

Solution

$$V_1 = \frac{e^2}{2\epsilon_0 h} = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.6 \times 10^{-34}} = 2.19 \times 10^6 \text{ m/s}$$

$$V_n = \frac{V_1}{n} \Rightarrow V_3 = \frac{V_1}{3} = \frac{2.19 \times 10^6}{3} = 0.73 \times 10^6 \text{ m/s}$$

$$r_n = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \text{ and } r_1 = \frac{h^2 \epsilon_0}{\pi m e^2} \Rightarrow r_n = n^2 r_1$$

$$\therefore r_3 = 9 \times 5.3 \times 10^{-11} \text{ m} = 47.7 \times 10^{-11} \text{ m}$$

$$\text{Number of revolutions made in one second} = \frac{V_3}{2\pi r_3} \text{ in the orbit } (n = 3)$$

$$= \frac{0.73 \times 10^6 \text{ m/s}}{2 \times \pi \times 47.7 \times 10^{-11} \text{ m}} = 0.002436 \times 10^{17} = 2436 \times 10^{11} \\ = 2.436 \times 10^{14} / \text{s}$$

$$\therefore \text{Number of revolutions made in } 10^{-8} \text{ s} = 2.436 \times 10^{14} \times 10^{-8} \text{ s} = 2.436 \times 10^6 \text{ revolutions.}$$

(Note: Bohr radius is the radius of the Bohr orbit corresponding to $n = 1$. It can be calculated from the expression $r_1 = \frac{h^2 \epsilon_0}{\pi m e^2}$ and the value comes to 5.3×10^{-11} m as given in the problem.)

7. A hydrogen-like atom has a stationary nucleus of charge Ze where Z is a constant and e is the electronic charge. It requires 68.0 eV to excite the electrons from the second Bohr orbit to third Bohr orbit. Find the following:

- the value of Z .
- the energy required to excite the electron from the third to the fourth Bohr orbit.
- the wavelength of electromagnetic radiation required to remove the electron from first Bohr orbit to infinity.
- the radius of the first Bohr orbit.

Solution

(a) For a hydrogen-like atom,

$$E_{n_2} - E_{n_1} = Z^2 E_0 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ eV}$$

where E_0 is the ionization energy of hydrogen atom.

$$\Delta E = Z^2 \times 13.6 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 68.0 \text{ eV}$$

$$Z^2 = \frac{68.0 \times 36}{13.6 \times 5} = 36 \Rightarrow Z = 6$$

$$(b) E_4 - E_3 = 6^2 \times 13.6 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) \text{ eV} = 23.8 \text{ eV}$$

$$(c) E_\infty - E_1 = 36 \times 13.6 \left(\frac{1}{1^2} - \frac{1}{\infty} \right) = 489.6 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{489.6 \times 1.6 \times 10^{-19}} \text{ m} = \frac{6.6 \times 3}{489.6 \times 1.6} \times 10^{-7} \text{ m} \\ = 0.0253 \times 10^{-7} \text{ m} \\ = 25.3 \times 10^{-10} \text{ m} \\ = 25.3 \text{ \AA}$$

$$(d) r_n = \frac{n^2 r_0}{Z}$$

For $n = 1$,

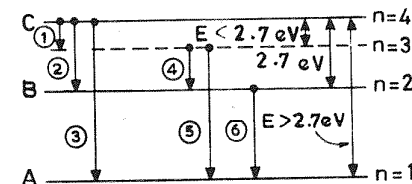
$$r_1 = \frac{1^2 \times 5.3 \times 10^{-11}}{6} = 0.88 \times 10^{-11} \text{ m}$$

8. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B. There are no atoms in any other level. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have more energy and some less than 2.7 eV. Find the following:

- the principal quantum number of the initially excited level B.
- the ionization energy for the gas atoms.
- maximum and minimum energies of radiation emitted.

Solution

The figure shows the energy levels A, B of the hydrogen-like atom. When light of photon energy 2.7 eV is absorbed, let the electrons go to an excited state C. Since subsequently the atom emits six different photons, state C should be such that six different transitions are possible. The possible transitions are shown in



the figure and it is obvious that energy level C must correspond to quantum number 4. The quantum number corresponding to state B must therefore be between 1 and 4. This means that it is either 2 or 3. Also,

$$E_C - E_B = 2.7 \text{ eV (given)}$$

If $n_B = 3$, there will be no subsequent radiations with energy less than 2.7 eV. But we are given that there are some subsequent radiations with energy less than 2.7 eV. This is possible only if there is some other energy state between B and C having a difference less than 2.7 eV. Therefore, n_B must be 2.

$$(a) \quad E_n = -\frac{13.6}{n^2} Z^2; \quad E_B = E_2 = -\frac{13.6}{2^2} Z^2 = -3.4 Z^2$$

$$\text{and} \quad E_C = E_4 = -\frac{13.6}{4^2} Z^2 = -0.852 Z^2$$

$$\therefore E_C - E_B = 2.7 \text{ eV} = -0.852 Z^2 - (-3.4 Z^2) = 2.55 Z^2$$

$$\therefore Z = 1$$

$$(b) \text{ The ionization energy} = E_1 = -13.6 Z^2 = -13.6 \text{ eV}$$

(c) The maximum energy of the emitted radiation $E_{\max}$ corresponds to a transition from $n = 4$ to $n = 1$ and is equal to

$$-0.852 - (-13.6) = 12.748 \text{ eV}$$

$$\text{and} \quad E_{\min} = E_4 - E_3 = 1.51 - 0.85 = 0.66 \text{ eV} \quad \left(E_3 = \frac{-13.6}{9} = -1.51 \text{ eV} \right)$$

9. Electrons of energies 10.20 eV and 12.09 eV can cause radiation to be emitted from hydrogen atoms. Calculate in each case the principal quantum number of the orbit to which the electron in the hydrogen atom is raised and the wavelength of the radiation emitted if it drops back to the ground state.

Solution

$$E_n = -\frac{13.6}{n^2} \text{ eV} \Rightarrow E_1 = -13.6, \quad E_2 = -3.4 \text{ eV}$$

$$\text{and} \quad E_3 = -1.51 \text{ eV}$$

$$E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

This corresponds to transitions between $n = 1$ to $n = 2$.

$$E_3 - E_1 = 1.51 - (13.6) = 12.09 \text{ eV}$$

This corresponds to transitions between $n = 1$ to $n = 3$.

Hence the electrons will be raised to orbits corresponding to principal quantum number $n = 2$ and $n = 3$.

Wavelength of radiation emitted in the transition from $n = 2$ to $n = 1$

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{2^2} \right] = R \times \frac{3}{4} \text{ m}^{-1} = 1.09 \times 10^7 \times \frac{3}{4}$$

$$\text{or} \quad \lambda = \frac{4}{3 \times 1.09 \times 10^7} = 1.223 \times 10^{-7} = 1223 \text{ \AA}$$

Similarly, for $n = 3$ to $n = 1$,

$$\frac{1}{\lambda} = R \left[\frac{1}{1^2} - \frac{1}{3^2} \right] = R \times \frac{8}{9} = 1.09 \times 10^7 \times \frac{8}{9}$$

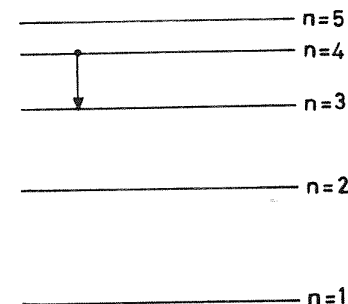
$$\text{or} \quad \lambda = \frac{9}{8 \times 1.09 \times 10^7} = 1032 \text{ \AA}$$

EXERCISES

- In case of a hydrogen atom, given that $R = 1.097 \times 10^7 \text{ m}^{-1}$, the wavelength of the radiation emitted when transition takes place from $n = 3$ to $n = 2$ is
(a) 6563 \AA (b) 5636 \AA
(c) 3656 \AA (d) 6356 \AA
- A photon is emitted by a hydrogen atom when the atom makes a transition from the ($n = 2$) to the ($n = 1$) state. The energy, momentum and the wavelength of the photon are respectively
(a) 10.2 eV, $2.7 \times 10^{-27} \text{ kg} \cdot \text{m/s}$ and 2420 \AA
(b) 13.6 eV, $2.7 \times 10^{-27} \text{ kg} \cdot \text{m/s}$ and 2420 \AA
(c) 10.2 eV, $5.4 \times 10^{-27} \text{ kg} \cdot \text{m/s}$ and 1210 \AA
(d) 13.6 eV, $5.4 \times 10^{-27} \text{ kg} \cdot \text{m/s}$ and 1210 \AA
- The first member of the Balmer series of a hydrogen atom has a wave number $1.526 \times 10^4 \text{ m}^{-1}$. The wavelength of its second member is
(a) 6553 \AA (b) 5448 \AA
(c) 4458 \AA (d) 4854 \AA
- The shortest wavelength of the Lyman series (i.e. the transitions in which the final state is $n = 1$) is
(a) $9.1 \times 10^{-7} \text{ m}$ (b) $9.1 \times 10^{-8} \text{ m}$
(c) $9.1 \times 10^{-10} \text{ m}$ (d) $9.1 \times 10^{-12} \text{ m}$
- The longest wavelength of the Lyman series is
(a) $1.2 \times 10^{-7} \text{ m}$ (b) $1.2 \times 10^{-8} \text{ m}$
(c) $1.2 \times 10^{-10} \text{ m}$ (d) $1.2 \times 10^{-9} \text{ m}$
- The shortest and longest wavelengths of the Balmer series (i.e. the transitions in which the final state is $n = 2$) are respectively
(a) $4.2 \times 10^{-7} \text{ m}$ and $5.2 \times 10^{-7} \text{ m}$
(b) $3.6 \times 10^{-8} \text{ m}$ and $6.6 \times 10^{-8} \text{ m}$
(c) $3.6 \times 10^{-7} \text{ m}$ and $6.6 \times 10^{-7} \text{ m}$
(d) $5.2 \times 10^{-7} \text{ m}$ and $6.6 \times 10^{-7} \text{ m}$
- The value of Planck's constant is $6.6 \times 10^{-34} \text{ J} \cdot \text{s}$, the mass of the electron is $9.1 \times 10^{-31} \text{ kg}$, its charge is $1.6 \times 10^{-19} \text{ C}$, and $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$. The ionization energy of the hydrogen atom is
(a) $2.165 \times 10^{-18} \text{ J}$ (b) $1.65 \times 10^{-18} \text{ J}$
(c) 13.6 J (d) $13.6 \times 10^{-18} \text{ J}$
- An α -particle of energy $5 \times 10^{-13} \text{ J}$, projected against a uranium target ($Z = 92$), is scattered through 90° . The distance of nearest approach between the scattering nucleus and the incident α -particle is
(a) $1.023 \times 10^{-12} \text{ m}$ (b) $1.023 \times 10^{-10} \text{ m}$
(c) $1.023 \times 10^{-13} \text{ m}$ (d) $1.023 \times 10^{-14} \text{ m}$
- Knowing the values of h , m_e , e , etc. from the table, the total energy of an electron in the second Bohr's orbit is

- (a) 4.3 eV (b) 9.4 eV
(c) 4.9 eV (d) 3.4 eV
10. The velocity of the electron in the first Bohr orbit as compared to that of light is about
(a) 1/300 (b) 1/500
(c) 1/287 (d) 1/187
11. The second ionization potential of helium is
(a) 23.54 V (b) 54.32 V
(c) 45.32 V (d) 35.52 V
12. In a hydrogen atom, the energy that must be supplied to put an electron from the ground state to the $n = 6$ state is
(a) 13.2 eV (b) 9.4 eV
(c) 6.2 eV (d) 10.4 eV
13. In a hydrogen atom, an electron is in the $n = 3$ state. In order to ionize the atom, the energy needed is
(a) 8.51 eV (b) 6.2 eV
(c) 1.51 eV (d) 3.02 eV
14. A hydrogen atom, initially in the $n = 4$ state, finally attains the ground state. In this process the possible wavelengths emitted are
(a) $\lambda_1 = 18700 \text{ \AA}$, $\lambda_2 = 12100 \text{ \AA}$, $\lambda_3 = 1940 \text{ \AA}$
(b) $\lambda_1 = 18700 \text{ \AA}$, $\lambda_2 = 6560 \text{ \AA}$, $\lambda_3 = 1025 \text{ \AA}$
 $\lambda_4 = 1210 \text{ \AA}$, $\lambda_5 = 4860 \text{ \AA}$, $\lambda_6 = 970 \text{ \AA}$
(c) $\lambda_1 = 970 \text{ \AA}$, $\lambda_2 = 10300 \text{ \AA}$, $\lambda_3 = 1940 \text{ \AA}$, $\lambda_4 = 24200 \text{ \AA}$
(d) $\lambda_1 = 12100 \text{ \AA}$, $\lambda_2 = 48600 \text{ \AA}$
15. The hydrogen gas is bombarded with electrons of energy 12.10 eV. The frequencies of the spectral lines emitted will be
(a) $2.93 \times 10^{15} \text{ s}^{-1}$, $2.47 \times 10^{15} \text{ s}^{-1}$, $4.6 \times 10^{15} \text{ s}^{-1}$
(b) $2.01 \times 10^{15} \text{ s}^{-1}$, $2.93 \times 10^{15} \text{ s}^{-1}$, $5.86 \times 10^{15} \text{ s}^{-1}$
(c) $1.93 \times 10^{15} \text{ s}^{-1}$, $5.86 \times 10^{15} \text{ s}^{-1}$
(d) $2.47 \times 10^{15} \text{ s}^{-1}$, $2.93 \times 10^{15} \text{ s}^{-1}$
16. The longest wavelength that a singly ionised helium atom in its ground state will absorb is ($R = 1.097 \times 10^7 \text{ m}^{-1}$)
(a) 151.5 $\text{\AA}$ (b) 227.2 $\text{\AA}$
(c) 259.4 $\text{\AA}$ (d) 303.9 $\text{\AA}$
17. In a hydrogen atom, the electron moves in an orbit of radius 0.5 $\text{\AA}$, making 10^{16} revolutions per second. The magnetic moment associated with the orbital motion of the electron is
(a) $2.5 \times 10^{-27} \text{ A} \cdot \text{m}^2$ (b) $2.5 \times 10^{-23} \text{ A} \cdot \text{m}^2$
(c) $1.25 \times 10^{-23} \text{ A} \cdot \text{m}^2$ (d) $1.25 \times 10^{-27} \text{ A} \cdot \text{m}^2$
18. The velocity of an electron in the second Bohr orbit of the hydrogen atom is
(a) $0.24 \times 10^7 \text{ m/s}$ (b) $4.2 \times 10^6 \text{ m/s}$
(c) $2.186 \times 10^6 \text{ m/s}$ (d) $1.093 \times 10^6 \text{ m/s}$
19. The third ionization potential of lithium ($Z = 3$) is
(a) 12.4 V (b) 122.4 V
(c) 102.4 V (d) 10.2 V
20. A hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 975 $\text{\AA}$. The resulting spectrum will have
(a) 5 lines (b) 8 lines
(c) 6 lines (d) 4 lines
21. The ionization energy of a hydrogen-like Bohr atom is 4 rydbergs. The wavelength of radiation emitted when the electron jumps from the first excited state to the ground state is (1 rydberg = $2.2 \times 10^{-18} \text{ J}$)

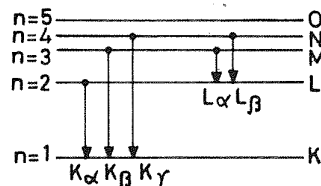
- (a) 600 $\text{\AA}$ (b) 300 $\text{\AA}$
(c) 400 $\text{\AA}$ (d) 200 $\text{\AA}$
22. The ionization energy of a hydrogen-like atom is 9 rydbergs. The electron jumps from its second excited state to the ground state. The frequency of the radiation emitted is (1 rydberg = $2.2 \times 10^{-18} \text{ J}$)
(a) $26 \times 10^{16} \text{ s}^{-1}$ (b) $2.6 \times 10^{18} \text{ s}^{-1}$
(c) $26 \times 10^{15} \text{ s}^{-1}$ (d) $6.6 \times 10^{13} \text{ s}^{-1}$
23. For electromagnetic radiations of frequency $3 \times 10^{15} \text{ Hz}$ the momentum of each photon is
(a) $3.3 \times 10^{-27} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$ (b) $1.1 \times 10^{-27} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$
(c) $9.9 \times 10^{-27} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$ (d) $6.6 \times 10^{-27} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$
24. The ratio of the radius of a hydrogen-like atom in the ground state and that of one in the second excited state is
(a) 1 : 9 (b) 1 : 4
(c) 1 : 3 (d) 1 : 2
25. If E_n and J_n denote the total energy and the angular momentum of an electron in the n th allowed orbit of a Bohr atom, then
(a) $E_n \propto J_n$ (b) $E_n \propto 1/J_n$
(c) $E_n \propto J_n^2$ (d) $E_n \propto 1/J_n^2$
26. The energy level diagram of a hypothetical atom is shown in the figure. A transition from $n = 4$ to $n = 3$ produces ultraviolet radiation. The possible transition to obtain infrared radiation is
(a) $5 \rightarrow 4$
(b) $4 \rightarrow 2$
(c) $2 \rightarrow 1$
(d) none of these
27. A doubly ionized lithium atom is hydrogen-like with atomic number 3. The electron is to be excited in Li^{++} from the first to the third Bohr orbit. The wavelength of radiation required to achieve this is (ionization energy of hydrogen atom is 13.6 eV)
(a) 1137 $\text{\AA}$ (b) 227.14 $\text{\AA}$
(c) 200 $\text{\AA}$ (d) 113.7 $\text{\AA}$
28. An electron in hydrogen and in a doubly ionized lithium atom are excited to the state $n = 2$. A photon is emitted when these electrons jump back to the ground state in each case. Then
(a) the energy of photon is the same in both the cases
(b) radiation emitted by lithium is more energetic than from the hydrogen atom
(c) radiation emitted by lithium is shifted towards the red as compared to the radiation from the hydrogen atom
(d) there is no definite relation between the radiation emitted by the two atoms.
29. An electron revolves round a nucleus of charge Ze . 32.39 eV of energy is required to excite an electron from the $n = 3$ state to the $n = 4$ state. The value of Z is
(a) 5 (b) 6
(c) 7 (d) 4
30. The H_β line of the Balmer series of the hydrogen atom has a wavelength of 4860 $\text{\AA}$. The wavelength of the H_α line is



- (a) 6561 Å (b) 5893 Å
(c) 6000 Å (d) 5493 Å
31. Mark the correct statement among the following:
- the rotational frequency of an electron in an orbit of radius r is inversely proportional to the cube of the radius of the orbit.
 - the linear momentum of the revolving electron is inversely proportional to the square root of the radius of the orbit.
 - the angular momentum of the electron is inversely proportional to the square root of the radius of the orbit.
 - the total energy of the system is directly proportional to the radius of the orbit.
32. Light of wavelength 4863 Å is emitted by a hydrogen atom. The transition of the hydrogen atom responsible for the radiation is
- $n = 3$ to $n = 2$
 - $n = 4$ to $n = 1$
 - $n = 4$ to $n = 2$
 - $n = 3$ to $n = 1$
33. A hydrogen atom undergoes a transition from the $n = 3$ state to $n = 1$. The energy and momentum of the photon emitted are
- $E = 6 \text{ eV}$; $p = 4.5 \times 10^{-27} \text{ kg} \cdot \text{m/s}$
 - $E = 12 \text{ eV}$; $p = 6.5 \times 10^{-27} \text{ kg} \cdot \text{m/s}$
 - $E = 12 \text{ eV}$; $p = 4.5 \times 10^{-27} \text{ kg} \cdot \text{m/s}$
 - $E = 6 \text{ eV}$; $p = 6.5 \times 10^{-27} \text{ kg} \cdot \text{m/s}$
34. It is desired to remove an electron from a hydrogen atom in the $n = 8$ state. The energy required to do so is
- 1.21 eV
 - 2.02 eV
 - 0.12 eV
 - 0.21 eV
35. A hydrogen atom is in a state having a binding energy of 0.85 eV. It makes a transition to a state with an excitation energy (the difference in energy between the state and the ground state) of 10.2 eV. The energy of the emitted photon is
- 2.6 eV
 - 1.3 eV
 - 2.92 eV
 - 3.2 eV

B. X-Rays

An inner electron of a many-electron system is rather well bound to the nucleus. To remove an electron from the ($n = 1$)-state, an energy of the order of a few thousand electron volts (keV) is needed. The removal of an electron from the state $n = 1$ leaves a hole in the state (the orbit $n = 1$ is also referred to as the K-shell). Therefore, an electron from the orbit $n = 2$ can now undergo a transition to the K-shell. During such a transition, the photon emitted has an energy of the order of 1 keV and a wavelength $\sim 10 \text{ Å}$. This photon is in the x-ray region, indicating that the atom emits x-rays. An electron from states higher than the ($n = 2$)-state can also undergo a transition to the K-shell, resulting in x-rays of varying energy. These discrete x-ray energies are called the K-lines as the transition is to the K-shell. K_α , K_β , etc. correspond to transitions from $n = 2$ to the K-shell, $n = 3$ to the K-shell and so on. Emission of K-lines and L-lines of x-rays are shown in the energy level diagram.



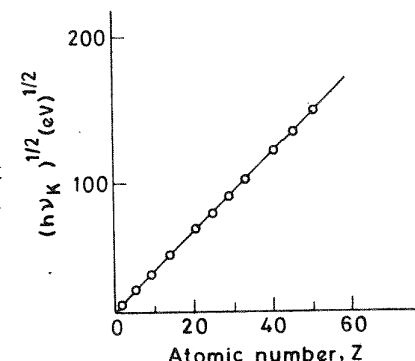
Moseley's Law

The square root of the frequency of any particular spectral line is proportional to the atomic number of the emitting element

$$\text{or } \nu = a(Z - b)^2$$

where a and b are constants. For the K_α -line, the value of a was found to be $\frac{3}{4} RC$ where R is the Rydberg constant and C is the velocity of light whereas b was found to be nearly = 1. Thus the above equation becomes

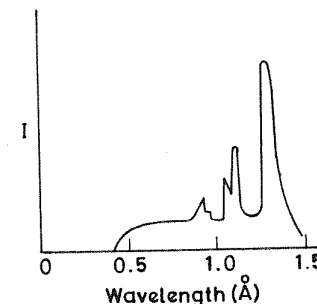
$$\begin{aligned} \nu &= \frac{3}{4} RC (Z - 1)^2 \\ &= RC (Z - 1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \end{aligned}$$



This equation is similar to the formula of hydrogen spectral lines derived on the Bohr atom model.

Continuous X-ray Spectrum

X-rays are produced by bombarding some target of a large atomic number with electrons having kinetic energy of the order of 10 keV. Such electrons knock out the inner electrons of the target atom, giving rise to photons. After they hit the target, these high-energy electrons get decelerated. A decelerated charge, as we know from classical electrodynamics, emits radiation having a *continuous spectrum*. A typical x-ray spectrum from a tungsten target is shown in the figure. As can be seen, the *line spectra* here are superimposed on a continuous spectrum. A continuous spectrum consists of radiations of all possible wavelengths within a certain range.



Characteristic or Sharp Line Spectrum

This consists of definite wavelengths superimposed on the continuous spectrum. The spectral lines are characteristic of the material used as the target. The group of spectral lines in the short wavelength region is termed the K_γ series, the next group in the direction of increasing wavelength the L-series, the one after this the M-series, and so on. There is a definite short wavelength limit to the continuous x-ray spectrum.

Minimum Wavelength and Accelerating Potential

The short wavelength limit of the continuous spectrum varies inversely as the accelerating potential across the tube

$$\text{or } \lambda_{\min} \propto \frac{1}{V}$$

Properties of X-rays

In addition to the properties pertaining to high-energy electromagnetic radiations, the two special features in the properties are: (i) x-ray diffraction, and (ii) scattering.

The Diffraction of X-rays X-rays of wavelength λ reflected from a crystal are described by the Bragg equation. Strong reflections are observed at grazing angle θ_n (θ is the angle between the face of the crystal and the reflected beam) given by

$$n\lambda = 2d \sin \theta_n$$

where d is the spacing between the reflecting planes in the crystal and $n = 1, 2, 3$ is the order of reflection.

Scattering of X-rays When the scattered x-rays have a wavelength identical to that of the incident x-rays, it is said to be a case of classical scattering or *Thomson scattering*. But Compton discovered that in addition to a component of some wavelength, the rays scattered by a light element also consisted of a wavelength slightly greater than that of the incident x-rays. Such scattering is called *Compton scattering*. If λ and λ' are the wavelengths of incident and scattered x-rays, and ϕ is the angle of scattering, then

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c} (1 - \cos \phi)$$

where m_0 is the rest mass of the electron. ($\Delta\lambda$ is called the Compton shift.) The quantity $h/m_0 c$ is called Compton wavelength of the scattering material.

ILLUSTRATIONS

1. Calculate approximately the energy needed to remove an electron from the K-shell of a sodium atom.

Solution

$$E = -\frac{Z^2 m c^2 \alpha^2}{2} = -Z^2 \times 13.6 \text{ eV}$$

The electron in the K-shell of a sodium atom, being the innermost electron, sees the entire nuclear charge of 11 units. Hence we have

$$E = -121 \times 13.6 \text{ eV} \approx 1.6 \text{ keV}$$

2. Calculate the energies required to create a K-vacancy and an L-vacancy in a copper atom and obtain the wavelength of the K_α -line and L_α -line.

Solution

The energy needed to create a vacancy in the K-shell, i.e. ($n = 1$)-orbit, is

$$Z^2 \times 13.6 \text{ eV} = (29)^2 \times 13.6 \text{ eV} \approx 11.6 \text{ keV}$$

The energy required to create a vacancy in the L-shell, i.e. ($n = 2$)-orbit, assuming that $Z_{\text{eff}} = 27$, is

$$\left(\frac{13.6}{4}\right) \times (27)^2 \approx 2.5 \text{ keV}$$

To calculate the wavelength for the K_α -line, we have

$$\begin{aligned} \lambda &= \frac{4}{3} \frac{1}{R(Z-1)^2} = \frac{4}{3} \times \frac{1}{1.09 \times 10^7 \times (29-1)^2} \\ &= \frac{4}{3 \times 1.09 \times (28)^2} \times 10^{-7} \text{ m} \\ &= 0.0015 \times 10^{-7} \text{ m} \approx 1.5 \text{ \AA} \end{aligned}$$

To calculate the wavelength for the L_α -line, we have

$$\begin{aligned} \lambda &= \frac{4}{3} \frac{1}{R(Z-1)^2} = \frac{4}{3 \times (27-1)^2 \times 1.09 \times 10^7} \\ &= \frac{4}{3 \times 1.09 \times (26)^2} \times 10^{-7} \\ &= 1.8 \times 10^{-10} \text{ m} = 1.8 \text{ \AA} \end{aligned}$$

3. A photon of energy 1.02 MeV is scattered through 90° by a free electron. Calculate the energy of the photon and the electron after the interaction.

Solution

$$\Delta\lambda = \frac{h}{m_0 c} (1 - \cos \phi) \text{ and as } \phi = 90$$

$$\Delta\lambda = \frac{h}{m_0 c} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8} = 2.42 \times 10^{-12} \text{ m}$$

$$\Delta\nu = \text{Change in frequency of photon} = \frac{c}{\Delta\lambda} = \frac{3 \times 10^8}{2.42 \times 10^{-12}} \text{ cycles/s}$$

$$\therefore \Delta E = h\Delta\nu = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.42 \times 10^{-12}} = \frac{6.6 \times 3}{2.42} \times 10^{-14} \text{ J}$$

$$= \frac{8.18 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^6} = 0.51 \text{ MeV}$$

This is the energy transferred to the free electron.

∴ Kinetic energy of electron after the interaction = 0.51 MeV

∴ Remaining energy of photon after the interaction

$$= 1.02 - 0.51 = 0.51 \text{ MeV}$$

4. Compute the potential difference through which an electron must be accelerated in order that the short wave limit of the continuous X-ray spectrum is exactly 0.1 nm.

Solution

$$\lambda_{\min} = 0.1 \text{ nm} = 10^{-10} \text{ m} \Rightarrow \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{10^{-10}} = 3 \times 10^{18} \text{ Hz}$$

$$\text{Energy of the photon} = h\nu = 6.6 \times 10^{-34} \times 3 \times 10^{18} = 19.9 \times 10^{-16} \text{ J}$$

This must be equal to the kinetic energy of the electron, $(1/2)mv^2$, which is also equal to the product of the electronic charge and the accelerating voltage V .

$$\frac{1}{2}mv^2 = eV = 19.9 \times 10^{-16} \text{ J}$$

$$\therefore V = \frac{19.9 \times 10^{-16}}{1.60 \times 10^{-19}} \text{ J} = 12400 \text{ V} = 12.4 \text{ kV}$$

5. An x-ray diffraction analysis of a crystal is made with x-rays of wavelength 0.58 Å. Reflections are observed at angles of: (a) 6.45°, (b) 9.15°, and (c) 13°. What are the interplanar spacings in the crystal?

Solution

$$2d \sin \theta = n\lambda$$

where d is interplanar spacing, θ is the angle of reflection, and n is the order of reflection.

$$(a) \frac{d}{n} = \frac{\lambda}{2 \sin \theta} = \frac{0.58}{2 \times \sin 6.45^\circ} = \frac{0.58}{2 \times 0.1129} = 2.568 \text{ Å}$$

$$(b) \frac{d}{n} = \frac{0.58}{2 \times \sin 9.15^\circ} = \frac{0.29}{0.1596} = 1.817 \text{ Å}$$

$$(c) \frac{d}{n} = \frac{0.58}{2 \times \sin 13^\circ} = \frac{0.29}{0.2250} = 1.288 \text{ Å}$$

It is observed here that the value of d/n in (a) is almost twice that of (c). It means that corresponding angles 6.45° and 13° represent the first order ($n = 1$) and second order ($n = 2$) reflection maxima from a set of parallel planes. Hence d can be determined either from (a) or from (c).

$$\frac{d}{1} = 2.568 \text{ Å} \Rightarrow d = 2.568 \text{ Å}$$

$$\text{or } \frac{d}{2} = 1.288 \text{ Å} \Rightarrow d = 2.576 \text{ Å}$$

Hence, interplanar spacing is $\sim 2.568 \text{ Å}$

6. If the potential difference applied across the x-ray tube is 10 kV and the current through it is 5 mA, calculate the number of electrons striking the target per second. What is their speed when they strike the target?

Solution

If n is the number of electrons striking the target per second, the current $I = ne$

$$\text{or } n = \frac{I}{e} = \frac{5 \times 10^{-3} \text{ A}}{1.6 \times 10^{-19} \text{ C}} = 3.125 \times 10^{16} / \text{s}$$

$$eV = \frac{1}{2}mv^2 \Rightarrow V = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 10000}{9.1 \times 10^{-31}}} \\ = 5.9 \times 10^7 \text{ m/s}$$

EXERCISES

- The wavelength of the K_α -line for the uranium atom is ($Z = 92$)
(a) 1.5 Å (b) 0.15 Å
(c) 0.5 Å (d) 2.0 Å
- The minimum energy of the electrons needed to produce a K_α -line from a molybdenum target is ($Z = 42$)
(a) 2399 eV (b) 2.399 V
(c) 2399 keV (d) $2.399 \times 10^5 \text{ eV}$
- Compton wavelength for an electron is
(a) 0.0242 Å (b) 0.242 Å
(c) 2.42 Å (d) 0.0042 Å
- An x-ray tube operates at 13.6 kV. The maximum speed of electrons striking the target is
(a) $0.9682 \times 10^8 \text{ m/s}$ (b) $0.8682 \times 10^8 \text{ m/s}$
(c) $0.6928 \times 10^8 \text{ m/s}$ (d) $0.2896 \times 10^8 \text{ m/s}$
- An x-ray tube operates at 50 kV. The shortest wavelength produced is
(a) 2.482 Å (b) 0.2482 Å
(c) 0.00248 Å (d) 0.2482 Å
- The cut-off wavelength when a potential difference of 25 kV is applied to an x-ray tube, is
(a) 0.248 Å (b) 0.496 Å
(c) 0.124 Å (d) 0.496 Å
- An x-ray tube operates at 30 kV and emits a continuous x-ray spectrum with a short wavelength limit $\lambda = 0.414 \text{ Å}$. If $e = 1.6 \times 10^{-19} \text{ C}$ and $c = 3 \times 10^8 \text{ m/s}$, the value of Planck's constant is
(a) $6.26 \times 10^{-34} \text{ J.s}$ (b) $66.2 \times 10^{-34} \text{ J.s}$
(c) $6.62 \times 10^{-34} \text{ J.s}$ (d) $2.66 \times 10^{-34} \text{ J.s}$
- In order that the short wavelength limit of the continuous x-ray spectrum be 1 Å, the potential difference through which an electron must be accelerated is
(a) 124 kV (b) 1.24 kV
(c) 12.4 kV (d) 1240 kV

9. Electrons bombard the target of an x-ray tube, producing x-rays of wavelength 5 Å. The energy of each electron at the moment of impact is
 - (a) 2484.4 eV
 - (b) 1242 eV
 - (c) 621 eV
 - (d) 1863 eV
10. An electron in a certain x-ray tube is accelerated from rest through a potential difference of 180 kV in going from the cathode to the anode. When it arrives at the anode, its kinetic energy in electron volts is
 - (a) 28.8×10^{-14}
 - (b) 288×10^{-14}
 - (c) 0.288×10^{-14}
 - (d) 2.88×10^{-14}
11. An x-ray tube is operating at 150 kV and 10 mA. If only 1% of the electric power supplied is converted into x-rays, the rate at which the target is heated in calories per second is
 - (a) 3.55
 - (b) 35.5
 - (c) 355
 - (d) 3550
12. X-rays with initial wavelength 0.5×10^{-10} m undergo Compton scattering. The wavelength of the scattered x-rays is greater than that of the incident x-rays by one per cent. The corresponding angle of scattering is
 - (a) 19.2°
 - (b) 46.4°
 - (c) 57.2°
 - (d) 37.6°
13. The minimum wavelength produced from a copper target in an x-ray tube that operates at 10000 V is
 - (a) 1.24×10^{-10} m
 - (b) 1.24×10^{-8} m
 - (c) 12.4×10^{-10} m
 - (d) 12.4×10^{-8} m
14. X-rays of wavelength 1 Å are scattered by a carbon target. The energy of the incident x-rays when they are scattered through 90° is
 - (a) 24.8 keV
 - (b) 49.16 keV
 - (c) 6.2 keV
 - (d) 12.4 keV
15. The momentum of an x-ray photon with $\lambda = 0.5$ Å is
 - (a) 13.26×10^{-26} kg. m/s
 - (b) 1.326×10^{-26} kg. m/s
 - (c) 13.26×10^{-24} kg. m/s
 - (d) 13.26×10^{-22} kg. m/s

C. Photoelectric Effect

When light is incident on the surface of a metal, under certain conditions electrons are ejected. When a photon of energy $E (= h\nu)$ collides with an electron at or near the surface of the material, then the photon transfers all its energy to the electron. In order that the electron may come out of the surface of the metal, a certain amount of energy ϕ is required (ϕ is known as the work function of the metal. The maximum kinetic energy $[(1/2) m V_{\max}^2]$ of the ejected electron is given by Einstein's photoelectric equation:

$$\frac{1}{2} m V_{\max}^2 = h\nu - \phi$$

Work Function (ϕ)

Work function is the minimum energy required to remove an electron from the surface of the substance.

Stopping Potential

Suppose the collector is given a negative potential, the photoelectric current

decreases and finally becomes zero at a particular value of negative potential. This is called the *stopping potential* or *cut-off potential*. If V_0 is the stopping potential,

$$eV_0 = h(\nu - \nu_0) = \frac{1}{2} m V_{\max}^2$$

Here, ν_0 is called the *threshold frequency*. (Electrons are emitted if the frequency of light is higher than a critical value, called threshold frequency.)

Important Features of Photoelectric Effect

1. The photoelectric current starts flowing as soon as light supply is begun. In other words, there is virtually no time lag between the shining of light and the emission of electrons.
2. If the light falling on the emitter has a fixed frequency, then the stopping potential V_0 does not depend on the intensity of light.
3. The stopping potential V_0 , on the other hand, does depend on the frequency (and hence on the wavelength) of the light used. When the frequency of light used is kept fixed, the current is directly proportional to the intensity of light.

ILLUSTRATIONS

1. A 100 W bulb emits light uniformly in all directions. A metallic sodium surface is kept at a distance of 1 m from the bulb. Assume that all the energy is absorbed by the top layer of the surface and that all the energy given to the Na atom is taken by one electron. Find the time needed by an electron in a sodium atom to receive an energy of 1 eV.

Solution

The rate of energy crossing a spherical surface of radius r , with the bulb at the centre, is then 100 J/s. For $r = 1$ m, the light intensity

$$I = \frac{100}{4\pi} \approx 8 \text{ J/m}^2 \cdot \text{s}$$

The number of sodium atoms N per unit area can be estimated approximately because the radius of the atom is about 10^{-10} m. Thus

$$N = \frac{1}{\pi \times (\text{radius of the Na atom})^2} \approx \frac{1}{\pi (10^{-10})^2} \approx 3 \times 10^{19} \text{ atoms/m}^2$$

Therefore, the power absorbed by one atom is then approximately

$$\frac{8 \text{ J/m}^2 \cdot \text{s}}{3 \times 10^{19} \text{ atoms/m}^2} \approx 2.6 \times 10^{-19} \text{ J/s} \approx 1.6 \text{ eV/s}$$

The time needed by the electron to gain an energy of 1 eV will be

$$\frac{1 \text{ eV}}{1.6 \text{ eV/s}} \approx 0.6 \text{ s}$$

Note: The assumption made here that the energy absorbed by an atom gets transferred to a single electron is not realistic. As such, the energy acquired by an electron will be much less.

2. The photoelectric workfunction for a material is 2.4 eV. Light of wavelength 6800 Å shines on the surface of the material. Find the incident and the threshold frequencies. Discuss the conditions of photoemission in this case.

Solution

$$\phi = 2.4 \text{ eV} = 2.4 \times 1.6 \times 10^{-19} \text{ J} = h\nu_0$$

$$\therefore \text{Threshold frequency} = \frac{2.4 \times 1.6 \times 10^{-19} \text{ J}}{6.6 \times 10^{-34} \text{ J.s}} = 5.8 \times 10^{14} \text{ Hz}$$

$$\text{Wavelength of incident light} = 6800 \text{ Å} = 6800 \times 10^{-10} \text{ m}$$

$$\therefore \text{Frequency } \nu = \frac{3 \times 10^8}{6800 \times 10^{-10}} = 4.411 \times 10^{14} \text{ Hz}$$

$$\frac{1}{2} mV_{\max}^2 = h(\nu - \nu_0)$$

We see that ν_0 is greater than ν and hence photoemission will not take place.

3. What are: (a) the frequency, (b) the wavelength, and (c) the momentum of a photon whose energy equals the rest energy of an electron?

Solution

$$\text{Rest energy of an electron} = m_0 c^2$$

$$= 9.1 \times 10^{-31} \text{ kg} \times (3 \times 10^8)^2 \left(\frac{\text{m}}{\text{s}} \right)^2$$

$$= 8.19 \times 10^{-14} \text{ J}$$

$$(a) \quad E = h\nu = h \frac{c}{\lambda}, \quad \nu = \frac{E}{h} = \frac{8.19 \times 10^{-14}}{6.6 \times 10^{-34}} = 1.24 \times 10^{20} \text{ Hz}$$

$$(b) \quad \lambda = \frac{h \cdot c}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{8.19 \times 10^{-14}} = \frac{6.6 \times 3}{8.19} \times 10^{-12} = 2.4 \times 10^{-12} \text{ m}$$

$$(c) \quad p = \frac{E}{c} = \frac{8.19 \times 10^{-14} \text{ J}}{3 \times 10^8 \text{ m/s}} = 2.73 \times 10^{-22} \frac{\text{J.s}}{\text{m}} = \frac{\text{kg.m}}{\text{s}}$$

4. You wish to pick a substance for a photo cell operable with visible light. Which of the following will do, given the work function for each:

- | | |
|-----------------|-------------------------|
| (i) Tantalum | $\phi = 4.2 \text{ eV}$ |
| (ii) Tungsten | $\phi = 4.5 \text{ eV}$ |
| (iii) Aluminium | $\phi = 4.2 \text{ eV}$ |
| (iv) Barium | $\phi = 2.5 \text{ eV}$ |
| (v) Lithium | $\phi = 2.3 \text{ eV}$ |

Solution

The visible range lies between the wavelengths 4000 and 7000 Å. The equation

$$\frac{1}{2} mV_{\max}^2 = h\nu - h\nu_0$$

suggests that ν should be greater than ν_0 . The frequencies corresponding to 4000 and 7000 Å are 0.75×10^{15} and 0.43×10^{15} .

The corresponding energies are

$$h\nu_1 = 6.6 \times 10^{-34} \times 0.75 \times 10^{15}$$

$$= 4.95 \times 10^{-19} \text{ J}$$

$$= 3.09 \text{ eV}$$

$$\text{and } h\nu_2 = 6.6 \times 10^{-34} \times 0.43 \times 10^{15}$$

$$= 1.77 \text{ eV}$$

This means that if the work function of a substance is greater than 3.09 eV, it will not emit photoelectrons. Hence only barium and lithium are suitable for the given range of light.

5. The photoelectric threshold of metallic copper is at $\lambda = 3000 \text{ Å}$. Find: (a) the maximum kinetic energy of photoelectrons emitted, (b) their maximum velocity, (c) the work function of the metal when ultraviolet light of 2536 Å falls on the metallic surface.

Solution

$$(a) \quad E_{\max} = \frac{1}{2} mV_{\max}^2 = h\nu - \phi$$

$$(\phi = h\nu_0 \text{ where } \nu_0 \text{ is the threshold frequency})$$

$$\text{or } E_{\max} = \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \cdot hc$$

$$= 6.62 \times 10^{-34} \times 3 \times 10^8 \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

$$= 19.86 \times 10^{-26} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

(here, energy is in joules and wavelengths are in metres)

$$\text{or } E_{\max} = \frac{19.86 \times 10^{-26}}{1.6 \times 10^{-19}} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV} = 12.4 \times 10^{-7} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) \text{ eV}$$

$$= 12.4 \times 10^{-7} \left(\frac{1}{2536} - \frac{1}{3000} \right) \times 10^{10}$$

$$= 12400 \left(\frac{464}{2536 \times 3000} \right) \text{ eV} = 0.7562 \text{ eV}$$

(b) For maximum velocity,

$$\frac{1}{2} m V_{\max}^2 = 0.7562 \text{ eV}$$

$$V_{\max}^2 = \frac{0.7562 \times 1.6 \times 10^{-19} \times 2}{9.1 \times 10^{-31}} = 0.266 \times 10^{12} \text{ (m/s)}^2$$

$$\therefore V_{\max} = 0.5157 \times 10^6 = 5.16 \times 10^5 \text{ m/s}$$

(c) Work function:

$$\begin{aligned} \phi = h\nu_0 &= \frac{hc}{\lambda_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10}} \\ &= \frac{6.6 \times 3 \times 10^{-26}}{3 \times 10^{-7}} = 6.6 \times 10^{-19} \text{ J} \\ &= \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} \\ &= 4.125 \text{ eV} \end{aligned}$$

6. A blue lamp emits light of mean wavelength $4500 \text{ \AA}$. The lamp is rated at 150 W and 8% efficiency. How many photons are emitted by the lamp per second?

Solution

Power of lamp = 150 W

The power of the lamp utilised for emission of light energy per second is

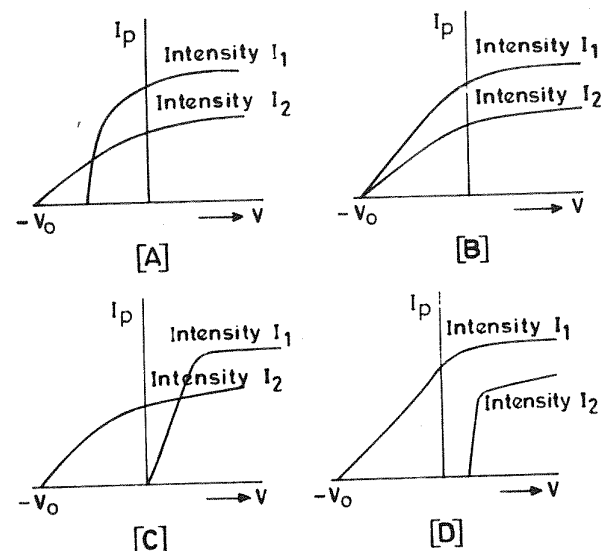
$$\frac{150 \times 8}{100} = 12.0 \text{ J/s}$$

Number of photons emitted per second by lamp

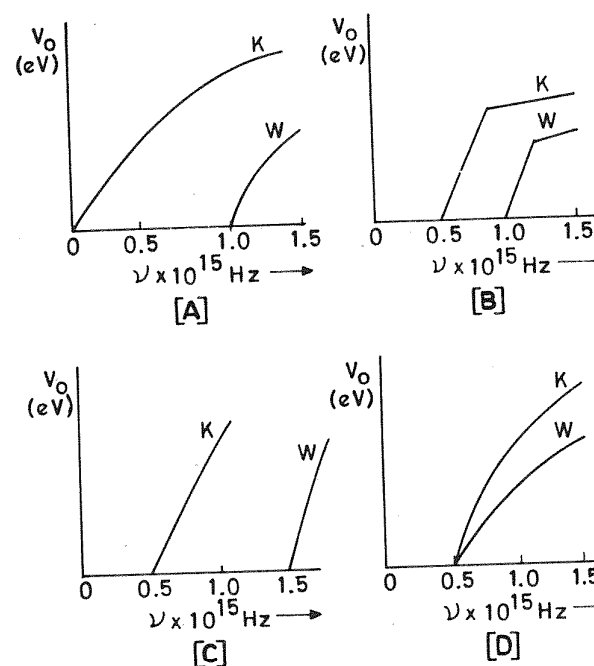
$$\begin{aligned} &= \frac{\text{useful power}}{\text{energy of photon}} = \frac{12 \times \lambda}{hc} = \frac{12 \times 4500 \times 10^{-10}}{6.6 \times 10^{-34} \times 3 \times 10^8} \\ &= 27.17 \times 10^{18} \end{aligned}$$

EXERCISES

- The curves (a), (b), (c) and (d) show the variation between the applied potential difference (V) and the photoelectric current, at two different intensities of light ($I_1 > I_2$). In which figure is the correct variation shown?

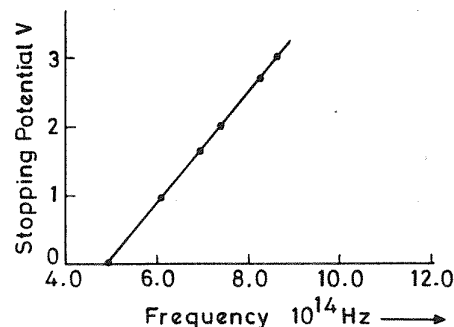


- Tick the figure showing the correct relationship between the stopping potential V_0 and the frequency ν of light for potassium and tungsten



- When the potential applied between the two plates is zero, and the stopping potential is 3 V , the maximum kinetic energy of the emitted electrons is

- (a) $9.6 \times 10^{-19} \text{ J}$ (b) $2.4 \times 10^{-19} \text{ J}$
 (c) $4.8 \times 10^{-12} \text{ J}$ (d) $4.8 \times 10^{-19} \text{ J}$
4. The work function of a substance is 1.6 eV. The longest wavelength of light that can produce photoemission from the substance is
 (a) 7734 Å (b) 3867 Å
 (c) 5800 Å (d) 2900 Å
5. The stopping potential vs frequency plot for a substance is shown in the figure. The work function for the substance is



- (a) 4.02 eV (b) 3.331 eV
 (c) 2.06 eV (d) 3.02 eV
6. X-rays of wavelength 2 Å fall on a metal plate. The kinetic energy of the photoelectrons emitted is (neglect work function)
 (a) $19.87 \times 10^{-16} \text{ J}$ (b) $9.9 \times 10^{-16} \text{ J}$
 (c) $4.95 \times 10^{-16} \text{ J}$ (d) $7.42 \times 10^{-16} \text{ J}$
7. For sodium metal, the photoelectric threshold wavelength is 6800 Å. The work function for sodium metal in electron volts is
 (a) 2.740 eV (b) 4.110 eV
 (c) 3.65 eV (d) 1.827 eV
8. The wavelength of incident light falling on a photosensitive surface is changed from 2000 to 2050 Å. The corresponding change in the stopping potential is
 (a) 0.012 V (b) 2.1 V
 (c) 1.2 V (d) 0.12 V
9. The photoelectric threshold for a certain metal is 4000 Å. The maximum energy of the ejected photoelectrons by a radiation of 2000 Å is
 (a) 2.13 eV (b) 3.12 eV
 (c) 1.23 eV (d) 3.82 eV
10. Light of wavelength 4000 Å falls on a photosensitive substance whose work function is 2.0 eV. Its stopping potential is
 (a) 1.09 V (b) 1.8 V
 (c) 1.26 eV (d) 0.8 V
11. The photoelectric work function of potassium is 2.0 eV. If light having a wavelength of 400 nm falls on it, the velocity of the electrons ejected is
 (a) $0.62 \times 10^5 \text{ m/s}$ (b) $0.31 \times 10^6 \text{ m/s}$
 (c) $0.62 \times 10^6 \text{ m/s}$ (d) $0.31 \times 10^5 \text{ m/s}$
12. The threshold wavelength for a photoelectric substance is 2300 Å. When ultraviolet radiation of wavelength 1800 Å is incident on the substance, electrons are ejected from the surface. The energy of electrons is

- (a) 1.48 eV (b) 1.08 eV
 (c) 2.05 eV (d) 1.82 eV
13. A photoelectric surface has a work function of 4.0 eV. Light of frequency $3 \times 10^{15} \text{ Hz}$ falls on the surface. The maximum velocity of the photoelectrons emitted is
 (a) $1.72 \times 10^6 \text{ m/s}$ (b) $1.72 \times 10^5 \text{ m/s}$
 (c) $1.72 \times 10^8 \text{ m/s}$ (d) $1.72 \times 10^{10} \text{ m/s}$
14. When ultraviolet light of wavelength 3000 Å falls upon a copper surface, the retarding potential necessary to stop the emission of photoelectrons is 0.60 V. The photoelectric threshold wavelength of copper is
 (a) 4012 Å (b) 2150 Å
 (c) 3512 Å (d) 3954 Å
15. For a certain photosensitive material, a stopping potential of 3.0 V is required for light of wavelength 300 nm, 2.0 V for 400 nm and 1.0 V for 600 nm. The work function of the material is
 (a) 2.5 eV (b) 1.5 eV
 (c) 2.0 eV (d) 1.0 eV
16. The effective area of a photosensitive material is 4 cm² and the anode is located at a distance of 1 m from a mercury arc lamp of 0.10 W, emitting an UV radiation of wavelength 2537 Å. The work function of the material is 2.22 eV. The photon energy, the flux of the photon at the cathode and the cut-off potential, respectively, are as follows:
 (a) 2.45 eV, 4.06×10^{10} photons/s and 3.54 V
 (b) 4.90 eV, 2.03×10^{12} photons/s and 2.64 V
 (c) 4.90 eV, 4.06×10^{12} photons/s and 2.64 V
 (d) 2.45 eV, 4.06×10^{12} photons/s and 3.54 V
17. Mark the correct statement among the following:
 (a) the energy of a photon with a wavelength 7000 Å is 2.4 eV
 (b) the wavelength of a photon with an energy 1 eV is 9000 Å
 (c) the work function of sodium metal is 2.3 eV, and the longest wavelength of light that can cause photoelectron emission from sodium is 540 nm.
 (d) a copper surface of work function 4.4 eV, when illuminated by visible light, will emit photoelectrons.
18. Ultraviolet light of wavelength 1000 Å is incident on molybdenum whose work function is 4.15 eV. The maximum velocity of the ejected photoelectrons is
 (a) $1.7 \times 10^5 \text{ m/s}$ (b) $1.7 \times 10^6 \text{ m/s}$
 (c) $1.7 \times 10^8 \text{ m/s}$ (d) $1.7 \times 10^7 \text{ m/s}$
19. The photoelectric threshold wavelength of tungsten is 2730 Å. The work function of tungsten in electron volts is
 (a) 4.553 (b) 0.455
 (c) 0.045 (d) 0.225
20. Photoelectrons are emitted with a maximum velocity of $7 \times 10^5 \text{ m/s}$ from a surface of a material when light of frequency $8 \times 10^{14} \text{ Hz}$ falls on it. The threshold frequency of the material is
 (a) $2.3 \times 10^{15} \text{ Hz}$ (b) $1.4 \times 10^{16} \text{ Hz}$
 (c) $2.3 \times 10^{14} \text{ Hz}$ (d) $4.6 \times 10^{14} \text{ Hz}$
21. The work function of a photosensitive substance is 2.3 eV. Its threshold wavelength is
 (a) 4390 Å (b) 5890 Å
 (c) 5394 Å (d) 4920 Å
22. A 1000 W transmitter works at a frequency of 880 kHz. The number of photons emitted per second is

- (a) 1.7×10^{28} (b) 1.7×10^{30}
 (c) 1.7×10^{23} (d) 1.7×10^{25}
23. A photon in a beam is to have the same momentum as an electron moving with a speed 5×10^5 m/s. The wavelength of the electromagnetic radiation should be
 (a) $14.56 \text{ \AA}$ (b) $1.456 \text{ \AA}$
 (c) $145.6 \text{ \AA}$ (d) $0.728 \text{ \AA}$
24. The potential difference that must be applied to stop the fastest photoelectrons emitted by a copper surface under the action of ultraviolet light of wavelength 200 nm is (ϕ for copper = 4.4 eV)
 (a) 0.9 V (b) 1.2 V
 (c) 1.8 V (d) 2.2 V

D. de Broglie Waves

A particle of mass m moving with a momentum p has associated with it a de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mV}$$

de Broglie conjectured that it is not just light but all matter that has a wave-like as well as a particle-like property. He asserted that the particle-wave nature is a general property of all objects.

ILLUSTRATIONS

1. Calculate the de Broglie wavelength associated with an electron having a kinetic energy of 10 eV.

Solution

$$E = \frac{p^2}{2m} \Rightarrow p = (2mE)^{1/2}$$

$$\begin{aligned} \text{de Broglie wavelength } \lambda &= \frac{h}{p} = \frac{h}{(2mE)^{1/2}} \\ &= \frac{6.6 \times 10^{-34}}{(2 \times 9.1 \times 10^{-31} \times 10 \times 1.6 \times 10^{-19})^{1/2}} \\ &\approx 3.8 \times 10^{-10} \text{ m} \approx 3.8 \text{ \AA} \end{aligned}$$

2. (a) What is the speed of a proton with a de Broglie wavelength of $0.1 \text{ \AA}$. (b) What is the de Broglie wavelength for a particle moving with a speed of 2×10^6 m/s if the particle is (i) an electron, (ii) a neutron and (iii) a 0.2 kg ball?

Solution

$$(a) \quad \lambda = \frac{h}{p} \text{ or } p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{0.1 \times 10^{-10}}$$

$$\begin{aligned} \therefore V &= \frac{h}{\lambda m} = \frac{6.6 \times 10^{-34}}{0.1 \times 10^{-10} \times 1.673 \times 10^{-27}} \\ &= 39.45 \times 10^3 = 3.94 \times 10^4 \text{ m/s} \end{aligned}$$

$$(b) \quad \lambda = \frac{h}{mV} = \frac{6.63 \times 10^{-34}}{m \times 2 \times 10^6} = \frac{3.3 \times 10^{-40}}{m}$$

$$(i) \quad \lambda_e = \frac{3.3 \times 10^{-40}}{9.1 \times 10^{-31}} = 0.36 \times 10^{-9} \text{ m}$$

$$(ii) \quad \lambda_n = \frac{3.3 \times 10^{-40}}{1.675 \times 10^{-27}} = 1.97 \times 10^{-13} \text{ m}$$

$$(iii) \quad \lambda_{\text{ball}} = \frac{h}{mV} = \frac{3.3 \times 10^{-40}}{m} = \frac{3.3 \times 10^{-40}}{0.2} = 16.5 \times 10^{-40} \text{ m}$$

EXERCISES

- An electron falls from rest through a potential difference of 500 V. Its de Broglie wavelength is
 (a) $0.1 \text{ \AA}$ (b) $0.5 \text{ \AA}$
 (c) $5.5 \text{ \AA}$ (d) $1.2 \text{ \AA}$
- A proton is accelerated with 60 kV. The de Broglie wavelength associated with it is
 (a) $1.166 \times 10^{-10} \text{ m}$ (b) $6.166 \times 10^{-13} \text{ m}$
 (c) $6.166 \times 10^{-10} \text{ m}$ (d) $1.666 \times 10^{-13} \text{ m}$
- An electron microscope is supposed to emit electrons of wavelength $0.5 \text{ \AA}$. The potential difference required for the purpose is
 (a) 6000 V (b) 2000 V
 (c) 600 V (d) 1000 V
- A particle of mass m is confined to a narrow tube of length L . The wavelengths of the de Broglie waves which will resonate in the tube are given by
 (a) $\lambda_n = \frac{L}{n}$ (b) $\lambda_n = \frac{2L}{n}$
 (c) $\lambda_n = nL$ (d) $\lambda_n = \frac{1}{2} \cdot \frac{L}{n}$
- A beam of electrons has kinetic energy = 100 eV. Its de Broglie wavelength is
 (a) $1.2 \text{ \AA}$ (b) $2.1 \text{ \AA}$
 (c) $0.12 \text{ \AA}$ (d) $0.21 \text{ \AA}$
- A bullet of mass 40 g travels at 1000 m/s. The wavelength that can be associated with it is
 (a) $1.7 \times 10^{-30} \text{ m}$ (b) $1.7 \times 10^{-32} \text{ m}$
 (c) $1.7 \times 10^{-15} \text{ m}$ (d) $1.7 \times 10^{-35} \text{ m}$
- The wavelength we associate with a beam of neutrons whose energy is 0.025 eV, is
 (a) $18.03 \times 10^{-11} \text{ m}$ (b) $18.03 \times 10^{-10} \text{ m}$
 (c) $18.03 \times 10^{-14} \text{ m}$ (d) $18.03 \times 10^{-8} \text{ m}$

8. If the de Broglie wavelength of a proton is 1.0×10^{-13} m, the electric potential through which it must have been accelerated is
 (a) 4.07×10^4 V (b) 8.15×10^4 V
 (c) 8.15×10^3 V (d) 4.07×10^5 V
9. Sodium ions are accelerated through a potential difference of 300 V. The momentum acquired by the ions is
 (a) 1.9×10^{-21} kg. m/s (b) 9.1×10^{-21} kg. m/s
 (c) 1.9×10^{-27} kg. m/s (d) 1.9×10^{-23} kg. m/s
10. An electron and a photon each have a wavelength of $2.0 \text{ \AA}$. Their momenta and energies respectively are as follows.
 (a) $p = 3.3 \times 10^{-24}$ kg. m/s for electron and photon both and $E = 38 \text{ eV}$ for electron and photon both.
 (b) $p = 3.3 \times 10^{-24}$ kg. m/s and 6.6×10^{-24} kg. m/s for electron and photon respectively and $E = 19 \text{ eV}$ for electron and 38 eV for photon.
 (c) $p = 3.3 \times 10^{-24}$ kg. m/s for electron and photon both and $E = 38 \text{ eV}$ for the electron and 6200 eV for the photon.
 (d) $p = 3.3 \times 10^{-24}$ kg. m/s and 16.5×10^{-42} kg. m/s for electron and photon and $E = 38 \text{ eV}$ and 4200 eV for electron and photon, respectively.
11. The accelerating voltage required for electrons in an electron microscope in order to obtain the same ultimate resolving power as can be obtained from a gamma ray microscope using 0.20 MeV gamma rays, is
 (a) 2×10^4 V (b) 2×10^5 V
 (c) 1000 V (d) 1×10^5 V
12. The wavelength of a hydrogen atom moving with a velocity corresponding to the mean kinetic energy for thermal equilibrium at 20°C , is
 (a) $2.5 \text{ \AA}$ (b) $0.5 \text{ \AA}$
 (c) $2.0 \text{ \AA}$ (d) $1.5 \text{ \AA}$

Answers

A.

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (a) | 2. (c) | 3. (d) | 4. (b) | 5. (a) |
| 6. (c) | 7. (a) | 8. (c) | 9. (d) | 10. (d) |
| 11. (b) | 12. (a) | 13. (c) | 14. (b) | 15. (a) |
| 16. (d) | 17. (c) | 18. (d) | 19. (a) | 20. (c) |
| 21. (b) | 22. (c) | 23. (d) | 24. (a) | 25. (a) |
| 26. (d) | 27. (d) | 28. (b) | 29. (c) | 30. (a) |
| 31. (b) | 32. (c) | 33. (b) | 34. (d) | 35. (a) |

B.

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (a) | 4. (c) | 5. (d) |
| 6. (b) | 7. (c) | 8. (c) | 9. (a) | 10. (d) |
| 11. (c) | 12. (d) | 13. (a) | 14. (d) | 15. (a) |

C.

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (d) | 4. (a) | 5. (c) |
| 6. (b) | 7. (d) | 8. (d) | 9. (b) | 10. (a) |
| 11. (c) | 12. (a) | 13. (a) | 14. (c) | 15. (d) |

- | | | | | |
|---------|---------|---------|---------|---------|
| 16. (c) | 17. (c) | 18. (b) | 19. (a) | 20. (d) |
| 21. (c) | 22. (b) | 23. (a) | 24. (c) | |

D.

- | | | | | |
|---------|---------|--------|--------|---------|
| 1. (b) | 2. (d) | 3. (c) | 4. (b) | 5. (a) |
| 6. (d) | 7. (a) | 8. (b) | 9. (a) | 10. (c) |
| 11. (b) | 12. (d) | | | |

Note: All problems in this chapter should take not more than three minutes each.

25

Nuclei and Radioactivity

Radioactivity: Emission from a Radioactive Substance: α , β and γ Radiation

α -particles are positively charged particles. They are nuclei of helium atoms with mass number 4 and atomic number 2. Such particles are also termed doubly ionised helium atoms. Being charged particles, they are deflected by electric and magnetic fields. Their velocities vary from 1/100 to 1/10 the velocity of light. Because of their highly energetic character, they cause ionization. They are scattered by metal foils.

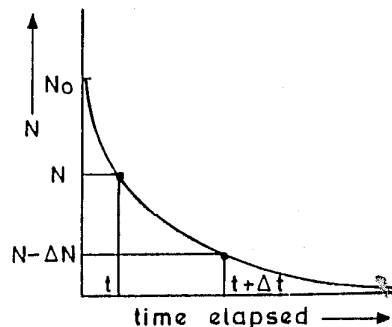
β -rays are fast moving electrons. Their penetrating power is about 100 times greater than that of α -particles. They are emitted with velocities 10 to 15 times greater than that of α -particles. Their ionizing power is 1/100 that of α -particles.

γ -rays are electromagnetic radiations and are not affected by electric or magnetic fields. They have a character similar to x-rays but their wavelength is smaller, of the order of 10^{-11} to 10^{-13} m. These radiations are highly penetrating (about 100 times more penetrating than β -particles.) Their ionizing power is small, being about 1/100 that of the β -particles.

Radioactive Decay

If, at an instant of time, $t = 0$, N_0 is the number of radioactive atoms of an element present, N_t is the number of atoms left after time t and the rate of disintegration at any time is given by $-dN/dt = \lambda N$ where λ is a constant called disintegration constant (the negative sign indicates that N is decreasing with time), then we have

$$N_t = N_0 e^{-\lambda t}$$



This equation shows that the number of atoms of a given radioactive element decreases exponentially with time.

As $t \rightarrow \infty$, $N_t \rightarrow 0$. This means that it takes an infinitely long time for a radioactive element to disintegrate completely.

Half-Life Period, $T_{1/2}$

It is the time in which half of the number of atoms of a radioactive element disintegrate.

$$\frac{N_0}{2} = N_0 \cdot e^{-\lambda T_{1/2}} \Rightarrow T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Average Life or Mean Lifetime (τ)

This is the average time for which the atoms of a radioactive element exist without disintegrating

$$\tau = \frac{\int_0^{N_0} \left(\frac{dN}{dt} \right) t dt}{N_0} = \frac{1}{\lambda}$$

This means that the average life expectancy of an individual radioactive atom is the reciprocal of its disintegration constant.

$$\tau = \frac{T_{1/2}}{0.693}$$

The disintegration rate (R) is given by

$$R = -\frac{dN}{dt} = \lambda N \quad \text{and}$$

$$R_t = R_0 \exp(-\lambda t)$$

The disintegration constant (λ) is given by $\lambda = 1/t$ where t is the time during which the number of atoms of a radioactive substance becomes $(1/e)$ of its original value.

Units of Radioactivity

The following two units for measuring the disintegration rate are commonly employed.

(i) *Curie*: The curie is a radioactive disintegration rate of 3.7×10^{10} nuclei per second.

(ii) *Rutherford*: This is a disintegration rate of 10^6 nuclei per second.

Radioactive Equilibrium

Each parent in a radioactive series decays into its daughter which decays in

turn, till a stable end product is formed. When the number of atoms of any member in a radioactive chain becomes constant, it is said to be in radioactive equilibrium with its parent. The condition of radioactive equilibrium is expressed as

$$\frac{dN}{dt} = \lambda_1 N_1 = \lambda_2 N_2 = \dots \lambda_n N_n$$

and in terms of mean lives, we have

$$\frac{N_1}{\tau_1} = \frac{N_2}{\tau_2} = \dots \frac{N_n}{\tau_n}$$

The Nucleus

In the central space of an atom is a nucleus. It is the space in which the entire positive charge is concentrated. The radius of a nucleus of an atom is approximately 10^{-14} m whereas the radius of the atom is about 10^{-10} m. Thus, the nucleus occupies an extremely small portion of an atom.

The mass of an atomic nucleus is very small and when expressed in ordinary units, it is about 10^{-25} kg. It is convenient to express it in terms of another unit, namely, the atomic mass unit (amu) or simply (u), defined as $1/12$ of the mass of the carbon atom ($1u = 1.66 \times 10^{-27}$ kg).

To obtain a nuclear mass, the total mass of the electrons has to be subtracted from the atomic mass, simultaneously taking into account the correction due to the electronic binding energy.

Elementary Atomic Particles

	Electron	Nucleon	
		proton	neutron
Mass (kg)	9.1×10^{-31}	1.6725×10^{-27}	1.6748×10^{-27}
Mass (u)	0.0005486	1.007825	1.008665
Charge (C)	-1.602×10^{-19}	1.602×10^{-19}	0

The number of protons in a nucleus is referred to as atomic number (Z). The mass number (A) is equal to the total number of protons and neutrons. Hence, the number of neutrons = $(A - Z)$.

An atomic number and a mass number are by convention denoted by a subscript and a superscript respectively. For example, ${}^{13}_6\text{C}$ denotes a nucleus containing 6 protons and 13 nucleons (neutrons + protons). The number of neutrons is 7.

Isotopes

In an element, each atom has the same number of electrons and the same number of protons. However, the atoms in an element may differ in respect of

the number of neutrons they contain. Such atoms are called *isotopes* (same Z but different neutron number N). For examples ${}^{15}_8\text{O}$, ${}^{16}_8\text{O}$, ${}^{17}_8\text{O}$ and ${}^{18}_8\text{O}$ are the isotopes of oxygen.

Isobars

Nuclei with the same number of nucleons but different N and Z are called isobars; for example, ${}^{14}_6\text{C}$, ${}^{14}_7\text{N}$, ${}^{14}_8\text{O}$ are the isobars corresponding to $A = 14$.

Binding Energy (BE)

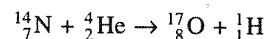
$\text{BE} = \Delta m \cdot C^2$, where Δm is the mass defect and C is the velocity of light. If M , M_n and M_p are the masses of a nucleus (${}^A_Z\text{X}$), a neutron and a proton respectively, then $\Delta m = N \cdot M_n + ZM_p - M$ and $\text{BE} = (NM_n + ZM_p - M) C^2$. BE in terms of atomic masses = $(NM_n + ZM_H - M_A)C^2$ where M_H is the mass of one atom of hydrogen, M_A is the mass of the atom (${}^A_Z\text{X}$). $\text{BE in amu} = 1.66 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 = 1.49 \times 10^{-10} \text{ J} = 931 \text{ MeV}$ ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$).

Nuclear Reaction Energy

In a nuclear reaction, in fact, there is no change in the total number of protons and neutrons. The rearrangement of nucleons before and after the reaction results in energy change. The energy liberated or absorbed during the course of a nuclear reaction is known as nuclear reaction energy and is represented by Q . The Q -value represents the energy balance of a nuclear reaction.

Artificial Transmutation or Disintegration of Elements

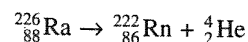
This is a process of producing new stable nuclei from other stable nuclei. When a nitrogen nucleus is hit by α -particles, an oxygen isotope is formed and a proton is emitted.



Different nuclear particles used for bombardment of the nuclei are:

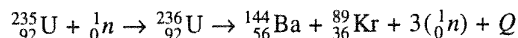
- α -particle (${}^4_2\text{He}$)
- Proton (${}^1_1\text{H}$)
- Deuteron (${}^2_1\text{D}$) or (${}^2_1\text{H}$)
- Neutron (${}_0^1n$)
- Triton (an isotope of hydrogen) (${}^3_1\text{T}$) or (${}^3_1\text{H}$)
- High energy photons (γ) with zero mass number and zero charge

In a balanced equation, the sum of subscripts (atomic number) must be the same on the two sides of the equation. The sum of the superscripts should also be same. For example,

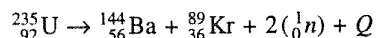


Nuclear Fission

Nuclear reactions involve absorption or emission of light particles such as protons, neutrons, and α -particles. In such reactions, generally, the Q -value is of the order of 10 MeV. In a nuclear fission process, the reaction can be induced by bombarding a certain nucleus with neutrons, e.g.

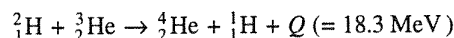
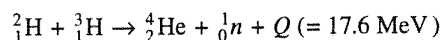
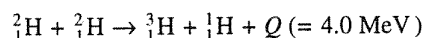
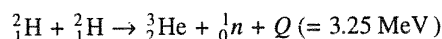


The Q -value in this reaction is about 200 MeV, i.e. twenty times the disintegration energy encountered in a normal nuclear reaction. This type of fission is known as *induced fission*. A heavy nucleus (with $A > 220$) fissions by itself, and is said to undergo spontaneous fission. For example,



Nuclear Fusion

A reaction in which two light nuclei combine with the release of energy is called a fusion reaction. A few examples are shown in the following:



We observe that the deuteron-triton reaction liberates a large amount of energy.

ILLUSTRATIONS

1. The mean life of radium (226) is 1600 years and that of radon (222) is 3.8 days. Calculate the volume of radon gas that would be in equilibrium with 1 g of radium.

Solution

$$\frac{\tau_1}{\tau_2} = \frac{1600 \text{ years}}{(3.8/365) \text{ years}} \text{ and } \frac{\tau_1}{\tau_2} = \frac{N_1}{N_2}$$

for radioactive equilibrium.

Number of atoms contained in 1 g of radium

$$= \left(\frac{6.02 \times 10^{23}}{226} \right) \times 1 = N_1$$

$$N_2 = N_1 \times \frac{\tau_2}{\tau_1} = \left(\frac{6.02 \times 10^{23}}{226} \right) \times \frac{(3.8/365)}{1600}$$

Weight of radon in equilibrium with radium

$$= \frac{222}{6.02 \times 10^{23}} \text{ g}$$

$$\therefore N_2(\text{g}) = \left(\frac{6.02 \times 10^{23}}{226} \right) \times \frac{(3.8/365)}{1600} \times \frac{222}{6.02 \times 10^{23}} \text{ g}$$

$$= \frac{222 \times 3.8}{226 \times 1600 \times 365} \text{ g}$$

222 g of radon at NTP occupies 22.4 litres = 22400 cc

Therefore, $\frac{222 \times 3.8}{226 \times 1600 \times 365}$ g of radon at NTP will occupy

$$\frac{22400 \times 3.8 \times 222}{226 \times 226 \times 1600 \times 365} \text{ cc} = 0.0006449 \text{ cc}$$

2. The half-life period of a radioactive source is 25 years. What will be its strength after 5 years if the initial strength of the source is 10 millicurie?

Solution

$$T_{1/2} = 25 \text{ years and } t = 5 \text{ years}$$

$$N_0 = 10 \text{ millicurie}$$

$$\ln \frac{N}{N_0} = -\lambda t = -\frac{0.693}{T_{1/2}} t = -0.693 \times \frac{5}{25} = -0.1386$$

$$\ln \frac{N}{N_0} = -0.1386 \Rightarrow \frac{N}{N_0} = \exp(-0.1386) = 0.8614$$

$$\therefore N = N_0 \times 0.8614 = 10 \times 0.8614 = 8.614 \text{ millicurie}$$

3. A count rate meter is used to measure the activity of a radioactive sample. At a certain instant the count rate was recorded as 4750 counts per minute. Five minutes later, the count rate showed 2700 counts per minute. Compute the disintegration constant and the half-life of the sample.

Solution

$$\frac{dN_1}{dt} = \lambda N_0 \quad \text{and} \quad \frac{dN_2}{dt} = \lambda N$$

$$\therefore \frac{N_0}{N} = \frac{dN_1/dt}{dN_2/dt} = \frac{4750}{2700} = 1.759$$

$$N = N_0 \cdot e^{-\lambda t} \text{ or } \ln \frac{N_0}{N} = \lambda t \Rightarrow \ln 1.759 = \lambda t$$

$$\therefore \lambda = \frac{\ln 1.759}{5 \text{ minutes}} = 0.1129 \text{ per minute}$$

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.1129} = 6.138 \text{ minutes}$$

4. A certain radioactive element has a half-life of 20 days. How long will it take for 75% of its atoms originally present to disintegrate? Calculate its disintegration constant and the average life of the element.

Solution

$$T_{1/2} = 20 \text{ days}, \lambda = \frac{0.693}{20} / \text{day} = 0.0346 / \text{day}$$

$$\frac{N}{N_0} = \frac{3}{4} \text{ (given), } \ln \frac{N}{N_0} = -\lambda t \text{ or } \ln \frac{3}{4} = -\frac{0.693}{20} t$$

$$\therefore t = \frac{-0.28768 \times 20}{-0.693}$$

$$= 8.30 \text{ days}$$

5. Calculate the average binding energy per nucleon in the ${}^4_2\text{He}$ nucleus using the neutron mass and the atomic masses of H and He, and neglecting the binding energy of the electron in the helium atom. Justify neglecting the BE of electrons in the helium atom.

Solution

The helium atom contains two protons and two neutrons in its nucleus and two electrons in the orbit. The combined mass of these particles, i.e.

$$NM_n + ZM_p + ZM_e (= NM_n + ZM_H) \text{ is given by}$$

$$2M_n + 2M_H = 2(1.008665 + 1.007825) = 4.032980$$

The experimentally measured mass of helium is $M_A(\text{He}) = 4.0026$

$$\therefore \Delta m = 4.03298 - 4.00260 = 0.03038 \text{ u}$$

$$1 \text{ u} = 1.66 \times 10^{-27} \times (3 \times 10^8)^2 = 14.94 \times 10^{-11} \text{ J} \approx 931 \text{ MeV}$$

$$\therefore \text{BE of } {}^4_2\text{He} = 0.03038 \times 931 = 28 \text{ MeV}$$

$$\therefore \text{Average BE per nucleon} = 7 \text{ MeV}$$

The total BE of ${}^4_2\text{He}$ is 28 MeV whereas the BE of the electron is 80 eV or $80 \times 10^{-6} \text{ MeV}$. Therefore, the error in the total BE is $\frac{80 \times 10^{-6}}{28} \times 100\% \approx 0.0003\%$. The error is very small as compared to the accuracy of measurement and thus when calculating the BE of a nucleus, the atomic BE is generally neglected.

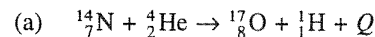
6. Calculate the Q value when:

(a) Nitrogen is bombarded by α -particle so as to produce an isotope of oxygen and a proton.

(b) Lithium is bombarded by a proton so as to produce α -particles.

Given: Mass of nitrogen	= 14.00753 u
Mass of α -particle	= 4.00386 u
Mass of oxygen isotope	= 17.00450 u
Mass of proton	= 1.00813 u
Mass of lithium	= 7.01816 u

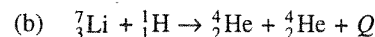
Solution



$$(14.00753 + 4.00386) = (17.00450 + 1.00813) + Q$$

$$Q = -0.00124 \text{ u} = -0.00124 \times 931 \text{ MeV} = -1.544 \text{ MeV}$$

This shows that the energy is absorbed in the reaction.

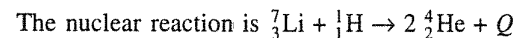


$$(7.01816 + 1.00813) = [2(4.00386)] + Q$$

$$Q = 0.01857 \text{ u} = 0.01857 \times 931 \text{ MeV} = +17.28867 \text{ MeV}$$

Since Q is positive, the energy will be liberated in the reaction.

(c) Calculate the energy generated in kilowatt-hours when 0.1 kg of ${}^7_3\text{Li}$ is converted into ${}^4_2\text{He}$ by proton bombardment.



$$7.0183 + 1.00813 = 2[4.00386] + Q$$

$$\Delta m, \text{ the mass defect} = (8.0264 - 8.00772) = 0.0187 \text{ u}$$

$$\text{Energy liberated by } 7.0183 \text{ kg of } {}^7_3\text{Li} = 0.0187 \times (3 \times 10^8)^2 \text{ J}$$

$$\text{Energy liberated by } 0.1 \text{ kg of } {}^7_3\text{Li}$$

$$= \frac{0.0187 \times 9 \times 10^{16}}{7.0183} \times 0.1 \text{ Ws}$$

$$= \frac{0.0187 \times 9 \times 10^{16}}{7.0183 \times 10 \times 60 \times 60 \times 1000} \text{ kWh}$$

$$= 66.6 \times 10^5 \text{ kWh}$$

7. Consider a typical reaction during the neutron-induced fission of ${}^{235}_{92}\text{U}$, namely ${}_0^1n + {}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U} \rightarrow {}^{98}_{40}\text{Zr} + {}^{136}_{52}\text{Te} + 2 {}^1_0n$. The resulting fission fragments are far from the stability region and decay into stable end products ${}^{98}_{42}\text{Mo}$ and ${}^{136}_{54}\text{Xe}$ by a successive emission of β -particle. Calculate the total energy that will be released in this fission reaction.

Given:

$$\text{Mass of neutron} = 1.0087 \text{ u}$$

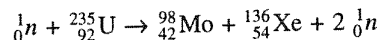
$$\text{Mass of } {}^{235}_{92}\text{U} = 236.0526 \text{ u}$$

Mass of $^{98}_{42}\text{Mo} = 97.9054 \text{ u}$

Mass of $^{136}_{54}\text{Xe} = 135.9170 \text{ u}$

Solution

The reaction is



$$(1.0087 + 235.0439) = (97.9054 + 135.9170 + 2.0174)$$

$$\Delta m = 0.2128 \text{ u}$$

$$\therefore \text{Energy released} = 0.2128 \times 931 \text{ MeV} = 198 \text{ MeV}$$

8. Calculate the amount of energy that will be released in completely fissioning 1 kg of $^{235}_{92}\text{U}$. Also how much power will be generated if the same quantity of $^{235}_{92}\text{U}$ is fissioned in one year?

Solution

The number of nuclei (N) in 1 kg of $^{235}_{92}\text{U}$ is

$$N = \frac{1000}{235} \times 6.03 \times 10^{23} = 2.56 \times 10^{24}$$

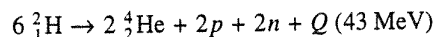
In Illustration 7, it is estimated that the energy released in fissioning one nucleus is about 200 MeV. Therefore, the total energy that will be released in fissioning N nuclei is

$$200 \times 2.56 \times 10^{24} = 5.12 \times 10^{26} \times 1.6 \times 10^{-13} \text{ J} = 8.2 \times 10^{13} \text{ J}$$

$$\text{One year} = 3.15 \times 10^7 \text{ s}$$

$$\therefore \text{Power output} = \frac{8.2 \times 10^{13} \text{ J}}{3.15 \times 10^7 \text{ s}} \approx 2.6 \text{ MW}$$

9. Consider the fusion reaction



Determine the amount of energy that will be generated by fusing all the nuclei in 1 kg of deuterium in such a reaction. Compare the energy so obtained with the energy that can be generated by fissioning 1 kg of $^{235}_{92}\text{U}$.

Solution

Since six deuterons fuse together to give an energy of 43 MeV, the total energy that will be emitted in fusing 1 kg of deuterium is

$$E = \left(\frac{43}{6}\right)N \quad \text{and} \quad N = \frac{6.02 \times 10^{26}}{2} = 3 \times 10^{26}$$

$$= \frac{43 \times 3 \times 10^{26}}{6}$$

$$= 2.15 \times 10^{27} \text{ MeV} = 3.44 \times 10^{14} \text{ J}$$

$$= \frac{3.44 \times 10^{14}}{3.6 \times 10^6} \approx 10^8 \text{ kWh}$$

Fission: Total energy that will be released from 1 kg of $^{235}_{92}\text{U}$ through fission will be

$$E = \frac{200 \times 6.02 \times 10^{26}}{235}$$

$$= 5.1 \times 10^{26} \text{ MeV}$$

$$= 2.3 \times 10^7 \text{ kWh}$$

The energy released is about four times more in fusion than in fission.

10. A railway engine develops an average power of 4 kW during a 15-hour run from one station to another. If the engine is driven by an atomic power plant that is 20% efficient, how much $^{235}_{92}\text{U}$ would be consumed during the run (given that each $^{235}_{92}\text{U}$ fission releases 200 MeV of energy)?

Solution

Energy needed by the engine during the run

$$= 4 \times 1000 \times 15 = 6 \times 10^4 \text{ kWh}$$

$$(1000 \text{ W} \times 3600 \text{ s} = 36 \times 10^5 \text{ J})$$

$$\therefore \text{Power developed} = 6 \times 10^4 \times 36 \times 10^5 \text{ J} = 216 \times 10^9 \text{ J}$$

The reactor output should be $216 \times 10^9 \text{ J}$.

As the reactor is 20% efficient, its output should be

$$\frac{216 \times 10^9 \times 100}{20} = 1080 \times 10^9 = 108 \times 10^{10} \text{ J}$$

Energy released per fission = 200 MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} = 320 \times 10^{-13} \text{ J}$$

$$\therefore \text{Number of } ^{235}\text{U fissions} = \frac{108 \times 10^{10}}{320 \times 10^{-13}} = \frac{108}{320} \times 10^{23} \\ = 0.3375 \times 10^{23} \text{ atoms}$$

6.03×10^{23} atoms are contained in 235 g of ^{235}U . Therefore, the quantity of ^{235}U needed will be

$$\frac{235}{6.03 \times 10^{23}} \times 3375 \times 10^{19} = \frac{235 \times 3375}{6.03} \times 10^{-4} \text{ g} \\ = 13.15 \text{ g}$$

11. It is proposed to use nuclear fusion reaction ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He}$ in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with 25% efficiency in the reactor, how many grams of deuterium fuel will be needed per day (the masses of ${}^2_1\text{H}$ and ${}^4_2\text{He}$ are 2.0141 and 4.0026 u respectively)?

Solution

$$\Delta m = (2 \times 2.014) - 4.0026 = 0.0256 \text{ u}$$

$$\therefore \Delta E' = 0.0256 \times 931 \text{ MeV} = 38.134 \times 10^{-13} \text{ J}$$

Since utilization efficiency is 25%

$$\Delta E = 38.134 \times 10^{-13} \text{ J} \times \frac{25}{100} = 9.531 \times 10^{-13} \text{ J}$$

The reactor rating is 200 MW. Hence, the total energy required per day is $(200 \times 10^6 \text{ J/s}) \times 24 \times 60 \times 60 = 172.8 \times 10^{11} \text{ J}$. Two deuterium atoms provide us with an energy of $9.531 \times 10^{-13} \text{ J}$. Therefore, the number of deuterium atoms needed per day is

$$n = \frac{E}{\Delta E} = \frac{172.8 \times 10^{11} \times 2}{9.531 \times 10^{-13}} = 36.25 \times 10^{24}$$

1 mole = $(6.02 \times 10^{23} \text{ atoms})$ of deuterium has a mass of 2.0141 g. Hence the mass of deuterium needed per day is

$$m = \frac{2.0141 \times 36.25 \times 10^{24}}{6.02 \times 10^{23}} = 121.3 \text{ g}$$

12. What is the minimum energy needed to break ${}^{12}_6\text{C}$ into three α -particles? If this energy is supplied by photons, calculate the maximum wavelength of photons (Mass of ${}^{12}_6\text{C} = 12.0038$, mass of α -particle = 4.00388)?

Solution

$$\text{Mass of } {}^{12}_6\text{C} = 12.0038$$

$$\text{Mass of 3 } \alpha\text{-particles} = 3 \times 4.00388 = 12.01164$$

$$\Delta m = 0.00784 \text{ u}$$

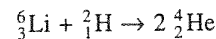
$$\therefore E = 0.00784 \times 931 \text{ MeV} = 7.2 \text{ MeV}$$

$$\text{Also } 1 \text{ amu} = 1.49 \times 10^{-10} \text{ J}$$

$$\therefore E = 1.49 \times 10^{-10} \times 0.00784 \text{ J} = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{0.00784 \times 1.49 \times 10^{-10}} = \frac{66 \times 3 \times 10^{-10}}{784 \times 149} \\ = 1695 \times 10^{-16} \text{ m}$$

13. An isotopic species of lithium hydride, Li^6H^2 , is a potential nuclear fuel, on the basis of the following reaction



Calculate the expected power production, in kilowatts, associated with the consumption of 1.00 g of Li^6H^2 per day. Assume 100% efficiency in the process (${}^6_3\text{Li} = 6.01702 \text{ u}$, ${}^2_1\text{H} = 2.01474 \text{ u}$, ${}^4_2\text{He} = 4.0388 \text{ u}$).

Solution

$$\left. \begin{array}{l} {}^6_3\text{Li} = 6.01702 \\ {}^2_1\text{H} = 2.01478 \end{array} \right\} = 8.03176 \text{ u}$$

$$2 {}^4_2\text{He} = 2 \times 4.00388 = 8.00776 \text{ u}$$

$$\Delta m = 0.02400 \text{ u}$$

For each gram of hydride consumed, the loss in mass is $0.02400/8.03 = 0.00299 \text{ g}$

Corresponding energy production

$$= \Delta m C^2 = \frac{(2.99 \times 10^{-6} \text{ kg})(3 \times 10^8)^2}{3.6 \times 10^6} \text{ kWh} = 7.48 \times 10^4 \text{ kWh}$$

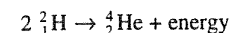
$$\text{Power per day} = \frac{7.48 \times 10^4}{24} = 3.12 \times 10^3 \text{ kW}$$

EXERCISES

- The mass of C^{12} is 12.0038 u and the masses of the proton and neutron respectively are 1.0081 and 1.0090 u. The binding energy of C^{12} is about
 - 184.0 MeV
 - 23 MeV
 - 46.0 MeV
 - 92.0 MeV
- In a reaction ${}^7_3\text{Li} (p, n) {}^7_4\text{Be}$, the atomic masses of ${}^7_3\text{Li} = 7.01822 \text{ u}$, ${}^1_1\text{H} = 1.00814 \text{ u}$, ${}_0^1n = 1.00899 \text{ u}$ and ${}^7_4\text{Be} = 7.01916 \text{ u}$. The overall energy balance of the reaction is
 - 1.67 MeV
 - 3.34 MeV
 - 1.11 MeV
 - 2.22 MeV
- Fill in the blanks in the following nuclear reaction:
 - ${}^{23}_{11}\text{Na} + {}^4_2\text{He} \rightarrow {}^{26}_{12}\text{Mg} + \text{_____}$
 - ${}^{64}_{29}\text{Cu} \rightarrow {}^0_1e + \text{_____}$
 - ${}^{106}_{47}\text{Ag} \rightarrow {}^{106}_{48}\text{Cd} + \text{_____}$
 - ${}^{10}_5\text{B} + {}^4_2\text{He} \rightarrow {}^{13}_7\text{Na} + \text{_____}$
 - ${}^{105}_{48}\text{Cd} + {}^0_1e \rightarrow \text{_____}$
 - ${}^{238}_{92}\text{Cd} \rightarrow {}^{234}_{90}\text{Th} + \text{_____}$

4. In the following nuclear reactions, d represents deuterium, n , a neutron, p , a proton, γ a photon and α , an alpha particle. Fill in the end products of the following reactions:
- ${}^{24}_{12}\text{Mg} (d, \alpha) \rightarrow \text{_____}$
 - ${}^{26}_{12}\text{Mg} (d, p) \rightarrow \text{_____}$
 - ${}^{40}_{18}\text{Ar} (\alpha, p) \rightarrow \text{_____}$
 - ${}^{12}_6\text{C} (d, n) \rightarrow \text{_____}$
 - ${}^{55}_{25}\text{Mn} (n, \gamma) \rightarrow \text{_____}$
 - ${}^{59}_{27}\text{Co} (n, \alpha) \rightarrow \text{_____}$
5. Complete the following nuclear reactions:
- ${}^4_2\text{He} + \text{_____} \rightarrow {}^{14}_7\text{N} + {}^1_1\text{H}$
 - ${}^{24}_{12}\text{Mg} + {}^1_0\text{n} \rightarrow {}^{24}_{11}\text{Na} + \text{_____}$
 - ${}^2_1\text{H} + {}^9_4\text{Be} \rightarrow {}^1_0\text{n} + \text{_____}$
 - $\text{_____} + {}^1_0\text{n} \rightarrow {}^{38}_{19}\text{K} + 2 {}^1_0\text{n}$
 - ${}^{65}_{29}\text{Cu} + {}^1_0\text{n} \rightarrow \text{_____} + \gamma$
6. A radioactive isotope ${}^{230}_{90}\text{Th}$ of activity $1 \mu\text{Ci}$ decays by α -emission to radium. If the half-life of decay is 76000 years, the activity of thorium after 76 years is
- $3.7 \times 10^4/\text{s}$
 - $3.7 \times 10^3/\text{s}$
 - $7.3 \times 10^4/\text{s}$
 - $7.3 \times 10^3/\text{s}$
7. One gram of cesium (${}^{137}_{55}\text{Cs}$) decays by β -emission with a half-life of 30 years. If the initial activity of cesium is 1.0 millicurie and atomic mass of cesium is 137 kg per kmol, tick the statement that is *incorrect*.
- the resulting isotope is ${}^{137}_{56}\text{Ba}$
 - the mean life is 43.29 years
 - the number of atoms left after five years $= 39 \times 10^{20}$
 - the activity of cesium after five years $= 0.329 \times 10^8/\text{s}$
8. If an atom of ${}^{235}_{92}\text{U}$, after absorbing a slow neutron, undergoes fission to form an atom of ${}^{139}_{54}\text{Xe}$ and an atom of ${}^{94}_{38}\text{Sr}$, the other particles produced are
- one proton and two neutrons
 - three neutrons
 - two neutrons
 - one proton and one neutron
9. The energy released in the following reaction
- $$\left. \begin{aligned} {}^3_1\text{H} + {}^2_1\text{H} &\rightarrow {}^4_2\text{He} + {}^1_0\text{n} \text{ is } \left\{ \begin{aligned} {}^3_1\text{H} &= 3.01700 \text{ u} & {}^4_2\text{He} &= 4.00388 \text{ u} \\ {}^2_1\text{H} &= 2.01474 \text{ u} & {}^1_0\text{n} &= 1.0090 \text{ u} \end{aligned} \right. \end{aligned} \right\}$$
- 16.2 MeV
 - 12.8 MeV
 - 14.6 MeV
 - 17.6 MeV
10. In the ${}^{14}_7\text{N} (n, p) {}^{14}_6\text{C}$ reaction, the proton is ejected with an energy of 0.6 MeV. The mass of the ${}^{14}_6\text{C}$ atom is
- 14.0182 u
 - 12.0014 u
 - 14.0077 u
 - 12.4681 u
11. A γ -ray photon disappears and an electron-positron pair is formed. If the total kinetic energy of electron-positron pair is 0.78 MeV, the energy of the photon is
- 1.80 MeV
 - 1.24 MeV
 - 2.24 MeV
 - 1.08 MeV

12. It is proposed to use the nuclear fusion reaction



to produce industrial electric power. If the power required to be produced daily is 50,000 kW and the energy of the above reaction is to be used with 30% efficiency, the amount of deuterium fuel needed per day is

- 100 g
 - 10 g
 - 25 g
 - 50 g
13. Taking 200 MeV as the energy released during the fission of a single uranium ${}^{235}_{92}\text{U}$ nucleus, the number of fission processes taking place each second in a reactor operating at a power level of 2 MW is
- 6.25×10^{16}
 - 6.25×10^{10}
 - 6.25×10^{12}
 - 6.25×10^{20}
14. The isotope of ${}^{209}_{83}\text{Bi}$ has a mass 208.98 amu, and ${}^{82}_{36}\text{Kr}$ has a mass of 81.913 amu. The difference in binding energy (in mega electron-volts) per nucleon for these two nuclei is ($m_p = 1.007825 \text{ u}$, $m_n = 1.008665 \text{ u}$):
- 7.845
 - 0.866
 - 8.660
 - 6.880
15. The total decrease each day in the total mass of the material in a uranium reactor operating at a power of 12 MW is
- 11.5 g
 - $11.5 \times 10^{-2} \text{ g}$
 - 115 g
 - $11.5 \times 10^{-3} \text{ g}$
16. Four hydrogen atoms are fused to form a helium atom according to the following reaction
- $$4 {}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\beta^+ + Q \quad (\beta^+ \text{ is a positron})$$
- The energy released in the process of fusion is
- 100 MeV
 - 42.8 MeV
 - 25.7 MeV
 - 72.4 MeV
17. A certain radioactive substance has a half-life of 30 days. The average life of the substance is
- 43.29 days
 - 29.34 days
 - 34.29 days
 - 9.24 days
18. A uranium sample U_1 has a half-life of 25 days. The time it will take for 90% of the sample to change to U_2 is
- 97.2 days
 - 28.97 days
 - 82.97 days
 - 41.45 days
19. The half-life of radium is 1500 years. The time taken to reduce a gram of the pure element to a centigram is
- 570 years
 - 9970 years
 - 7990 years
 - 7099 years
20. A sample of radon loses half of its activity in 3.86 days. The time in which the activity is reduced to 5% is
- 16.68 days
 - 6.68 days
 - 8.34 days
 - 12.44 days
21. A certain radioactive element disintegrates for an interval of time equal to its mean life. The fraction that has disintegrated is
- $\frac{1}{e}$
 - $\left(1 - \frac{1}{e}\right)$
 - $\left(\frac{e-1}{2}\right)$
 - $\left(e - \frac{1}{e}\right)$

22. One gram of a radioactive substance takes 50 s to lose 1 centigram. The half-life period of the substance is
 (a) 42 hrs (b) 57 hrs
 (c) 57 min. (d) 3.2 days
23. The activity of ^{226}Ra is 3.7×10^{10} disintegrations/s/g. The half-life and average life respectively are
 (a) 1582 years and 2283 years (b) 2283 years and 1582 years
 (c) 791 years and 1142 years (d) 1142 years and 791
24. A sample of 1 g of ^{209}Bi with a half-life of 2.7×10^7 years decays into a stable isotope of thallium by emitting α -particles. Its activity is
 (a) 6.3 mCi (b) 0.0063 mCi
 (c) 0.63 mCi (d) 0.064 mCi
25. The half-life of a certain radioactive material is 15 hr. The time it will take for 93.75% of this material to disintegrate, is
 (a) 120 hrs (b) 30 hrs
 (c) 60 hrs (d) 4 hrs
26. A radioactive material initially contains 3 mg of $^{234}_{92}\text{U}$. If the half-life of $^{234}_{92}\text{U}$ is 2.47×10^5 years, the amount of $^{234}_{92}\text{U}$ that will remain unchanged after 62000 years is
 (a) 1.00 mg (b) 2.50 mg
 (c) 1.05 mg (d) 0.025 mg
27. The radioactive disintegration constant of a substance is $1.13 \times 10^{-9}/\text{s}$. The time it would take for the material to be reduced to one-tenth its original quantity, is
 (a) 23500 days (b) 4 years
 (c) 65 years (d) 235 days
28. The half life of radium is 1590 years. The time it would take for 1 g of this pure element to reduce to 1 centigram, is
 (a) 565 years (b) 10565 years
 (c) 9565 years (d) 6955 years
29. One gram of a radioactive substance takes 45 s to lose 1 centigram. Its half-life period is
 (a) 26654 s (b) 265 s
 (c) 456 s (d) 66254 s
30. The half-life of radon is three days. The number of days it will take so that only 1/16 of the sample is left is
 (a) 6 (b) 4
 (c) 20 (d) 12
31. The density of a nucleus is of the order of
 (a) 10^8 kg/m^3 (b) 10^{17} kg/m^3
 (c) 10^{10} kg/m^3 (d) 10^{22} kg/m^3
32. Fill in the blanks:
 (a) $^6_3\text{Li} + ^2_1\text{H} \rightarrow ^4_2\text{He} + \text{_____}$
 (b) $^{35}_{17}\text{Cl} + \text{_____} \rightarrow ^{32}_{16}\text{S} + ^4_2\text{He}$
 (c) $^9_4\text{Be} + ^4_2\text{He} \rightarrow ^1_0\text{n} + \text{_____}$
 (d) $^{14}_7\text{N} + ^1_0\text{n} \rightarrow ^{14}_6\text{C} + \text{_____}$
 (e) $^2_1\text{H} + ^{16}_8\text{O} \rightarrow ^{17}_7\text{N} + \text{_____}$
 (f) $^2_1\text{H} + ^{10}_5\text{B} \rightarrow ^{11}_6\text{C} + \text{_____}$
- (g) $^9_4\text{Be} + ^1_1\text{H} \rightarrow ^8_4\text{Be} + \text{_____}$
 (h) $^7_3\text{Li} + ^2_1\text{H} \rightarrow ^6_3\text{Li} + \text{_____}$
 (i) $^{12}_6\text{C} + ^1_0\text{n} \rightarrow ^{11}_6\text{C} + \text{_____}$
33. The energy released in the following nuclear reaction is
 $^{10}_5\text{B} + ^2_1\text{H} \rightarrow ^{11}_5\text{B} + ^1_1\text{H}$
 ($^{10}_5\text{B} = 10.0165 \text{ u}$, $^{11}_5\text{B} = 11.01286 \text{ u}$, $^2_1\text{H} = 2.01472 \text{ u}$ and $^1_1\text{H} = 1.00812 \text{ u}$)
 (a) 95.33 MeV (b) 9.533 MeV
 (c) 953 MeV (d) 9533 MeV
34. In the nuclear reaction $^{14}_7\text{N} + ^1_0\text{n} \rightarrow ^{14}_6\text{C} + ^1_1\text{H} + 0.55 \text{ MeV}$, the mass of $^{14}_6\text{C}$ is ($^{14}_7\text{N} = 14.003074$, $^1_0\text{n} = 1.008665$)
 (a) 14.003323 u (b) 14.0345 u
 (c) 14.3013 u (d) 14.11222 u
35. The binding energy of α -particle is ($m_p = 1.00758 \text{ u}$ and $m_n = 1.00897 \text{ u}$)
 (a) 56.42 MeV (b) 2.821 MeV
 (c) 28.21 MeV (d) 282.1 MeV
36. If the atomic masses of ^6_3Li and ^2_1H are 6.015126 and 3.016049 u respectively, the energy released in the reaction $^6_3\text{Li} + ^2_1\text{H} \rightarrow ^4_2\text{He} + ^3_1\text{H}$, is
 (a) 47.8 MeV (b) 478 MeV
 (c) 0.478 MeV (d) 4.78 MeV
37. A deuteron is to be disintegrated into a proton and a neutron. If the mass of deuteron is 2.014735 u, the minimum energy required for disintegration is
 (a) 4.4482 MeV (b) 2.2053 MeV
 (c) 1.1120 MeV (d) 6.6641 MeV
38. The mass of lithium atom is 7.0878 u. The binding energy of lithium in electron-volts is
 (a) 25.779 MeV (b) 2.5779 MeV
 (c) 257.7903 MeV (d) 0.25779 MeV
39. If the energy per fission is 200 MeV, the energy released in the fission of 0.1 kg of $^{235}_{92}\text{U}$ in kilowatt-hours is
 (a) 2.278×10^3 (b) 22.78×10^4
 (c) 2.278×10^4 (d) 0.2278×10^4
40. When $^{235}_{92}\text{U}$ undergoes fission, about 0.1% of the original mass is released as energy. In a nuclear reactor, it is desired to produce 100 megawatt of electric power. The quantity of $^{235}_{92}\text{U}$ required per day is
 (a) 70 mg (b) 1.2 g
 (c) 9.6 g (d) 96 g
41. In the following nuclear reaction,
 $^{14}_7\text{N} + ^4_2\text{He} \rightarrow ^{17}_8\text{O} + ^1_1\text{H}$, the Q -value is
 (a) 1.198 MeV (b) 11.98 MeV
 (c) -1.198 MeV (d) -11.98 MeV
42. The number of fissions of $^{235}_{92}\text{U}$ required to produce a power of 1 W is
 (a) 3.1×10^{10} (b) 3.1×10^{13}
 (c) 3.1×10^{19} (d) 3.1×10^8
43. The half-life of a radioactive substance is 15 minutes. The fraction of this sample that will remain undecayed after 75 minutes is

- (a) $\frac{1}{2}$ (b) $\frac{1}{32}$
 (c) $\frac{1}{16}$ (d) $\frac{1}{64}$
44. A radioactive substance disintegrates for a time equal to its mean life period. The fraction of the original substance disintegrated is
 (a) 0.368 (b) 0.836
 (c) 0.632 (d) 0.236
45. An ancient radioactive specimen shows an activity of 2.4 disintegrations per second per gram. If the activity of fresh specimen is 38.4 disintegrations/s/g, the age of the specimen is ($T_{1/2}$ of specimen = 5000 years)
 (a) 2004 years (b) 2500 years
 (c) 1000 years (d) 20004 years
46. A reactor has a reserve of nuclear fuel of ${}^{235}_{92}\text{U} = 100$ kg and in a neutron-induced fissioning, about 200 MeV of energy is released. The reactor generates 200 MW of power continuously. The stock of nuclear fuel will last for
 (a) 231.5 days (b) 1 year
 (c) 400 days (d) 200 days
47. The binding energy per nucleon for deuteron (${}^2_1\text{H}$) and helium (${}^4_2\text{He}$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus is
 (a) 47.12 MeV (b) 23.6 MeV
 (c) 11.8 MeV (d) 34.4 MeV
48. During a negative beta decay,
 (a) an atomic electron is ejected
 (b) an electron which is already present within the nucleus is ejected
 (c) a neutron in the nucleus decays emitting an electron
 (d) a part of the binding energy of the nucleus is converted into an electron
49. During a nuclear fusion reaction,
 (a) two light nuclei combine to give a heavier nucleus and possibly other products
 (b) a heavy nucleus breaks into two fragments by itself
 (c) a heavy nucleus bombarded by thermal neutrons breaks up
 (d) a light nucleus bombarded by thermal neutrons breaks up
50. A radioactive material at a given instant emits 4750 particles per minute. Five minutes later it emits 2700 particles per minute. Its half-life is
 (a) 12.268 min. (b) 4.089 min.
 (c) 6.138 min. (d) 10.022 min.

Answers

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (*) | 4. (*) | 5. (*) |
| 6. (a) | 7. (a) | 8. (b) | 9. (d) | 10. (c) |
| 11. (a) | 12. (c) | 13. (a) | 14. (b) | 15. (d) |
| 16. (c) | 17. (a) | 18. (c) | 19. (b) | 20. (a) |
| 21. (b) | 22. (c) | 23. (a) | 24. (d) | 25. (c) |
| 26. (b) | 27. (a) | 28. (b) | 29. (a) | 30. (d) |
| 31. (b) | 32. (*) | 33. (b) | 34. (a) | 35. (c) |
| 36. (d) | 37. (b) | 38. (a) | 39. (c) | 40. (d) |
| 41. (c) | 42. (a) | 43. (b) | 44. (c) | 45. (d) |
| 46. (a) | 47. (b) | 48. (c) | 49. (a) | 50. (c) |

*See below:

3. (a) ${}^1_1\text{H}$ (b) ${}^{64}_{28}\text{Ni}$ (c) ${}^0_{-1}e$ (d) ${}_0^1n$ (e) ${}^{105}_{47}\text{Ag}$ (f) ${}^4_2\text{He}$
4. (a) ${}^{24}_{12}\text{Mg} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + {}^{22}_{11}\text{Na}$ (b) ${}^{26}_{12}\text{Mg} + {}^2_1\text{H} \rightarrow {}^1_1\text{H} + {}^{27}_{12}\text{Mg}$
 (c) ${}^{40}_{18}\text{Ar} + {}^4_2\text{He} \rightarrow {}^1_1\text{H} + {}^{43}_{19}\text{K}$ (d) ${}^{12}_6\text{C} + {}^2_1\text{H} \rightarrow {}^1_0n + {}^{13}_7\text{N}$
 (e) ${}^{55}_{25}\text{Mn} + {}^1_0n \rightarrow {}^0_0\gamma + {}^{56}_{25}\text{Mn}$ (f) ${}^{59}_{27}\text{Co} + {}^1_0n \rightarrow {}^4_2\text{He} + {}^{56}_{25}\text{Mn}$
5. (a) ${}^{11}_6\text{C}$ (b) ${}^1_1\text{H}$ (c) ${}^{10}_5\text{B}$ (d) ${}^{39}_{19}\text{K}$ (e) ${}^{66}_{29}\text{Cu}$
32. (a) ${}^4_2\text{He}$ (b) ${}^1_1\text{H}$ (c) ${}^{12}_6\text{C}$ (d) ${}^1_1\text{H}$ (e) ${}^4_2\text{He}$ (f) ${}_0^1n$
 (g) ${}^2_1\text{H}$ (h) ${}^3_1\text{H}$ (i) $2 {}^1_0n$

Note: All problems which do not involve numerical calculations should take not more than two minutes. All the rest should take three to five minutes.

26

Electronics: Thermionic and Solid State

A. Thermionic Electronics

Thermionic Emission

The thermionic current per unit area of the emitting surface is given by

$$I = AT^2 e^{-b/T}$$

where I is the current in amperes per square centimetre of the emitting surface, T is the absolute temperature of the emitter, and A and b are constants depending upon the nature of the emitter. (This is known as Richardson's formula).

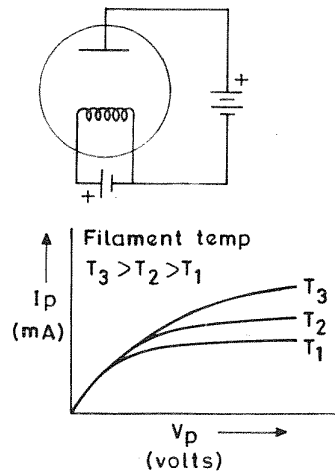
Diode Valve and Its Characteristics The diode is the simplest of thermionic valves and consists of only two elements: (i) a cathode to serve as electron emitter, and (ii) an anode to serve as electron collector. The relationship between the plate potential and the plate current is known as the characteristic of the diode.

An analytical expression (Langmuir Child Law) has been obtained between the plate current and the plate potential.

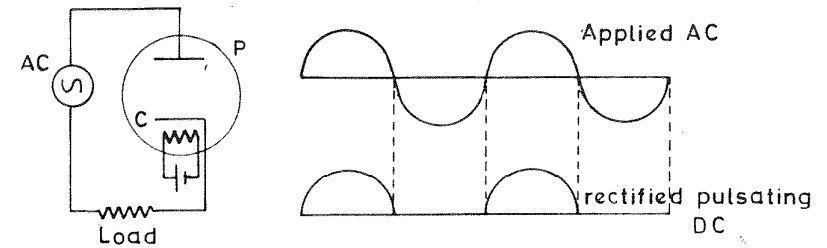
$$I_p = K(E_p)^{3/2}$$

where K is a constant of the tube. This law is not accurate in actual practice and is only a guiding relation.

Vacuum Diode as a Rectifier The property of the diode to conduct current in one direction only makes it work as a rectifier. This property can convert an alternating current into direct current.

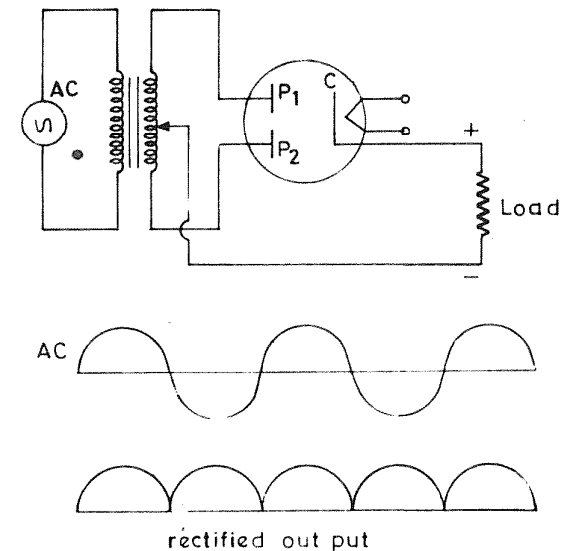


Half-wave Rectification



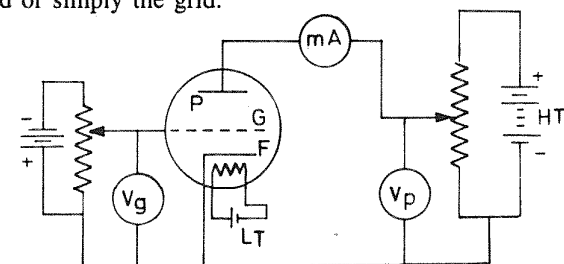
Full-Wave Rectification

For such rectification, a double diode is employed. During the positive half-cycle, P_1 is positive and during the negative half-cycle, P_2 is positive.



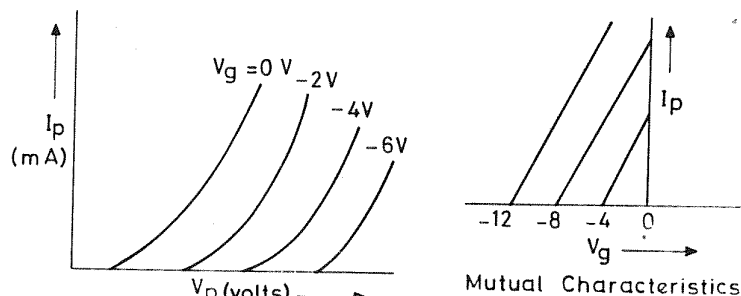
Triode Valve

A triode valve has three elements: (i) the cathode, (ii) the plate, and (iii) the control grid or simply the grid.



Static Characteristics of a Triode

The three important variables in a triode are the plate potential V_p , the grid potential V_g and the plate current I_p . A graph showing the relationship between the plate current and the plate potential at a constant grid potential is called the anode characteristic or the plate characteristic of the valve. The graph showing the relationship between plate current and grid voltage keeping plate voltage constant, known as mutual characteristic, is shown in the figure on the right.



Tube Constants

The main parameters of a triode are:

1. Amplification factor (μ),
2. Plate resistance (R_p), and
3. Transconductance or mutual conductance (g_m).

The amplification factor (μ) is the ratio of the change in plate voltage (ΔV_p) to the change in grid voltage (ΔV_g) at constant plate current, or

$$\mu = \left(\frac{\Delta V_p}{\Delta V_g} \right)_{I_p}, \quad \text{where } I_p \text{ is constant.}$$

The ratio of change in plate voltage (ΔV_p) to change in plate current (ΔI_p) at constant grid voltage is known as plate resistance (R_p).

$$R_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g}, \quad \text{where } V_g \text{ is constant.}$$

The ratio of change in plate current (ΔI_p) to change in grid voltage (ΔV_g) at constant plate voltage is called transconductance or mutual conductance.

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right)_{V_p}, \quad \text{where } V_p \text{ is constant.}$$

Relation between μ , R_p and g_m :

$$\mu = R_p \times g_m$$

ILLUSTRATIONS

1. In a triode, a current of 8 mA flows when the plate voltage is 200 V and grid voltage is -2 V. When the grid voltage is changed to -4 V, the plate current falls to 4 mA but is restored to 8 mA by increasing the plate voltage to 220 V. Determine the constants of the valve.

Solution

$$\mu = \left(\frac{\Delta V_p}{\Delta V_g} \right)_{I_p} = \left(\frac{220 - 200}{-2 + 4} \right) = \frac{20}{2} = 10$$

$$R_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g} = \frac{(220 - 200) \text{ V}}{(8 - 4) \text{ mA}} = \frac{20 \text{ V}}{4 \times 10^{-3} \text{ A}} = 5000 \Omega$$

At constant plate potential of 200 V,

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right) = \frac{4 \text{ mA}}{2 \text{ V}} = 2 \text{ mA/V}$$

Also,

$$\mu = R_p \times g_m = 5000 \Omega \times 2 \times 10^{-3} \text{ A/V} = 10$$

2. The following observations are made while studying the characteristics of a triode.

V_g (V)	V_p (V)	I_p (mA)
-5	300	8 mA
-6.5	300	4 mA
-6.5	200	8 mA

Find the values of the tube parameters.

Solution

(a) At constant plate current of 8 mA

$$\mu = \left(\frac{\Delta V_p}{\Delta V_g} \right)_{I_p} = \frac{(300 - 200) \text{ V}}{-5 - (-6.5) \text{ V}} = \frac{100}{1.5} = 66.6$$

(b) At constant grid voltage of -6.5 V,

$$R_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g} = \frac{(300 - 200) \text{ V}}{(8 - 4) \text{ mA}} = \frac{100 \text{ V}}{4 \times 10^{-3} \text{ A}} = 25 \times 10^3 \Omega$$

(c) At constant plate voltage of 300 V

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right)_{V_p} = \frac{(8 - 4) \text{ mA}}{-5 - (-6.5) \text{ V}} = \frac{4 \text{ mA}}{1.5 \text{ V}} = 2.66 \times 10^{-3} \text{ mho}$$

3. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of -1 V, the anode current I (in mA) may be obtained in terms of plate voltage V (in volts) according to the algebraic relation

$$I = 0.125 V - 7.5$$

For a grid voltage of -3 V, the current at an anode voltage of 300 V is 5 mA. Determine the plate resistance, transconductance and the amplification factor of the triode.

Solution

From the given equation, it follows that, at constant grid voltage,

$$di = 0.125 dV \text{ or } \frac{dV}{di} = \frac{1}{0.125} = 8$$

By definition, R_p is given by

$$R_p = \left(\frac{\Delta V_p}{\Delta I_p} \right)_{V_g}; \text{ hence } R_p = 8 \times 10^3 \Omega \text{ (as current is in milliamperes)}$$

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right)_{V_p}; \text{ at } V_g = -1 \text{ V, } V_p = 300 \text{ V}$$

The plate current is given by

$$I_p = [0.125 \times 300 - 7.5] \text{ mA} = 30.0 \text{ mA}$$

It is also given that

$$V_g = -3 \text{ V, } V_p = 300 \text{ V and } I_p = 5 \text{ mA}$$

Therefore, for a constant value of $V_p (= 300 \text{ V})$, the change in the plate current $dI = (30 - 5) \text{ mA} = 25 \text{ mA}$ for a change of $[-1 - (-3)] \text{ V}$, i.e., 2 V of grid voltage.

Hence,

$$g_m = \left(\frac{\Delta I_p}{\Delta V_g} \right) = \frac{25}{2} \times 10^{-3} = 12.5 \times 10^{-3} \text{ mho}$$

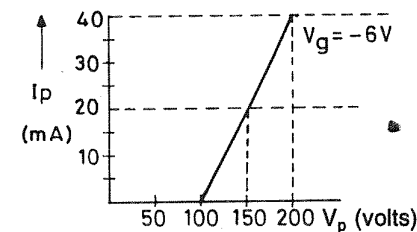
$$\mu = R_p \times g_m = 8 \times 10^3 \times 12.5 \times 10^{-3} = 100$$

EXERCISES

1. The anode slope resistance of a triode is 10000Ω and the amplification factor is 100 . Its mutual conductance is

- (a) 0.1 mho (b) 100 mho
(c) 0.001 mho (d) 0.01 mho

- 2*. A triode gives a plate current of 5 mA at a plate voltage of 150 V and grid voltage of -2 V . If the grid voltage is reduced to -3.5 V , plate current falls to 3.5 mA . It is then restored to its original value of 5 mA by increasing the plate potential by 45 V . The values of μ , R_p and g_m respectively are
(a) $30, 25000 \Omega$ and 0.001 mho
(b) $20, 20000 \Omega$ and 0.010 mho
(c) $30, 20000 \Omega$ and 0.0015 mho
(d) $20, 15000 \Omega$ and 0.00133 mho
3. For a certain triode, the slope of the plate voltage and plate current characteristic is 20 V/mA and the amplification factor is 25 . The mutual conductance of the valve is
(a) 0.125 mho (b) 0.00125 mho
(c) 0.0125 mho (d) 125 mho
4. The anode slope resistance of a triode is 25000Ω and its amplification factor is 50 . Its mutual conductance is
(a) 0.2 mhos (b) 0.002 mhos
(c) 0.02 mhos (d) 2000 mhos
- 5*. A triode valve gives a plate current of 5 mA at a plate voltage of 200 V and grid potential of -1 V . The plate voltage is increased to 250 V and the plate current goes up to 9.5 mA . The grid potential is decreased to -1.7 V and the plate current is restored to 5 mA . The amplification factor of the valve is
(a) 100 (b) 95
(c) 71.4 (d) 50
6. The plate resistance of a triode valve is $5 \times 10^3 \Omega$ and its mutual conductance is $2 \times 10^{-3} \text{ A/V}$. The amplification factor of the triode is
(a) 10 (b) 2.5
(c) 10^7 (d) 2
7. A triode is operated at 300 V and the grid voltage is kept at -15 V . The plate current is 10 mA . If the plate voltage is reduced to 200 V , the plate current is correspondingly reduced. In order to get the same current back, the grid voltage has to be set at -10.0 V . The amplification factor of the valve is
(a) 100 (b) 50
(c) 10 (d) 20
8. In an experimental set-up on the study of the characteristics of a triode, the grid voltage was set at -8 V and the plate current recorded was 16 mA . The subsequent readings were $V_g = -16 \text{ V}$ and $I_p = 2.4 \text{ mA}$. The mutual conductance of the triode is
(a) $1.7 \times 10^{-5} \text{ mhos}$ (b) $1.7 \times 10^{-3} \text{ mhos}$
(c) 1700 mhos (d) $1.7 \times 10^{-4} \text{ mhos}$
9. The V_p - I_p characteristic of a triode is drawn at a constant grid voltage of -6 V (shown in the figure). It is given that the amplification factor of the valve is 50 . The mutual conductance of the valve is
(a) 0.02 mho
(b) $2 \times 10^{-3} \text{ mho}$
(c) $2 \times 10^{-4} \text{ mho}$
(d) 0.2 mho



10*. The following is the table for the observations recorded in the experiment on triode characteristic studies

V_g	V_p	I_p
- 4 V	200 V	6 mA
- 6 V	200 V	4 mA
- 6 V	250 V	6 mA

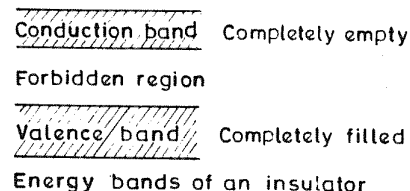
The μ , R_p and g_m values of the valve are

- (a) $25, 50 \times 10^3 \Omega$ and 5×10^{-3} mho
 (b) $50, 75 \times 10^3 \Omega$ and 5×10^{-3} mho
 (c) $25, 25 \times 10^3 \Omega$ and 1×10^{-3} mho
 (d) $50, 50 \times 10^3 \Omega$ and 1×10^{-3} mho

B. Solid State Electronics

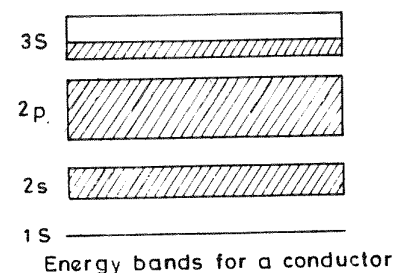
Classification of Solids

(a) **Insulators** We can understand the behaviour of an insulator using the band theory of solids. The allowed energy bands of a solid are shown in the figure. Electrons in a solid occupy the lowest allowed levels. The number of electrons is such that they fill only the lowest energy levels, i.e. the next allowed energy band is completely empty. Many solids, e.g. diamond, silicon and germanium, have this property. The topmost completely filled energy band is called the *valence band* and the next band of allowed energy is known as the *conduction band*. In an insulator, the region between the conduction

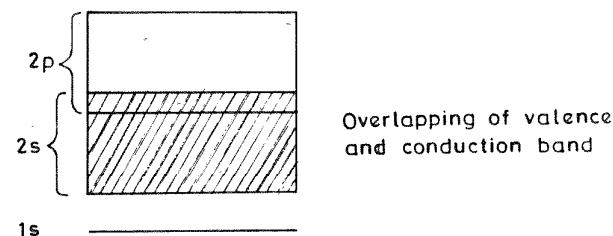


band and the valence band, called the *forbidden region*, is large and it requires a large amount of energy to take electrons from the valence band to the conduction band. In insulators, the valence electrons are bound very tightly to their parent atoms. For example, in glass, the valence band is completely full at 0 K and the energy gap between the valence band and the conduction band is of the order of 10 eV. At very high temperatures, it may be possible that some electrons may cross the forbidden region to provide some feeble conduction.

(b) **Conductors** Consider a solid in which the conduction band is not completely filled. For example, in sodium, one electron from each of the atoms belonging to the 3 s state moves freely in the solid. The valence band in sodium is only half-filled. The electrons in the valence band can be accelerated because they can move freely in the unfilled state itself. A similar state exists in metals like Cu, Ag and Au, and all these are good conductors.



In some solids, the valence band and the conduction band overlap as shown in the figure. In such a solid, even when one band is completely filled, the energy can change in a continuous manner and the solid can conduct electricity.



(c) **Semiconductors** In a solid, if the energy gap between the bottom of the conduction band and the top of the valence band is about 1 eV, then the solid is said to be a semiconductor; pure germanium and silicon are examples of semiconductors. Such a semiconductor is referred to as an *intrinsic semiconductor*. Semiconductors can also be made by adding a small number of impurity atoms to certain substance. Such a semiconductor is referred to as a *extrinsic semiconductor*.

Intrinsic Semiconductors In pure materials, germanium and silicon are the most common semiconductors. The band gaps in germanium and silicon are 0.7 and 1.1 eV respectively. The conduction takes place because some electrons from the valence band move over to the conduction band due to thermal excitation. When an electric field is applied, the electrons in the conduction band are free to absorb energy as this band is almost empty.

When an electron moves out of the valence band, it leaves a *hole* in the valence band. This hole is positively charged as the electron has left the valence band and the original system of the electron and the core was initially electrically neutral. A neighbouring electron can now move into the hole, leaving a hole at the place from which it moves. Thus, the motion of the electrons can be thought of as a hole moving in the direction opposite to that of the electrons.

There are thus two contributions to the electric current when an external electric field is applied to solid germanium or silicon. Of these, one is due to the motion of electrons in the conduction band and the other due to the

motion of electrons in the valence band. The latter is more conveniently described in terms of the motion of holes. The total current is the sum of the two currents.

Extrinsic Semiconductors The current produced in a semiconductor as described above is not adequate for many useful purposes. The conductivity can, however, be enormously increased by adding a suitable impurity in a very small proportion (1 in 10^6 parts) in the pure semiconductor. Such *doped* semiconductors are known as *extrinsic semiconductors*.

n-type If a small quantity (1 ppm) of a pentavalent impurity like arsenic ($Z = 33$) or antimony ($Z = 51$), having five valence electrons, is introduced in germanium, it replaces an equal number of germanium atoms in the crystal lattice without changing its physical structure. Each of the four out of five valence electrons of the impurity, say the arsenic atom, then enters into covalent bonds with one valence electron of each of the nearest four germanium atoms; the fifth valence electron of arsenic is free to move from one atom to the other in the crystal lattice in a random manner. The impurity is called the *donor impurity* since it donates electrons to the germanium crystal and the crystal is called *n-type germanium*.

p-type When a trivalent impurity like indium ($Z = 49$) or gallium ($Z = 31$) is added to a germanium crystal, the situation is quite different. However, the net result of increasing the conductivity is similar to what is achieved in an *n-type* semiconductor. Since an indium atom has only three electrons in its outer orbit, it borrows one electron from the nearby germanium band so as to complete four covalent bonds with the nearest germanium atoms. As soon as the borrowed electron moves from its place, it creates a hole at the place which it originally occupied. This hole can again be filled by another electron, simultaneously producing another hole, and the process can go on. Thus a hole can move freely through the crystal lattice. As the impurity accepts an electron from the nearby germanium bond, it is said to be an *acceptor impurity* and the crystal is called a *p-type* crystal.

Salient features of n-type semiconductor:

- (i) The donor atom, after giving its one valence electron, becomes a positively charged ion, but cannot take part in the conduction as it is tightly bound to the crystal lattice.
- (ii) There is an excess of electrons and the number of holes is small. The majority carriers are electrons and the minority carriers are holes.
- (iii) Though it contains an excess of electrons, it is electrically neutral on the whole. Although a neutral pentavalent impurity is added, there is no addition of either negative or positive charges.
- (iv) When an electric field is applied, the excess electrons donated by the impurity atoms will travel towards the positive electrode.

Salient features of p-type semiconductor:

- (i) In such semiconductors, the majority carriers are positive holes while the minority carriers are electrons.
- (ii) This also remains electrically neutral.
- (iii) When an electric field is applied across a p-type semiconductor, electrons move towards the positive electrode while holes move towards the negative electrode.

Conductivity of Semiconductor Materials

On the basis of the free-electron theory, the conductivity σ of a metal is given by $\sigma = n \cdot e \mu_e$ where n is the number of free electrons per unit volume of the conductor (i.e. electron density), e is the electronic charge and μ_e is electron mobility. Let n be the electron density in the conduction band per unit volume, p the positive hole density in the valence band, and μ_e and μ_h be the electron and hole mobility respectively. Then the conductivity of the semiconductor due to electrons is

$$\sigma_e = ne\mu_e$$

and that due to holes is

$$\sigma_p = pe\mu_h$$

Hence, the total conductivity is given by

$$\sigma = \sigma_e + \sigma_h = e(n\mu_e + p\mu_h)$$

Conductivity of Intrinsic Semiconductor

In the case of an intrinsic semiconductor, the number of conduction electrons is equal to the number of holes, i.e.

$$n = p = n_i \Rightarrow \sigma_i = en_i(\mu_e + \mu_h)$$

Conductivity of *n-type* semiconductor = $\sigma_n = e(n\mu_e + p\mu_h)$

But in an *n-type* semiconductor, $n \gg p$ or $n\mu_e \gg p\mu_h$

$$\therefore \sigma_n = en_d\mu_e$$

(Here the electron concentration is represented by n_d , i.e. the concentration of donor atoms.)

Similarly, for a *p-type* semiconductor,

$$\sigma_p = en_a\mu_h$$

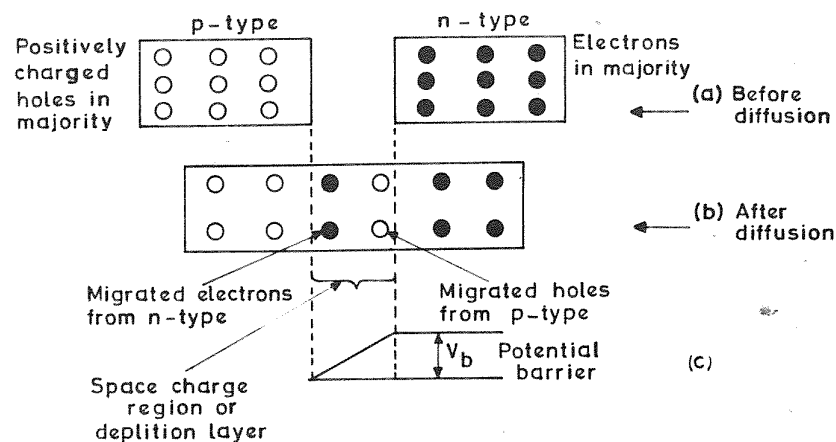
(Here the hole concentration is represented by n_a , i.e. the concentration of acceptor atoms.)

(Mobility μ is defined as drift velocity per unit electric field; $\mu = V/E$.)

The Junction Diode

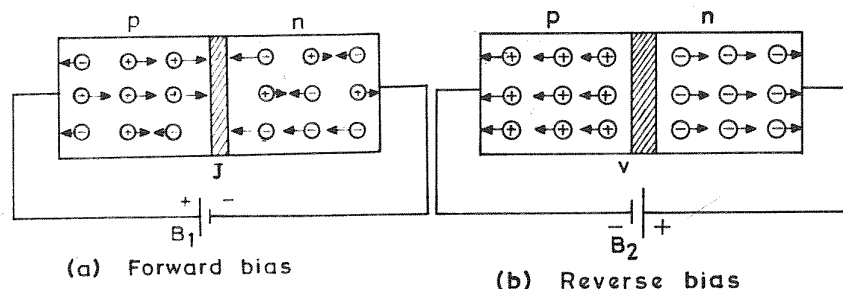
A *p-type* crystal can be joined to an *n-type* crystal by a suitable technique.

When this is done, a thin layer is formed at the junction. This arrangement is known as a junction diode. The figures here illustrate the functioning of such a diode.



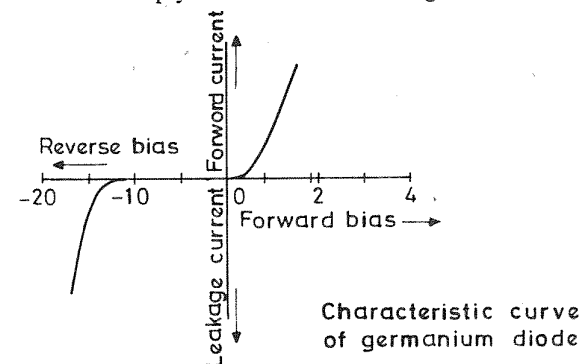
Due to high concentration, some free electrons in the n -type crystal diffuse through the junction to the p -region and, likewise, some holes in the p -type crystal diffuse into the n -region. The transfer of these charges to either region disturbs the neutrality of that region, making the p -region negative and the n -region positive. Consequently, an electric field is established at the junction as if the positive terminal of a battery were connected to the n -type crystal and the negative terminal to the p -type crystal. The potential thus produced is called *potential barrier* (V_b) and is of the order of 0.3 to 0.7 V, depending upon the extent of the impurity added to the crystal. This barrier voltage opposes any further movement of charges through the junction, and must be overcome in order that some current may flow through the junction. When no external voltage is applied to the diode, it is called *unbiased*.

Forward Bias and Reverse Bias If an external dc voltage is applied to a diode through an external battery B_1 , connecting its positive terminal to the p -end and the negative terminal to the n -end as shown in Fig. (a), it will oppose



the barrier voltage. In case it exceeds the barrier voltage, the free electron in the n -type crystal and the holes in the p -type crystal will migrate from one end of the crystal across the junction to the other end. Consequently, a current, called the *forward current*, will start flowing in the crystal. The external voltage that forces the free charges in germanium to move towards the junction is called *forward bias*, and results in a lowered resistance at the junction. The typical characteristic for a junction diode with forward bias is shown in the figure. Note that practically no current flows until the barrier voltage is overcome. The curve then has a linear rise and the current increases with the forward bias voltage.

Let us now reverse the polarity (shown in (b)) with reverse bias, i.e. let us attach the negative terminal of battery B_2 to the p -type crystal, and the positive terminal to the n -type crystal. This will increase the barrier voltage and thus prevent the flow of forward current in the crystal, resulting in high resistance at the junction. Such a voltage is called *reverse biased voltage*. However, a few minority charge carriers (holes in the n -type and electrons in the p -type crystal) on being accelerated by this reverse biased voltage will cross the junction and will constitute a current in the opposite direction. This is known as *leakage current*. When a voltage of about 25 V is applied to the crystal, the crystal gets heated excessively and destroys the covalent bond structure in germanium, and the reverse current rises sharply. The breakdown voltage is called Zener voltage.

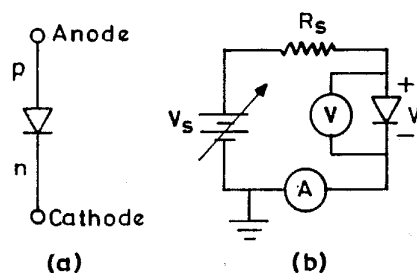


In the characteristic curve for the germanium diode, we observe that, near the breakdown region, large changes in diode current produce only small changes in diode voltage. For this reason, such a diode is used as a constant-voltage device. Diodes used in this manner are called Zener diodes.

Minority-Carrier Current or Saturation Current Even after the depletion layer adjusts to its new width, there exists a very small current because of the minority carriers. The reverse current caused by the minority carriers is called *saturation current*. Increasing the reverse voltage will not increase the number of thermally created minority carriers. Only an increase in temperature can increase the saturation current. A silicon diode has a much smaller saturation current at a particular temperature than a germanium diode. This is one of the reasons why silicon dominates the field of semiconductor devices.

572 Problems in Physics

The schematic symbol for a rectifier diode is shown in Fig. (a). The p -side is called the anode and the n -side the cathode. Forward bias can produce a large electron flow from the n -side to the p -side. This is equivalent to having a large conventional flow from the p -side to the n -side. The arrow on the diode symbol points along the direction of the conventional current. Figure (b) shows a circuit that can be set in the laboratory to measure the current and voltage of a diode.



Equation For the Load Line Suppose the forward bias voltage is V_s , and let us call the current limiting resistor R_s . The voltage from the right end of the resistor to the ground is V , the anode voltage, and the voltage from the left end of the resistor to the ground is V_s , the source voltage. Therefore, the difference of potential across the resistor is $V_s - V$, and the current is

$$I = \frac{V_s - V}{R_s}$$

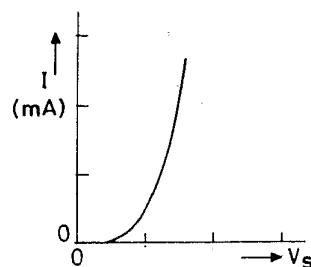
Current in Forward-Biased Diode

$$I = I_0 \left[\exp \left(\frac{eV}{KT} \right) - 1 \right], \text{ where } I_0 \text{ is the saturation current, } K \text{ is}$$

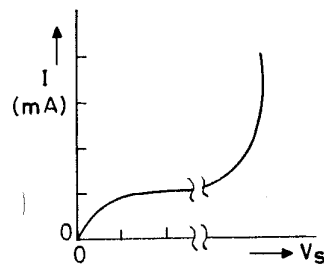
Boltzmann constant and T is junction temperature in $^{\circ}\text{K}$.

This equation shows that the current increases exponentially.

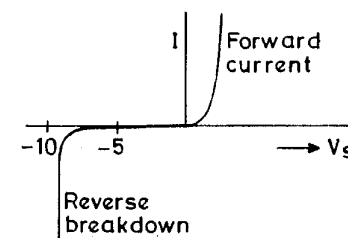
The graph between the external voltage V_s and the forward current is shown in Fig. (a). The current in the reverse biased diode is shown in Fig. (b). Here V_s is the reverse-biased voltage. The complete I - V characteristics of a p - n diode are shown in Fig. (c).



(a) Current in forward bias



(b) Reverse bias with breakdown region



(c)

 Complete I - V characteristic

ILLUSTRATIONS

1. Find the mobility of electrons in copper assuming that each atom contributes one free electron for conduction (given that the resistivity of copper is $1.7 \times 10^{-6} \Omega \text{ cm}$, atomic weight is 63.54, and density is 8.96 g/cc).

Solution

$$\begin{aligned} \text{Number of free electrons per unit volume} &= \frac{N_0 \times \rho}{\text{at. wt}} \\ &= \frac{6.025 \times 10^{23} \times 8.96}{63.54} = 0.85 \times 10^{23} = 8.5 \times 10^{22} \end{aligned}$$

The mobility of the electron is given by

$$\mu = \frac{\sigma}{ne}, \quad \text{where } \sigma \text{ is the conductivity.}$$

$$\text{Resistivity} = \frac{1}{\sigma}$$

Therefore, mobility of electrons is

$$\begin{aligned} \mu &= \frac{1}{1.7 \times 10^{-6} \times 8.5 \times 10^{22} \times 1.6 \times 10^{-19}} = 0.0433 \times 10^3 \\ &= 433.25 \text{ cm}^2/\text{V.s} \end{aligned}$$

2. The following data is given for pure silicon crystal

Electron mobility	$= 0.135 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$
Hole mobility	$= 0.048 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$
Length of the crystal	$= 0.1 \text{ m}$
Area of crystal	$= 1 \times 10^{-4} \text{ m}^2$
Voltage applied across it	$= 2 \text{ V}$
Intrinsic conductivity of the crystal	$= 4.4 \times 10^{-4} (\text{Ohm. m})^{-1}$
Intrinsic carrier concentration	$= 1.5 \times 10^{16} \text{ m}^{-3}$

Find the electron and hole currents. Also find the magnitude of total current.

Solution

$$\text{Electron drift velocity } v_e = \mu_e \times E = 0.135 \times \frac{2}{0.1} = 2.7 \text{ m/s}$$

$$\text{Hole drift velocity } v_h = \mu_h \times E$$

$$= 0.048 \times \frac{2}{0.1} = 0.96 \text{ m/s}$$

$$I_e = n_e \times A \times e \times v_e$$

$$= 1.5 \times 10^{16} \times 1 \times 10^{-4} \times 1.6 \times 10^{-19} \times 2.7 = 6.48 \times 10^{-7} \text{ A}$$

$$I_h = 1.5 \times 10^{16} \times 1 \times 10^{-4} \times 1.6 \times 10^{-19} \times 0.96 = 2.3 \times 10^{-7} \text{ A}$$

$$I = I_e + I_h = 8.78 \times 10^{-7} \text{ A}$$

3. The concentration of carriers in pure silicon at room temperature is $1.6 \times 10^{16} \text{ m}^{-3}$. If the electron mobility is $0.150 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$ and the hole mobility is $0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, find the conductivity of silicon.

Solution

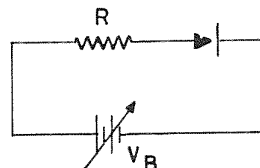
In the case of a pure semiconductor, conductivity is given by

$$\sigma_i = en_i (\mu_e + \mu_h)$$

$$= 1.6 \times 10^{-19} \times 1.6 \times 10^{16} (0.150 + 0.05)$$

$$= 0.512 \times 10^{-3} (\Omega \text{ m})^{-1}$$

4. A silicon diode requires a minimum current of 1 mA to be above the knee point (0.7 V) of its I - V characteristics. Assume that the voltage across the diode is independent of current above the knee point.



(i) If $V_B = 8 \text{ V}$, what should be the maximum value of R so that the voltage is above the knee point?

(ii) If $V_B = 8 \text{ V}$, what should be the value of R to establish a current of 6 mA in the circuit?

(iii) Calculate the power dissipated in the resistance R and in the diode when a current of 6 mA flows in the circuit at $V_B = 8 \text{ V}$?

Solution

$$(a) \quad \frac{8 \text{ V} - 0.7 \text{ V}}{R} = 1 \text{ mA or } R = \frac{7.3 \text{ V}}{1 \text{ mA}} = 7.3 \times 10^3 = 7.3 \text{ k}\Omega$$

$$(b) \quad R = \frac{7.3 \text{ V}}{6 \times 10^{-3}} = \frac{7.3}{6} \times 10^3 = 1.216 \times 10^3 = 1216 \Omega$$

$$(c) \quad \begin{aligned} \text{Total power dissipated} &= I \times V = 6 \text{ mA} \times 8 \text{ V} \\ &= 6 \times 10^{-3} \text{ A} \times 8 \text{ V} \\ &= 48 \times 10^{-3} \text{ W} = 48 \text{ mW} \end{aligned}$$

$$\text{Power dissipated in } R = I^2 R = 36 \times 10^{-6} \times 1216 = 43.78 \text{ mW}$$

$$\text{Power dissipated in diode} = 4.2 \text{ mW}$$

5. In a p - n junction diode, the current can be expressed as

$$I = I_0 \left[\exp \left(\frac{eV}{kT} \right) - 1 \right]$$

where I_0 is called the reverse saturation current, V is the voltage across the diode and is positive for forward bias and negative for reverse bias, I is the current through the diode, k is the Boltzmann constant $= 8.6 \times 10^{-5} \text{ eV/K}$, and T is the absolute temperature. If, for a diode, $I_0 = 52 \times 10^{-12} \text{ A}$ and $T = 300 \text{ K}$,

(i) Find the forward current at forward voltage of 0.6 V.

(ii) What will be the increase in the current if the voltage across the diode is increased to 0.7 V?

(iii) What will be the current if the reverse bias voltage changes from 1 to 2 V?

Solution

$$(i) \quad I = I_0 \left[\exp \left(\frac{0.6}{kT} \right) - 1 \right] \quad (kT = 8.6 \times 10^{-5} \times 300 = 0.0258 \text{ eV})$$

$$= I_0 \left[\exp \left(\frac{0.6}{0.0258} \right) - 1 \right]$$

$$= 5 \times 10^{-12} (1.26 \times 10^{10} - 1) = 5 \times 1.26 \times 10^{-2}$$

$$= 6.3 \times 10^{-2} \text{ A}$$

(ii) In the second case, when the voltage across the diode is increased to 0.7 V

$$I = I_0 \left[\exp \left(\frac{0.7}{kT} \right) - 1 \right] = 5 \times 10^{-12} (6.06 \times 10^{11})$$

$$= 30.3 \times 10^{-1} = 3.0333 \text{ A}$$

$\therefore$ Increase in current due to this change in diode voltage

$$= 3.03 - 6.3 \times 10^{-2} \text{ A} = 3.03 - 0.063$$

$$= 2.967 \text{ A}$$

(iii) $V = -2$ for reverse bias

$$I(-2) = I_0 \left[\exp \left(\frac{-2e}{kT} \right) - 1 \right]$$

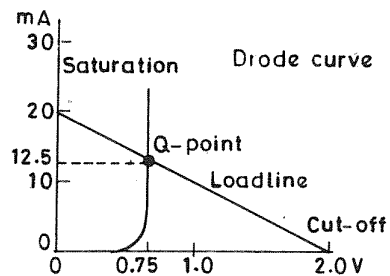
$$I(-2) = I_0 \left[\exp \left(\frac{-2e}{0.0258} \right) - 1 \right] \approx -I_0$$

Similarly, for $I(-1)$, the current reaches the saturation value of $-I_0$ very quickly when $V < 0$.

6. The source voltage in a diode is 2 V and the current limiting resistance is $100\ \Omega$. What is the current at the upper end of the load line and what is the Q -point?

Solution

To draw the load line in the given I - V characteristic curve of a diode, we use the linear relation between the current and voltage



$$I = \frac{V_S - V}{R_S} = \frac{2 - V}{100}$$

This relation, when plotted, gives a straight line.

If $V = 0$, then

$$I = \frac{2\text{ V} - 0\text{ V}}{100} = 20\text{ mA}$$

We plot this point on the vertical axis. This point is called the *saturation* point because it represents maximum current.

To obtain another point, let $V = 2\text{ V}$. Then,

$$I = \frac{2\text{ V} - 2\text{ V}}{100} = 0$$

We plot this point ($I = 0$, $V = 2\text{ V}$) on the horizontal axis (this point is called the *cut-off* point.)

The maximum current, as seen in the graph, is therefore 20 mA.

The point of intersection of the load line and the diode curve gives the *operating* point or Q -point.

The operating point is 12.5 mA.

EXERCISES

1*. For a sample of a pure germanium plate, the following data is given.

Plate area = 0.0001 m^2 and thickness = 0.3 mm

Voltage applied across the faces = 2 V

Concentration of free electrons in Ge = $2 \times 10^{19}/\text{m}^3$

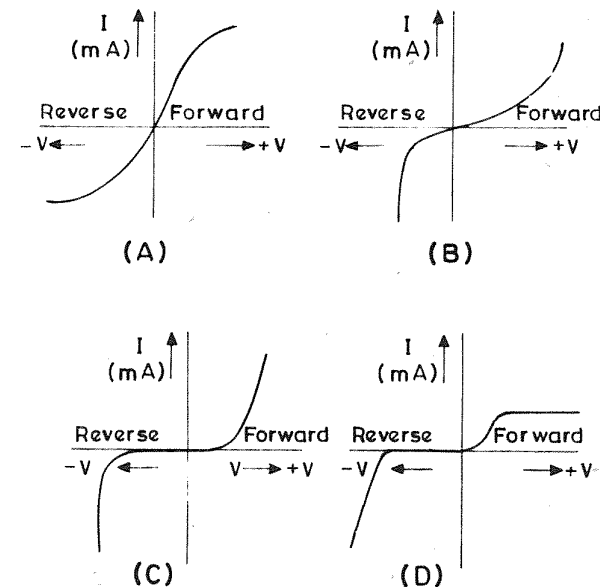
Mobility of electrons = $0.36\text{ m}^2/\text{V} \cdot \text{s}$

Mobility of holes = $0.17\text{ m}^2/\text{V} \cdot \text{s}$

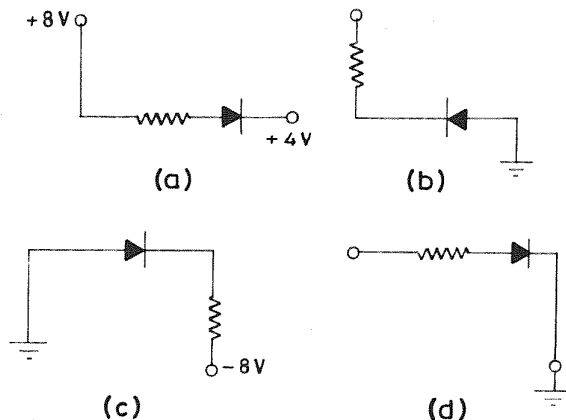
The current produced in the specimen is

- (a) 3.11 A (b) 0.311 A
(c) 1.13 A (d) 0.113 A

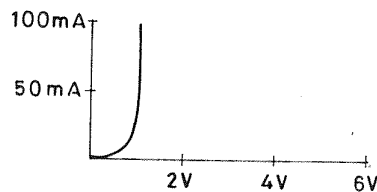
2. In case of a reverse bias applied to a p - n junction diode, mark the statement that is incorrect.
- in a reverse bias the positive of battery is connected to n -type crystal and negative is connected to the p -type crystal
 - the effect of reverse bias is to decrease the potential barrier
 - when the junction is reverse-biased, the electrons in the n -type semiconductor and holes in the p -type semiconductor are attracted away from the junction under the action of the applied voltage
 - there is no recombination of electron-hole pairs and the junction has high resistance
3. In case of a forward bias applied to a p - n junction diode, mark the correct statement.
- in a forward bias, the positive terminal of a battery is connected to the n -type semiconductor and the negative terminal to the p -type semiconductor
 - the effect of forward bias is to increase the potential barrier and decrease the current
 - in this situation, the resistance of the junction is very high
 - the holes from the p -type semiconductor are repelled by the positive battery terminal towards the junction and simultaneously, the electrons in the n -type semiconductor are repelled by the negative battery terminal towards the junction.
4. The diode current vs forward and reverse bias are plotted and shown in the figures below. Mark the one which is correct.



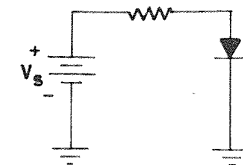
5. Semiconductors containing donor atoms and free electrons belong to the type
 (a) n (b) p
 (c) hole (d) none of the above
6. In an n -type semiconductor, the majority carriers are
 (a) positive holes (b) electrons
 (c) both in same proportion (d) none of these
- 7*. The electron concentration and hole concentration in a semiconductor are $8 \times 10^{19}/\text{m}^3$ and $5 \times 10^{18}/\text{m}^3$ respectively. If the electron mobility is $2.3 \text{ m}^2/\text{V} \cdot \text{s}$ and the hole mobility is $0.01 \text{ m}^2/\text{V} \cdot \text{s}$, then the semiconductor is
 (a) n -type and its resistivity is $0.33 \Omega \text{ m}$
 (b) p -type and its resistivity is $0.033 \Omega \text{ m}$
 (c) n -type and its resistivity is $0.033 \Omega \text{ m}$
 (d) p -type and its resistivity is $3.30 \Omega \text{ m}$
8. In the following figures, the diodes are either forward-biased or reverse-biased. Choose the correct statement.



- (a) (a) and (b) are forward biased, and (c) and (d) are reverse biased
 (b) (a) and (d) are forward biased, and (b) and (c) are reverse biased
 (c) (a) and (c) are forward biased, and (b) and (d) are reverse biased
 (d) (b) and (c) are forward biased, and (a) and (d) are reverse biased
9. When forward biased in a particular circuit, a diode has a current of 50 mA. When reverse biased, the current drops to 20 mA. The ratio of forward to reverse current is
 (a) 2.6×10^6 (b) 6.2×10^5
 (c) 2.6×10^8 (d) 2.6×10^4
10. A source voltage of 8 V drives a diode through a current-limiting resistor of 100Ω . If the diode has a characteristic as shown in the figure, the current at the upper end of the load line is
 (a) 40 mA
 (b) 100 mA
 (c) 80 mA
 (d) 20 mA

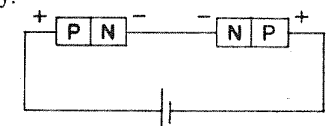
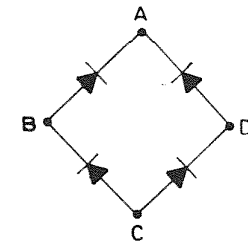


11. The source voltage is 9 V and the source resistance is $1 \text{ k}\Omega$. The current through the diode is
 (a) 3.8 mA
 (b) 4.2 mA
 (c) 0.833 mA
 (d) 8.3 mA

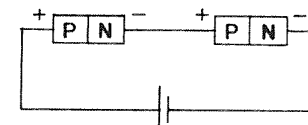


Fill in the blanks:

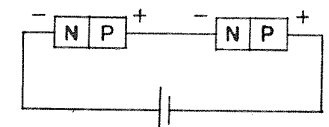
12. If the circuit shown here is to act as a full-wave rectifier, the ac input should be connected across _____ and _____ and the dc output will appear across _____ and _____.
13. _____ biasing of the p - n junction offers high resistance to current flow across the junction. This biasing is obtained by connecting the p -side to the _____ terminal of the battery.
14. Two identical p - n junctions may be connected in series with a battery in three ways. The potential drops across the two p - n junctions are equal in



Circuit 1



Circuit 2



Circuit 3

- (a) circuit 1 and circuit 2 (b) circuit 2 and circuit 3
 (c) circuit 3 and circuit 1 (d) circuit 1 only
15. A piece of copper and another of germanium are cooled from room temperature to 80 K. The resistance of
 (a) each of them increases
 (b) each of them decreases
 (c) copper increases and germanium decreases
 (d) copper decreases and germanium increases
- 16*. The intrinsic conductivity of germanium at 27°C is $2.13 \Omega^{-1}\text{m}^{-1}$ and the mobilities of electrons and holes are 0.38 and $0.18 \text{ m}^2/\text{V} \cdot \text{s}$ respectively. The density of carriers is
 (a) $0.2378 \times 10^{19}/\text{m}^3$ (b) $3.287 \times 10^{19}/\text{m}^3$
 (c) $7.832 \times 10^{19}/\text{m}^3$ (d) $8.472 \times 10^{18}/\text{m}^3$
- 17*. An n -type semiconductor is to have a resistivity of $0.1 \Omega \cdot \text{m}$. The number of donor atoms which must be added to achieve this is ($\mu_e = 0.05 \text{ m}^2/\text{V} \cdot \text{s}$)
 (a) 12.5×10^{23} (b) 1.25×10^{23}
 (c) 12.5×10^{20} (d) 1.25×10^{17}
18. In germanium, the energy gap is about 0.75 eV . The wavelength of light which germanium starts absorbing is
 (a) $5000 \text{ \AA}$ (b) $16500 \text{ \AA}$
 (c) $20000 \text{ \AA}$ (d) $51600 \text{ \AA}$

Answers

A. Thermionic Electronics

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (d) | 2. (a) | 3. (b) | 4. (b) | 5. (b) |
| 6. (a) | 7. (d) | 8. (b) | 9. (a) | 10. (c) |

B. Solid State Electronics

- | | | | | |
|---------|----------------------|----------------------|---------|---------|
| 1. (c) | 2. (b) | 3. (d) | 4. (c) | 5. (a) |
| 6. (a) | 7. (c) | 8. (c) | 9. (a) | 10. (c) |
| 11. (d) | 12. b and d, a and d | 13. Reverse Negative | | |
| 14. (b) | 15. (d) | 16. (a) | 17. (c) | 18. (b) |

Note: Problems marked with an asterisk should take about five minutes to solve, and the rest should not take more than three minutes.

Self-Assessment

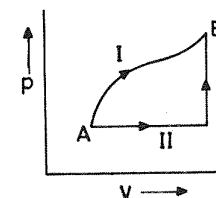
Now that you have had sufficient practice in solving problems, you are ready to attempt these test papers in a given time and grade yourself. The answers are given at the end of the book. In these sets, more than one answer may be correct.

TEST PAPER I

Max. Marks 100

Time: 1 hr

- Two plates of the same thickness form a composite insulating plate, the temperature on one side of which is 0°C and that on the other side 60°C . If the ratio of thermal conductivities is 3 : 1 and the more conducting plate faces 0°C , then the temperature of the interface is
 (a) 45°C (b) 40°C
 (c) 20°C (d) 15°C
- There are two solar energy collectors made of thin aluminium sheet. They are flat and uncoated. Collector A is rectangular, 20 cm long and 15 cm wide. Collector B is circular, having a 20 cm diameter. If they are kept in the sun for an hour, then
 (a) both collectors will show nearly the same temperature
 (b) collector A will show a higher temperature than B
 (c) collector B will show a higher temperature than A
 (d) there will be no rise in temperature of the two collectors as they are uncoated.
- A certain mass of gas is taken from an initial thermodynamic state A to another state B by process I and II. In process I, the gas does 5 J of work and absorbs 4 J of heat energy. In process II, the gas absorbs 5 J of heat. The work done by the gas in process II is
 (a) + 6 J (b) - 6 J
 (c) + 4 J (d) - 4 J
- Two vessels A and B contain water at temperatures T_A and T_B of 10°C and 2°C respectively. If water in both the vessels is compressed adiabatically, then T_A and T_B will



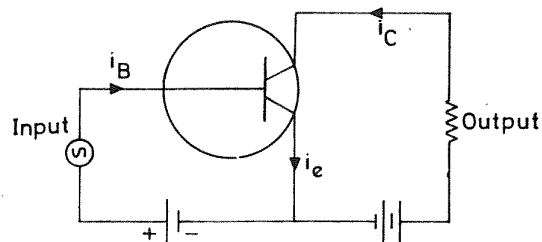
- (a) increase and decrease respectively
 (b) decrease and increase respectively
 (c) both increase
 (d) both decrease
5. According to the kinetic theory of gases, the pressure of a gas is given by the expression $p = 2E/3V$ where E and V are the energy and volume respectively of the gas molecules. In this expression, the energy E is the total
- (a) mechanical energy of the molecules
 (b) translational kinetic energy of the molecules
 (c) rotational and translational kinetic energy of the molecules
 (d) none of the above
6. The temperature, in kelvin, at which the average speed of hydrogen molecules will be the same as that of nitrogen molecules at 35°C , is
- (a) 22 (b) 42
 (c) 295 (d) 495
7. The order of magnitude of the mean free path of a diatomic molecule at STP is
- (a) 1 cm (b) 10^{-2} cm
 (c) 10^{-5} cm (d) 10^{-8} cm
8. A string vibrates in four segments at a frequency of 400 Hz. The frequency that will cause it to vibrate in eight segments is
- (a) 600 Hz (b) 800 Hz
 (c) 1000 Hz (d) 1200 Hz
9. The equation of a transverse wave is given by
- $$y = 10 \sin (0.5\pi x + 4.0\pi t)$$

The conclusions are:

- (a) the amplitude and wavelength are 10 units and 4 units respectively
 (b) the amplitude and frequency are 10 units and 2 Hz respectively
 (c) the velocity is 8 units and the wave travels in the positive x -direction
 (d) the velocity is 10 units but its maximum velocity is 126 units
10. An object is located at the bottom of a vessel containing two different liquids which do not mix with each other. The object is viewed vertically from above. The lower and upper liquids are respectively of depths h_1 and h_2 and their refractive indices are μ_1 and μ_2 . The object appears to be located, from the top surface, at a depth of
- (a) $\frac{h_2 - h_1}{\mu_1 + \mu_2}$ (b) $\frac{h_1}{\mu_1} + \frac{h_2}{\mu_2}$
 (c) $\frac{h_2}{\mu_2} - \frac{h_1}{\mu_1}$ (d) $\frac{h_1 + h_2}{\mu_1 \mu_2}$
11. Two prisms of crown and flint glass are to be combined so as to give zero mean deviation. The refractive indices of crown and flint glass are 1.530 and 1.795 respectively. If the crown glass prism has a refracting angle of 9° , the angle of the flint glass prism is
- (a) 7° (b) 4°
 (c) 11° (d) 6°

12. A convex lens of refractive index 1.52 and focal length 10 cm is placed in a liquid of refractive index 1.68. The lens will behave
- (a) as a convex lens of focal length 54.6 cm
 (b) as a concave lens of focal length 54.6 cm
 (c) as a concave lens of focal length 32.4 cm
 (d) as a convex lens of focal length 32.4 cm
13. Farad/metre is the unit of
- (a) electric field (b) electric potential
 (c) permittivity (d) permeability
14. Two parallel wires of radii a are separated by a distance d , and carry equal currents in opposite directions. The inductance of length l of such a pair of wires is (neglect the flux within the wires; μ_0 is the permeability of free space).
- (a) $\frac{\mu_0 l}{2\pi} \ln \frac{d}{a}$ (b) $\frac{\mu_0 l}{\pi} \ln \frac{d-a}{a}$
 (c) $\frac{\mu_0 l}{4\pi} \ln \frac{d-a}{a}$ (d) $\frac{\mu_0 l}{2\pi} \ln \frac{d-a}{a}$
15. An alternating current is given by $I = I_0 \sin \omega t$. The average value of the current in the first half-cycle starting with $t = 0$ is
- (a) zero (b) $\frac{\sqrt{2} I_0}{\pi}$
 (c) $\frac{2I_0}{\pi}$ (d) $\frac{\pi I_0}{2}$
16. The wavelength of a radio wave tuned circuit containing an inductance of $(L/2\pi)$ H and capacitance of $(C/2\pi)$ F is given by
- (a) $3 \times 10^8 \sqrt{LC}$ m (b) $\frac{3 \times 10^8}{2\pi \sqrt{LC}}$ m
 (c) $\frac{3 \times 10^8}{\sqrt{LC}}$ m (d) $\frac{\sqrt{LC}}{3 \times 10^8}$ m
17. If B = magnetic induction and R = radius of the path, the energy of a charged particle of charge q and mass m coming out of a cyclotron is given by
- (a) $qB^2 R^2/2m$ (b) $q^2 B R^2/2m$
 (c) $q^2 B^2 R/2m$ (d) $q^2 B^2 R^2/2m$
18. When an electron jumps from an orbit with $n = 1$ to another with $n = 2$ in a singly ionised helium atom, the change of orbital angular momentum of the electron is
- (a) $h/2\pi$ (b) $h/4\pi$
 (c) $3h/4\pi$ (d) zero
19. A nucleus of ${}^{242}_{94}\text{Pu}$ decays to ${}^{206}_{82}\text{Pb}$ by emitting
- (a) 9α and 12β -particles (b) 9α and 6β -particles
 (c) 6α and 9β -particles (d) 6α and 12β -particles

20. A deuteron
 (a) cannot be disintegrated by photon
 (b) can be disintegrated by photons of 1.02 MeV
 (c) can be disintegrated by photons of 2.22 MeV
 (d) can be disintegrated only by photons of 8 MeV
21. The circuit shown here is that of an



- (a) oscillator (b) modulator
 (c) amplifier (d) rectifier
22. The dimensions of each quantity in list I are given in list II. Match the two lists to select the correct answers and write them in the boxes provided below.

List I

1. Angular momentum
2. Constant of universal gravitation
3. Surface tension
4. Permeability
5. Permittivity

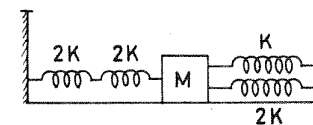
List II

- (a) $M^{-1}L^3T^{-2}$
- (b) $ML^{-1}T^{-1}$
- (c) $ML^{-1}T^{-2}$
- (d) ML^2T^{-1}
- (e) MT^{-2}
- (f) MLQ^{-2}
- (g) $M^{-1}L^{-3}T^2Q^2$

1		2		3	
4		5			

23. A small drop of oil falls through air with a terminal velocity of 4×10^{-4} m/s. Given that the viscosity of air is 1.8×10^{-5} N.s./m², density of oil is 900 kg/m^3 and $g = 10 \text{ m/s}^2$, and neglecting density of air in the calculation, the radius of the drop is
 (a) $0.55 \times 10^{-6} \text{ m}$ (b) $1.9 \times 10^{-6} \text{ m}$
 (c) $19 \times 10^{-6} \text{ m}$ (d) $95 \times 10^{-6} \text{ m}$
24. The velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1m. Neglecting end corrections, the air column in the pipe will resonate for sound of frequency
 (a) 400 Hz (b) 240 Hz
 (c) 320 Hz (d) 80 Hz
25. The energy of most energetic photoelectrons emitted from a metal target depends on the

- (a) intensity of the incident radiation
 (b) wavelength of the incident radiation
 (c) photoelectric work function of the metal
 (d) threshold frequency of the metal
26. The activity of a radioactive sample goes down to about 6% in a time of 2 hours. The half-life of the sample in minutes is about
 (a) 30 (b) 15
 (c) 60 (d) 120
27. A thin conducting spherical shell of radius R is charged with Q coulombs. The potential at its centre and at a distance $R/2$ from the centre respectively are
 (a) $\frac{Q}{4\pi\epsilon_0 R}$ and $\frac{Q}{4\pi\epsilon_0 R}$ (b) 0 and $\frac{Q}{4\pi\epsilon_0 R}$
 (c) $\frac{Q}{4\pi\epsilon_0 R}$ and $\frac{Q}{2\pi\epsilon_0 R}$ (d) $\frac{Q}{2\pi\epsilon_0 R}$ and $\frac{Q}{2\pi\epsilon_0 R}$
28. An insulated vessel containing a small amount of water at 0°C is being evacuated. The evaporation causes a gradual freezing of some of the water present. If the latent heat of fusion of ice is 80 cal/g and latent heat of vaporization of water is 540 cal/g , the percentage of the original amount of water that can be converted to ice in this process is
 (a) 15.2 (b) 72.5
 (c) 87.1 (d) 50
29. The temperature of a body is increased from 27°C to 127°C . The amount of radiation emitted by the body changes by a factor of
 (a) 256/81 (b) 4/3
 (c) 49/16 (d) 181/27
30. A uniform rod of length L , hinged at one end, is held horizontally and released. The angular velocity of the rod when it becomes vertical is
 (a) $\sqrt{5g/L}$ (b) $\sqrt{g/L}$
 (c) $\sqrt{3g/L}$ (d) $\sqrt{g/2L}$
31. A parallel-plate capacitor is charged to a fixed potential and the charging battery is then disconnected. If the plates of the capacitor are now moved farther apart, then
 (a) the capacitance increases
 (b) the voltage across the capacitor increases
 (c) the charge on the capacitor increases
 (d) the energy stored in the capacitor increases
32. Four massless springs of force constant $2K$, $2K$, K and $2K$ are connected to a mass M lying on a smooth horizontal surface as shown in the figure. If the mass M is displaced horizontally through a small distance and released, the system will oscillate with a frequency



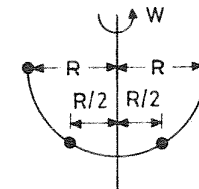
- (a) $\frac{1}{2\pi} \sqrt{\frac{7K}{M}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{4K}{M}}$
 (c) $\frac{1}{2\pi} \sqrt{\frac{K}{3M}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{K}{4M}}$
33. A proton with velocity $\vec{V}$ enters a region of uniform magnetic induction $\vec{B}$ with $\vec{V}$ perpendicular to $\vec{B}$ and describes a circle of radius R . If an α -particle enters this region with the same velocity $\vec{V}$, it describes a circle of radius
- (a) $4R$ (b) $\sqrt{2} R$
 (c) $R/4$ (d) $2R$
34. In a Young's double slit experiment, the fringes are displaced by a distance x when a glass plate of refractive index 1.5 is introduced in the path of one of the beams. When this plate is replaced by another plate of the same thickness, the shift of fringes is $2x$. The refractive index of the second plate is
- (a) 2.5 (b) 1.75
 (c) 2.0 (d) 3.0
35. A double convex lens of refractive index μ_1 is immersed in a liquid of refractive index μ_2 . This lens will act as a
- (a) concave lens if $\mu_1 > \mu_2$ (b) concave lens if $\mu_1 < \mu_2$
 (c) convex lens if $\mu_1 > \mu_2$ (d) convex lens if $\mu_1 < \mu_2$
36. The typical ionization energy of a donor in silicon is
- (a) 0.1 eV (b) 2.0 eV
 (c) 0.001 eV (d) 10.0 eV
37. The energy released in a typical nuclear fusion reaction is approximately
- (a) 25 MeV (b) 1050 MeV
 (c) 800 MeV (d) 200 MeV
38. Oil floating on water shows coloured fringes due to interference of light. The order of magnitude of the thickness of oil for such an effect to be visible is
- (a) 100 Å (b) 1000 Å
 (c) 1 mm (d) 1 cm
39. The rest mass of an electron is m_0 . If it is moving with a velocity $V = 0.8 C$ where C is the velocity of light, its mass becomes
- (a) $m_0 \times 0.8$ (b) $m_0 \times 0.6$
 (c) $m_0/0.8$ (d) $m_0/0.6$
40. The momentum of a photon of wavelength 5000 Å will be
- (a) $1.3 \times 10^{-27} \text{ kg ms}^{-1}$ (b) $1.3 \times 10^{-28} \text{ kg ms}^{-1}$
 (c) $4 \times 10^{-29} \text{ kg ms}^{-1}$ (d) $4 \times 10^{-13} \text{ kg ms}^{-1}$

TEST PAPER II

Max. Marks 100

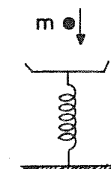
Time: 1 hr

1. Two beads of mass m each can slide freely without friction on a semicircular wire having a moment of inertia I about the vertical axis. If the system rotates initially with angular velocity ω with the beads at a distance R from the axis on either side of it, its angular velocity when the beads are at a distance $R/2$ from the axis is

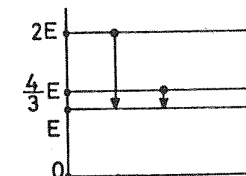


- (a) $\frac{I + mR^2}{I + (mR^2/4)}$ (b) 4ω
 (c) $\frac{(I + (mR^2/4))}{I + mR^2}$ (d) $\frac{I + 2mR^2}{I + (mR^2/2)}$

2. A light elastic spring supports a massless pan P attached to it as shown in the figure. When a piece of clay of mass 0.1 kg falls on the pan from a height of 0.24 m and sticks to it, the compression in the spring is 0.01 m. The height in metres from which the same piece of clay has to fall to create a compression of 0.04 m is

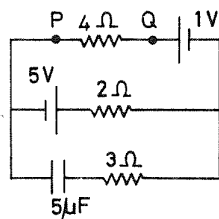


- (a) 3.96 (b) 2.18
 (c) 1.4 (d) 0.80
3. The potential drop across a rectangular resistor is 20 V when a current of I A passes through it. If all the dimensions of the resistor (L , B and t) and also the current passing through it are doubled and if one neglects the temperature effects, if any, the potential drop across the resistor is
- (a) 15 V (b) 40 V
 (c) 20 V (d) 25 V
4. The figure shows energy levels of a certain atom. When the system moves from level $2E$ to E , a photon of wavelength λ is emitted. The wavelength of photon produced during its transition from level $4E/3$ to E level is
- (a) 3λ
 (b) $3\lambda/4$
 (c) $\lambda/4$
 (d) 2λ



original length and the mass suspended is 2 m. The frequency of oscillation now will be

- (a) $2v$ (b) $4v$
 (c) v (d) $v/2$
6. A wire has a linear mass density of 5.0×10^{-3} kg/m and is stretched between two rigid supports under a tension of 50 N. The wire resonates at a frequency of 300 Hz. The next higher frequency at which the wire resonates is 360 Hz. The length of the wire is
 (a) 183 cm (b) 83 cm
 (c) 54 cm (d) 123 cm
7. A hollow metallic tube of length L closed at one end produces resonance with a tuning fork of frequency v . The entire tube is then uniformly heated so that its length changes by l . If the change in velocity V of sound is v , the resonance will now be produced by a tuning fork whose frequency is
 (a) $(V + v)/4 (L + l)$ (b) $(V - v)/4 (L - l)$
 (c) $(V + v)/4 (L - l)$ (d) $(V - v)/4 (L + l)$
8. Fill in the blanks in the following statements
 (a) The binding energy per nucleon for a ${}^4\text{He}$ nucleus is 7.06 MeV. If the energy released during the nuclear reaction ${}^7\text{Li} + p \rightarrow {}^4\text{He} + {}^4\text{He}$ is 17.28 MeV, the binding energy per nucleon for the ${}^7\text{Li}$ nucleus is _____ MeV.
 (b) A thin straight wire of length 0.2 m and mass 10^{-4} kg just floats in a uniform magnetic induction while carrying a current of 10 A. The minimum magnitude of the magnetic induction required for this to happen is _____ tesla.
 (c) A ring of radius R , made of an insulating material, carries a charge Q uniformly distributed along the circumference. If the ring rotates about its normal axis with a constant angular velocity $\vec{\omega}$, the magnetic moment of the ring is _____.
 (d) The internal energy of $2/9$ mole of argon gas at 300 K and at atmospheric pressure is _____ J. (The universal gas constant, $R = 8.32$ J/mole-K.)
 (e) If a convex lens produces an image the size of which is the same as that of the object, the distance between the object and the image is _____ times the focal length of the lens.
 (f) The force on a bar magnet of magnetic moment $\vec{M}$ due to a uniform magnetic field of induction $\vec{B}$ is _____.
 (g) In the given circuit, the potential difference at the points P and Q, $V_P - V_Q =$ _____ volts in the steady state.
9. In column I are given five physical quantities numbered 1



to 5. Choose the appropriate matching dimensions of these quantities from column II and write your answer in the boxes provided below.

I	II
1. Curie	(a) MLT^{-2}
2. Light year	(b) M
3. Dielectric strength	(c) dimensionless
4. Atomic weight	(d) T
5. Decibel	(e) ML^2T^{-2}
	(f) MT^{-3}
	(g) T^{-1}
	(h) L
	(i) $\text{MLT}^{-3}\text{T}^{-1}$
	(j) LT^{-1}

I	II	I	II	I	II	I	II	I	II
1		2		3		4		5	

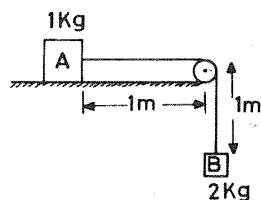
10. Two travelling waves

$$y_1 = A \sin K(x - Vt) \text{ and } y_2 = A \sin K(x + Vt)$$

are superposed on a string. The distance between the adjacent nodes is

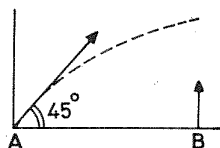
- (a) π/K (b) $2\pi/K$
 (c) Vt/π (d) $Vt/2\pi$
11. The time taken by a particle executing SHM of period T to move from the mean position to half the maximum displacement is
 (a) $T/8$ (b) $T/12$
 (c) $T/4$ (d) $T/2$
12. A wire of uniform cross-sectional area A and Young's Modulus Y is stretched within elastic limits. If S is the stress in the wire, the elastic energy stored in the wire in terms of the given parameters is
 (a) $\frac{S^2}{2Y}$ (b) $\frac{S^2}{Y}$
 (c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$
13. The impurity atoms with which pure silicon should be doped to make it a p -type semiconductor are those of
 (a) boron (b) antimony
 (c) aluminium (d) phosphorus
14. If the dimension of length is expressed as $G^x C^y h^z$, where G , C and h are the universal gravitational constant, speed of light and Planck's constant respectively, then
 (a) $x = \frac{1}{2}, y = \frac{1}{2}$ (b) $x = \frac{1}{2}, z = \frac{1}{2}$
 (c) $y = \frac{1}{2}, z = \frac{3}{2}$ (d) $y = -\frac{3}{2}, z = \frac{1}{2}$

15. A block A of mass 1 kg and another block B of mass 2 kg are connected by a uniform massless string of length 2 m. Block A rests on a smooth horizontal table and the rope passes over a smooth light pulley fixed at the end of the table.



Block B hangs as shown in the figure. The system is released from a situation when block A is 1 m from the pulley. The block A will hit the pulley with a speed of

- (a) 1.5 m/s (b) 2.5 m/s
(c) 3.6 m/s (d) 4.5 m/s
16. A ball (A) is projected from a point A with a speed of 7 m/s at an angle of 45° with the horizontal. Another ball (B) is projected vertically upward from a point B which lies in the same horizontal plane as A. If the ball B hits A when it is moving horizontally, the distance AB is equal to



- (a) 0.5 m (b) 1.5 m
(c) 2.5 m (d) 5 m
17. Two waves of unequal intensity interfere to give interference fringes. The maximum intensity is four times the minimum intensity in the fringe pattern. The ratio of the intensity of the less intense wave to that of the more intense wave is
- (a) 1 : 1 (b) 1 : 2
(c) 1 : 4 (d) 1 : 9
18. Two transparent media A and B are separated by an interface. Light travelling in medium A is incident on the interface. The part reflected is found to have undergone a sudden phase change of π . If the speed of light in medium A is 2.5×10^8 m/s, that in medium B may be
- (a) 1.8×10^8 m/s (b) 2.2×10^8 m/s
(c) 2.6×10^8 m/s (d) 2.8×10^8 m/s
19. Two coherent sources of light S_1 and S_2 are placed at a small separation. The intensity at a point on the perpendicular bisector of the line S_1S_2 and far away from it is zero.
- (a) the two sources are in phase
(b) the two sources emit light of the same intensity
(c) the two sources emit light of the same wavelength
(d) there is a phase difference of π between the two sources.
20. Two of the natural frequencies of an organ pipe are 180 Hz and 240 Hz. The organ pipe
- (a) is necessarily a closed one

- (b) is necessarily an open one
(c) must have a fundamental frequency of 60 Hz
(d) may have a fundamental frequency less than 60 Hz
21. Consider a mixture of several ideal gases in thermal equilibrium. The quantities which have the same value for all molecules are
- (a) average speed
(b) rms speed
(c) average magnitude of momentum
(d) average translational kinetic energy
22. A battery of emf 60 V is connected across a parallel combination of a $8\text{-}\Omega$ resistor and a thick wire of negligible resistance. If the current observed is 6 A, the internal resistance of the battery is
- (a) $2\text{ }\Omega$ (b) $4\text{ }\Omega$
(c) $6\text{ }\Omega$ (d) $10\text{ }\Omega$
23. n moles of an ideal gas undergo a process in which the temperature changes with volume as $T = KV^2$. The work done by the gas as the volume changes from V_0 to $2V_0$ is
- (a) $3nRKV_0^2$ (b) $\frac{5}{2}nRKV_0^2$
(c) $\frac{3nRKV_0^2}{2}$ (d) zero
24. A charge Q is circulating with a constant speed V in a semicircular wire loop of radius R . The magnetic moment of the loop is
- (a) $\left(\frac{\pi}{\pi+2}\right)QVR$ (b) $\frac{1}{2}QVR$
(c) $\left[\frac{\pi}{2(\pi+2)}\right]QVR$ (d) QVR
25. The magnetic field B due to a single electron moving with a velocity V , at a distance r on the normal to its trajectory, is proportional to
- (a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$
(c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$
26. Three point objects, each carrying a small charge $+q$, are joined pairwise through three identical springs each of force constant K and unstretched length L (the charges are at the vertices of the triangle formed by the spring). In the equilibrium position, the extension of each spring is approximately
- (a) $\frac{2}{\sqrt{3}}\left(\frac{q^2}{4\pi\epsilon_0 L^2 K}\right)$ (b) $\frac{\sqrt{3} q^2}{4\pi\epsilon_0 L^2 K}$
(c) $\frac{1}{\sqrt{3}}\left(\frac{q^2}{4\pi\epsilon_0 L^2 K}\right)$ (d) $\frac{q^2}{4\pi\epsilon_0 L^2 K}$

27. One end of a uniform wire of length L and weight W is attached rigidly to a point in the roof and a weight W_1 is suspended at its lower end. If A is the area of cross-section of the wire, the stress in the wire at a height $3L/4$ from its lower end is

- (a) $\frac{W_1}{A}$ (b) $\frac{\left(W_1 + \frac{W}{4}\right)}{A}$
 (c) $\frac{\left(W_1 + \frac{3W}{4}\right)}{A}$ (d) $\frac{3(W_1 + W)}{4A}$

28. Chromatic aberration of a convex lens is due to

- (a) the varying thickness of the lens
 (b) the variations in the radius of curvature of the surface of the lens
 (c) the variation in the refractive index of the material of the lens with wavelength of light
 (d) the presence of nonparaxial rays

29. A capacitor of capacitance $2\mu\text{F}$ is charged to a potential difference of 12 V. It is then connected across an inductor of inductance 0.6 mH. The current in the circuit at a time when the potential difference across the capacitor is 6.0 V is

- (a) 1.2 A (b) 2.4 A
 (c) 3.0 A (d) 0.6 A

30. If the potential difference between the plates of a parallel-plate capacitor of capacitance C farad changes from 12 V to 8 V when a dielectric slab of dielectric constant K is introduced to fill the entire space between the plates, then

- (a) the change in the charge on the capacitor is 1.50 C
 (b) the change in the charge on the capacitor is 0.66 C
 (c) the electric field in the capacitor decreases by 33.33%
 (d) the value of K is 1.5

31. Out of the given waves (a), (b), (c) and (d)

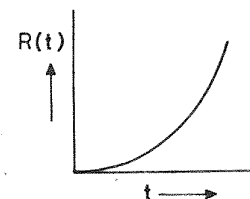
- $y = a \sin(kx - \omega t)$ (a)
 $y = a \sin(\omega t - kx)$ (b)
 $y = a \cos(kx + \omega t)$ (c)
 $y = a \cos(\omega t - kx)$ (d)

emitted by four different sources S_1 , S_2 , S_3 and S_4 respectively, perfect interference will be observed in space under appropriate conditions when

- (a) S_1 emits waves (a) and S_2 emits waves (b)
 (b) S_2 emits waves (c) and S_4 emits waves (d)
 (c) S_3 emits waves (b) and S_4 emits waves (d)
 (d) S_4 emits waves (b) and S_3 emits waves (c)

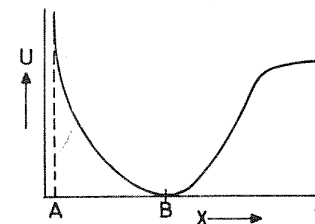
32. The resistance $R(t)$ of a conductor varies with temperature t as shown in the figure here. If the variation is represented by

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$



- (a) α and β are both negative
 (b) α is positive but β is negative
 (c) α and β , both are positive
 (d) α is negative but β is positive

33. The potential energy U between two molecules as a function of the distance X between them has been shown in the figure here. The two molecules



- (a) attract when X lies between A and B and repel when X lies between B and C
 (b) attract when X lies between B and C and repel when X lies between A and B
 (c) repel when they approach B from C
 (d) attract when they approach B from A

34. A helium nucleus makes a full rotation in a circle of radius 0.8 m in 2 s. The value of the magnetic field B at the centre of the circle will be (μ_0 is permeability constant)

- (a) $(10^{-19}/\mu_0)$ tesla (b) $(10^{-19}\mu_0)$ tesla
 (c) $(2 \times 10^{-19}\mu_0)$ tesla (d) $(2 \times 10^{-19}/\mu_0)$ tesla

35. If a current given by $I = I_0 \sin(\omega t - \pi/2)$ flows in a circuit which has an ac potential given by $E = E_0 \sin \omega t$, applied across the circuit, then the power consumption P in the circuit will be

- (a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P = \frac{E_0 I_0}{2}$
 (c) $P = \frac{EI}{2}$ (d) $P = 0$

36. An inductance and a resistance are connected in series with an ac potential. In this circuit

- (a) the current and the potential difference across the resistance lead the potential difference across the inductance by $\pi/2$.
 (b) the current and the potential difference across the resistance lag behind the potential difference across the inductance by $\pi/2$.
 (c) the current across the resistance leads and the potential difference across the resistance lags behind the potential difference across the inductance by $\pi/2$.

- (d) the current across the resistance lags behind and the potential difference across the resistance leads the potential difference across the inductance by $\pi/2$.
37. A man weighing 60 kg is standing at the centre of a flat boat of mass 300 kg. The man who is at a distance of 12 m from the shore walks 3 m towards it and stops. Assume the boat to be of uniform thickness. Neglect friction of any kind. The distance of the man from the shore is
 (a) 6.4 m (b) 8.2 m
 (c) 9.5 m (d) 7.5 m
38. The excitation energy of a hydrogen-like ion in its first excited state is 40.8 eV. The energy needed to remove the electron from the ion is
 (a) 54.4 eV (b) 72.6 eV
 (c) 62.6 eV (d) 58.6 eV
39. The wavelength of the radiation emitted when the electron in a hydrogen atom jumps from $n = \infty$ to $n = 2$ is
 (a) 185 nm (b) 365 nm
 (c) 530 nm (d) 235 nm
40. The terminal velocity of a steel ball ($\rho_s = 7.8 \times 10^3 \text{ kg/m}^3$) of radius 2 mm falling through glycerine ($\rho_g = 1.2 \times 10^3 \text{ kg/m}^3$ and $\eta = 0.83 \text{ Pa.s}$) is
 (a) 0.7 m/s (b) 0.5 m/s
 (c) 0.35 m/s (d) 0.07 m/s

TEST PAPER III

Max. Marks 100

Time: 1.5 hr

1. A steel ball of mass 10 g is moving towards the east with a speed of 10 m/s and another steel ball of mass 20 g is moving towards the north with a speed of 5 m/s. The velocity of the centre of mass is
 (a) 4.7 m/s E-45° S (b) 4.7 m/s E-45° N
 (c) 2.35 m/s E-45° N (d) 2.35 m/s E-45° S
2. A radioactive nucleus initially at rest decays by emitting an electron and an antineutrino at right angles to each other. The momentum of the electron is $1.5 \times 10^{-23} \text{ kg} \cdot \text{m/s}$ and that of the antineutrino is $6.25 \times 10^{-23} \text{ kg} \cdot \text{m/s}$. The magnitude and direction of the momentum of the recoiling nucleus is
 (a) $3.2 \times 10^{-23} \text{ kg} \cdot \text{m/s}$ 76.5° with the direction of the electron
 (b) $3.2 \times 10^{-23} \text{ kg} \cdot \text{m/s}$ 76.5° with the direction of the antineutrino
 (c) $6.4 \times 10^{-23} \text{ kg} \cdot \text{m/s}$ 35° with the direction of the electron
 (d) $6.4 \times 10^{-23} \text{ kg} \cdot \text{m/s}$ 76.5° with the direction of the electron
3. The length of an equivalent simple pendulum for a vibrating loop of mass M and radius R supported from a point on its periphery is
 (a) $3R$ (b) $1.5R$
 (c) $2R$ (d) $2.5R$

4. A ball is dropped from a height h and after falling on the floor, it rebounds and falls again. The process continues till the ball comes to rest. If e is the coefficient of restitution, then the total distance covered by the ball is

(a) $\left(\frac{1+2e^2}{1-2e^2}\right)h$ (b) $\left(\frac{1+e^2}{1-e^2}\right)h$
 (c) $(1+e^2)h$ (d) $(\sqrt{2}+e^2)h$

5. A particle of mass m_0 is moving with a velocity V . The following are the relations between momentum p , the total energy E of the particle (c is the velocity of light)

(a) $pc = \frac{EV}{c}$ (b) $E^2 = p^2c^2 + m_0^2c^4$
 (c) $E = p^2 + m_0c^2$ (d) $p = m_0V + m_0c^2$

6. Different symmetric solid bodies like a cylinder, sphere or a disc are released one by one from a height h_0 over an inclined plane. The mass of the rolling body is M , its radius R and radius of gyration K . When the body is at a height h from the floor, its velocity is

(a) $\frac{g(h_0 - h)R^2}{2R^2 + k^2R^2}$ (b) $\frac{2g(h_0 - h)}{(1 + k^2/R^2)}$
 (c) $\frac{4g(h_0 - h)R^2}{4R^2 + k^2R^2}$ (d) $\frac{(h_0 - h)g}{(1 + k^2/R^2)}$

7. A thermally isolated vessel contains 0.5 kg of water at 0°C. When air above the water is pumped out some water freezes and some evaporates at 0°C itself. If, finally, no water is left in the vessel the amount of ice formed is (latent heat of vaporization of water at 0°C = $2 \times 10^6 \text{ J/kg}$ and latent heat of fusion of ice = $3.5 \times 10^5 \text{ J/kg}$)

(a) 425 g (b) 225 g
 (c) 270 g (d) 325 g

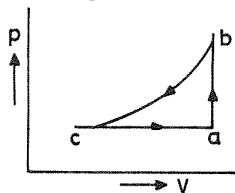
8. The temperature of 5 moles of a gas which was held at constant volume was changed from 100°C to 120°C. The change in internal energy was found to be 80 J. The total heat capacity of the gas at constant volume is

(a) 4.0 J/K (b) 2.0 J/K
 (c) 8.0 J/K (d) 0.4 J/K

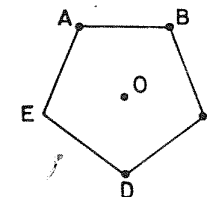
9. A simple pendulum consists of a point mass m attached to a light inextensible string of length L . The pendulum is given an initial angular displacement of 60° from the mean position and released from rest. During the period it moves from 60° to 30°.

(a) work done by gravity is $\frac{1}{4}mgL(\sqrt{3}-1)$
 (b) work done by the string tension is $\frac{1}{4}mgL(\sqrt{3}-1)$
 (c) work done by gravity is $\frac{1}{2}mgL(\sqrt{3}-1)$
 (d) total work done by gravity and the string tension is $\frac{1}{2}mgL(\sqrt{3}-1)$

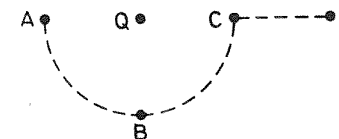
10. Two moles of an ideal gas with molecular mass m have volume V at pressure P . The root mean square speed of the molecules of the gas is (N_0 is Avogadro number)
- (a) $\left(\frac{2PV}{3mN_0}\right)^{1/2}$ (b) $\left(\frac{3PV}{mN_0}\right)^{1/2}$
 (c) $\left(\frac{3PV}{2mN_0}\right)^{1/2}$ (d) $\left(\frac{6PV}{mN_0}\right)^{1/2}$
11. Two sound waves of equal intensity produce beats. The maximum intensity of sound produced in beats will be
- (a) $4I$ (b) $2I$
 (c) $I/2$ (d) $I/\sqrt{2}$
12. A capacitor C is connected to a battery of emf V . The double switch S is then connected to the point 2 connecting the charged capacitor in parallel to another uncharged capacitor $3C$. In the final situation, if V_f is the potential difference across each capacitor and E is the energy finally stored in capacitor C , then
- (a) $V_f = \frac{1}{2}V$ (b) $V_f = \frac{V}{4}$
 (c) $E = \frac{1}{32}CV^2$ (d) $E = \frac{1}{8}CV^2$
13. A particle of charge $+Q$ is released from rest and moves under the influence of a uniform horizontal electric field and gravity. The quantities/quantity which continuously increase (s) during its motion are/is
- (a) electric charge
 (b) gravitational potential energy
 (c) electric potential energy
 (d) kinetic energy
14. A system can be taken from the initial state p_i, V_i to the final state p_f, V_f by two different methods. Let ΔQ and ΔW represent the heat given to the system and the work done by the system. The quantity/quantities that remains/remains same in both the methods is/are
- (a) ΔQ (b) ΔW
 (c) $\Delta Q + \Delta W$ (d) $\Delta Q - \Delta W$
15. A sample of an ideal gas is taken through the cyclic process $abca$ shown in the figure. It absorbs 50 J of heat during the part ab , does not absorb or reject heat during bc and rejects 70 J of heat during ca . 40 J of work is done on the gas during the part bc . If the internal energy at a is 1500 J, the internal energies at b and c respectively will be



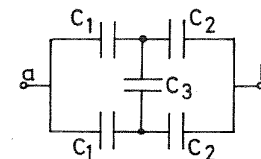
- (a) 1520 J and 1550 J (b) 1550 J and 1590 J
 (c) 1550 J and 1500 J (d) 1590 J and 1620 J
16. A black body of surface area 0.005 m^2 is placed inside an enclosure. The enclosure has a constant temperature 27°C and the black body is maintained at 327°C by heating it electrically. The electric power needed to maintain the temperature of the black body is [Stefan Boltzmann constant $\sigma = 6.0 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4$]
- (a) 73.0 W (b) 36.5 W
 (c) 25.0 W (d) 50.0 W
17. Four particles, each having a charge Q , are placed on any four vertices of a regular pentagon. The distance of each corner from the centre is a . The electric field at the centre of the pentagon is



- (a) $\frac{Q}{2\sqrt{2}\epsilon_0 a^2}$ along $\overrightarrow{EO}$
 (b) $\frac{Q}{2\sqrt{2}\epsilon_0 a^2}$ along $\overrightarrow{OE}$
 (c) $\frac{Q}{4\pi\epsilon_0 a^2}$ along $\overrightarrow{OE}$ (d) $\frac{Q}{4\pi\epsilon_0 a^2}$ along $\overrightarrow{EO}$
18. The electric field in a region is given by $\vec{E} = \left(\frac{A}{x^3}\right)\vec{i}$. The SI units for A are
- (a) Vm^2 (b) $\frac{\text{V}}{\text{m}^2}$
 (c) $\frac{\text{m}^2}{\text{V}}$ (d) V^2m
19. Consider three points A, B and C situated around a point charge Q on a semicircle as shown in the figure. A point charge is brought from P to C, P to B and P to A and the corresponding work done in each case is W_C , W_B and W_A .
- (a) $W_A < W_B < W_C$ (b) $W_A = W_B = W_C$
 (c) $W_A > W_B > W_C$ (d) none of these



20. A network of five capacitors is shown in the figure here. The equivalent capacitance between a and b is

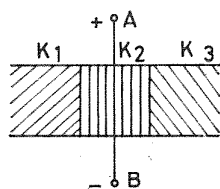


- (a) $\frac{2C_1 C_2}{C_1 + C_3}$
 (b) $\frac{C_1 C_2 + C_1 C_3}{C_1 + C_2 + C_3}$

(c) $\frac{2C_1 C_2}{C_1 + C_2}$

(d) $\frac{C_1 + C_2}{2C_1 C_2}$

21. Three dielectric slabs of identical size fill the space between the plates of a parallel-plate capacitor of capacitance C . If the dielectric constant of three slabs is K_1 , K_2 and K_3 , the new capacitance will be



- (a) $(K_1 + K_2 + K_3)C$ (b) $\left(\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_1 K_3}\right)C$
 (c) $(K_1 + K_2 + K_3)\frac{C}{3}$ (d) $\left(\frac{K_1 K_2 K_3}{K_1 K_2 + K_2 K_3 + K_1 K_3}\right)\frac{C}{3}$

22. Column I gives three physical quantities. Select the appropriate matching entries for each of them from among the ones given in columns II and III. Write your answers in the boxes provided.

I (Quantity)	II (Dimensions)	III (Units)
1. Angular momentum	(a) MLT^{-1}	(e) Ns
2. Torque	(b) MLT^{-2}	(f) NmS^{-1}
3. Impulse	(c) ML^2T^{-1}	(g) Nm
	(d) ML^2T^{-2}	(h) Js
(1)	(2)	(3)

23. 1. Radiowaves (a) 10^{-10} m (e) 10^{-8} eV
 2. X-rays (b) 10^{-7} m (f) 10^{-2} eV
 (c) 10^{-4} m (g) 10 eV
 (d) 10^2 m (h) 10^4 eV

(1) (2)

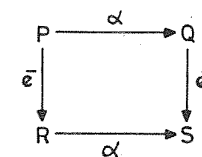
24. A particle is moving on a circle with a speed increasing at constant rate. The force (s) acting on the particle is/are
 (a) a constant tangential force only
 (b) a constant centripetal force and a constant tangential force
 (c) a constant tangential force and a centripetal force increasing with time
 (d) a tangential force increasing with time at a constant rate
25. In two different experiments a mass of ideal gas at a given initial pressure and temperature is expanded to twice its initial volume isothermally and adiabatically. If T_i and T_a are the final temperatures in the two processes and P_i and P_a are the two corresponding final pressures, then
 (a) $T_i > T_a$
 (b) $T_i < T_a$
 (c) $P_a < P_i$
 (d) work done by the gas is the same in the two cases

26. A planar conducting loop of area A and resistance R moves in a positive X -direction with a constant velocity V ($\vec{V}$ is perpendicular to the plane of the loop) in the presence of a magnetic field of induction $\vec{B}(X) = KX\hat{i}$ where K is a positive constant and $\hat{i}$ is a unit vector along the positive X -axis. The induced current in the loop is

- (a) zero (b) $\frac{KAXV}{R}$
 (c) $\frac{KAV}{R}$ (d) $\frac{KAX}{R}$

27. A lens made from a material of refractive index 1.25 behaves as a converging lens when placed in air. When placed in water, it will
 (a) still behave as a converging lens
 (b) have its focal length increased
 (c) have its focal length decreased
 (d) behave as a divergent lens
28. For a p - n junction diode and a vacuum diode, which of the following statement(s) is/are true?
 (a) at sufficiently large forward bias, current saturates in both the diodes
 (b) in both, the electrons are produced by thermionic process
 (c) in a reverse biased p - n junction diode there is a small current flowing but there is no current in the corresponding situation (plate negative) for the vacuum tube diode
 (d) In a forward biased p - n junction a large number of electrons flow from the n -region to the p -region.

29. Four nuclei P, Q, R and S, participate in processes involving α and β decay as shown here. The mass number and atomic number of P are 210 and 83 respectively. Which of the following statement(s) is/are true?



- (a) mass number of R is 210
 (b) mass number of Q is 208
 (c) mass number of S is 206 and its atomic number is 82
 (d) the energy released in the resultant process $P \rightarrow S$ via Q and via R is the same
30. A metal sphere of radius R is given a charge Q . The energy stored in the electric field produced is

- (a) $\frac{Q^2}{2\pi\epsilon_0 R}$ (b) $\frac{Q^2}{4\pi\epsilon_0 R}$
 (c) $\frac{Q^2}{\pi\epsilon_0 R}$ (d) $\frac{Q^2}{8\pi\epsilon_0 R}$

31. A current of 1.0 A exists in a wire of cross-sectional area 1.0 mm². If each cubic metre of the wire contains 6.0×10^{28} free electrons, the drift velocity of an electron is

(a) 1×10^{-4} m/s

(c) 1×10^4 m/s

(b) 0.5×10^{-4} m/s

(d) 0.5×10^4 m/s

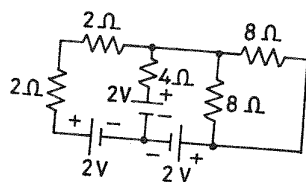
32. In the given network, the current in the $2\ \Omega$ resistors is

(a) 1.0 A

(b) 2.0 A

(c) 2.5 A

(d) zero



33. The electric field between the plates of a parallel-plate capacitor of capacitance $1.0\ \mu\text{F}$ drops to one fourth of its initial value in $4.0\ \mu\text{s}$ when the plates are connected by a thin wire. The resistance of the wire is $[\ln 4 = 1.4]$

(a) $1.6\ \Omega$

(b) $2.8\ \Omega$

(c) $0.8\ \Omega$

(d) $3.2\ \Omega$

34. A proton, deuteron and an α -particle moving with equal kinetic energies, enter perpendicularly into a magnetic field of induction B . If r_p , r_d and r_α are the respective radii of their circular paths, the following are their ratios:

(a) $\frac{r_p}{r_\alpha} = 1$

(b) $\frac{r_p}{r_\alpha} = \frac{\sqrt{2}}{1}$

(c) $\frac{r_d}{r_p} = \frac{1}{\sqrt{2}}$

(d) $\frac{r_d}{r_p} = \frac{\sqrt{2}}{1}$

35. A beam of protons with a velocity of 4×10^5 m/s enters a uniform magnetic field of 0.4 T. The beam makes an angle of 30° with the magnetic field. The time taken to complete one complete revolution in the plane perpendicular to $\vec{B}$ is

(a) $\pi \times 10^{-8}$ s

(b) $5\pi \times 10^{-8}$ s

(c) $2\pi \times 10^{-6}$ s

(d) $2\pi \times 10^{-4}$ s

36. A long solenoid is formed by winding 20 turns/cm. The current necessary to produce a magnetic field of 20 mT inside the solenoid is

(a) 10.0 A

(b) 16.0 A

(c) 4.0 A

(d) 8.0 A

37. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field

(a) increases linearly from the axis to the surface

(b) is constant inside the tube

(c) is zero just outside the tube

(d) is zero at the axis

38. A conducting circular loop is placed in a uniform magnetic field $B = 0.040$ T with its plane perpendicular to the field. If the radius of the loop starts shrinking at a constant rate of 2.0 mm/s, the induced emf in the loop at an instant when the radius is 1.0 cm is

(a) 5 μV

(b) 10 μV

(c) 2 μV

(d) 1 μV

39. A coil of inductance 5.0 H and resistance $500\ \Omega$ is connected to a battery

of emf 50 V. The energy stored in the magnetic field associated with the coil at an instant 10 ms after the circuit is switched on, is

(a) 40 mJ

(b) 5 mJ

(c) 10 mJ

(d) 0.4 mJ

40. The energy of an atom or ion in its ground state is -54.4 eV. It may be

(a) He^+

(b) Li^{++}

(c) hydrogen

(d) deuterium

PHYSICAL CONSTANTS

Speed of light in free space	c	3×10^8 m/s
Acceleration due to gravity (normal)	g	9.807 m/s ²
Gravitational constant	G	6.67×10^{-11} Nm ² /kg ²
Universal gas constant	R	8.31 J/mol. K
Permeability constant	μ_0	$4\pi \times 10^{-7}$ H/m
Permittivity constant	ϵ_0	8.85×10^{-12} F/m
Avogadro's constant	N_0	6.02×10^{23} molecules/mol.
Boltzmann's constant	K	1.38×10^{-23} J/K
Standard atmosphere		1.0132×10^5 N/m ²
Density of water (20°)		1.00×10^3 kg/m ³
Density of air (STP)		1.29 kg/m ³
Planck's constant	h	6.626×10^{-34} J.s
Elementary charge	e	1.60×10^{-19} C
Electron rest mass	m_e	9.109×10^{-31} kg
Electron charge to mass ratio	e/m_e	1.76×10^{11} C/kg
Proton rest mass	m_p	1.67×10^{-27} kg
Neutron rest mass	m_n	1.6749×10^{-27} kg
Alpha particle rest mass		6.64×10^{-27} kg
Atomic mass unit	u	1.66×10^{-27} kg
Rest energy of 1u		931.5 MeV

SELECTED NUMERICAL CONSTANTS

$$\pi = 3.14, \pi^2 = 9.87 \quad e = 2.72 \quad e^{-1} = \frac{1}{e} = 0.368 \quad \ln 2 = 0.693$$

$$\log e = 0.434$$

$$\sqrt{2} = 1.41 \quad \sqrt{3} = 1.73$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2} = 0.500$$

$$\cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\tan 30^\circ = \cot 60^\circ = \frac{1}{\sqrt{3}} = 0.577$$

$$\tan 60^\circ = \cot 30^\circ = \sqrt{3} = 1.732$$

$$\sin 45^\circ = \cos 45^\circ = \sqrt{2}/2 = 0.702$$

$$\tan 45^\circ = \cot 45^\circ = 1.00$$

SELECTED CONVERSION FACTORS

$$180^\circ = \pi \text{ rad}$$

$$1 \text{ rad} = 57.3^\circ = 0.159 \text{ rev.}$$

$$1 \text{ atomic mass unit} = 1.66 \times 10^{-27} \text{ kg}$$

$$1 \text{ \AA} = 10^{-10} \text{ m} = 0.1 \text{ m}\mu$$

$$1 \text{ millimicron} = 10^{-9} \text{ m}$$

$$1 \text{ cal} = 4.19 \text{ J}$$

$$1 \text{ J} = 0.239 \text{ cal} = 2.78 \times 10^{-7} \text{ kWh}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$1 \text{ hp} = 746 \text{ W}$$

$$1 \text{ Wb/m}^2 = 1 \text{ T} = 10^4 \text{ G (gauss)}$$

$$1 \text{ bar} = 10^5 \text{ Pa}, 1 \text{ N/m}^2 = 1 \text{ Pa}, 1 \text{ torr} = 133.3 \text{ Pa}$$

$$1 \text{ cm Hg} = 1333 \text{ Pa}$$

$$1 \text{ dyne/cm}^2 = 10^{-1} \text{ Pa}$$

Answers

Test Paper I

- | | | | | |
|------------------|-----------------------------|---------------|---------------|---------|
| 1. (d) | 2. (a) | 3. (a) | 4. (c) | 5. (b) |
| 6. (a) | 7. (d) | 8. (b) | 9. (a and b) | 10. (b) |
| 11. (d) | 12. (b) | 13. (c) | 14. (b) | 15. (c) |
| 16. (a) | 17. (d) | 18. (a) | 19. (b) | 20. (c) |
| 21. (c) | 22. 1—D, 2—A, 3—E, 4—F, 5—G | | | 23. (b) |
| 24. (a, b and d) | 25. (b, c and d) | 26. (a) | | 27. (a) |
| 28. (c) | 29. (a) | 30. (c) | 31. (b and d) | 32. (b) |
| 33. (d) | 34. (c) | 35. (b and c) | 36. (a) | 37. (a) |
| 38. (b) | 39. (d) | 40. (a) | | |

Test Paper II

- | | | | | |
|---------|---------|---------------|------------------|---------------|
| 1. (d) | 2. (a) | 3. (c) | 4. (a) | 5. (c) |
| 6. (b) | 7. (a) | 8. (*) | 9. (**) | 10. (a) |
| 11. (b) | 12. (a) | 13. (a) | 14. (b and d) | 15. (c) |
| 16. (c) | 17. (d) | 18. (a and b) | 19. (b, c and d) | 20. (b and d) |
| 21. (d) | 22. (d) | 23. (c) | 24. (c) | 25. (b) |
| 26. (d) | 27. (c) | 28. (c) | 29. (d) | 30. (c and d) |

- | | | | | |
|---------|---------|---------|---------|---------|
| 31. (c) | 32. (c) | 33. (b) | 34. (b) | 35. (d) |
| 36. (b) | 37. (c) | 38. (a) | 39. (b) | 40. (d) |

*(a) 5.6 MeV, (b) 4.8×10^{-4} , (c) $\frac{Q\bar{\omega}}{2} R^2$, (d) 832, (e) 4 (f) zero (g) - 4 V

**1. G, 2. H, 3. J, 4. C, 5. C

Test Paper III

- | | | | | |
|---------|---------------|---------------|------------------|---------------|
| 1. (b) | 2. (d) | 3. (c) | 4. (b) | 5. (a and b) |
| 6. (b) | 7. (a) | 8. (a) | 9. (c and d) | 10. (c) |
| 11. (a) | 12. (b and c) | 13. (d) | 14. (d) | 15. (b) |
| 16. (b) | 17. (c) | 18. (a) | 19. (b) | 20. (c) |
| 21. (c) | 22. (*) | 23. (**) | 24. (c) | 25. (a and c) |
| 26. (c) | 27. (b and d) | 28. (c and d) | 29. (a, c and d) | 30. (d) |
| 31. (a) | 32. (d) | 33. (b) | 34. (a and d) | 35. (b) |
| 36. (d) | 37. (b and d) | 38. (a) | 39. (c) | 40. (a) |

*1. c and h 2. d and g 3. a and e

**1. d and e 2. a and h